Magnetism and Electromagnetic Induction

CHAPTER 1

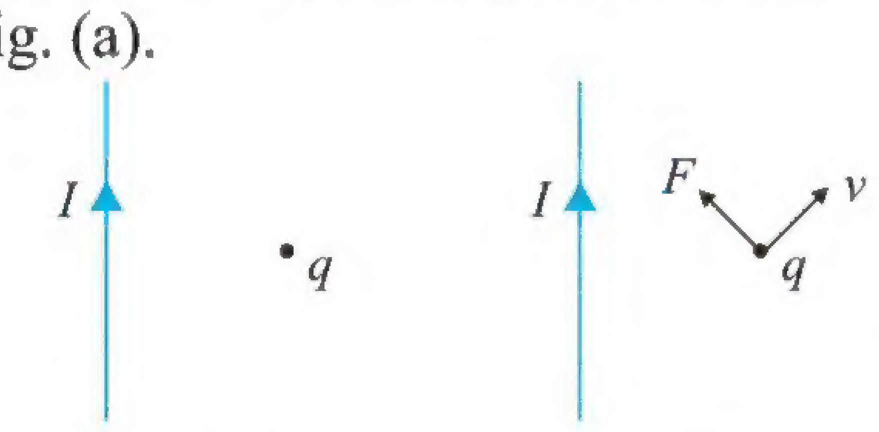
Magnetic Field and Magnetic Forces 1.1–1.56

1 Magnetic Field and Magnetic Forces

INTRODUCTION

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any electric charge. In addition to containing an electric field, the region of space surrounding any moving electric charge also contains a magnetic field. A magnetic field also surrounds a magnetic substance making up a permanent magnet. Historically, the symbol $\vec{B}$ has been used to represent a magnetic field, and we use this notation in this book. The direction of the magnetic field $\vec{B}$ at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with magnetic field lines.

A moving charge is a source of magnetic field. Consider this experiment: A point charge is kept near a current carrying wire as shown in Fig. (a).



(a) (b)

It is found that no force acts on charge if the charge is at rest. It means a current carrying wire does not produce electric field.

Now if the charge is given some velocity v in some direction as shown in Fig. (b), then a force is found to act on the charge as shown.

Now the question arises where from this force comes. Basically this force is due to the magnetic field produced by current carrying wire. From the above experiment we can conclude the following:

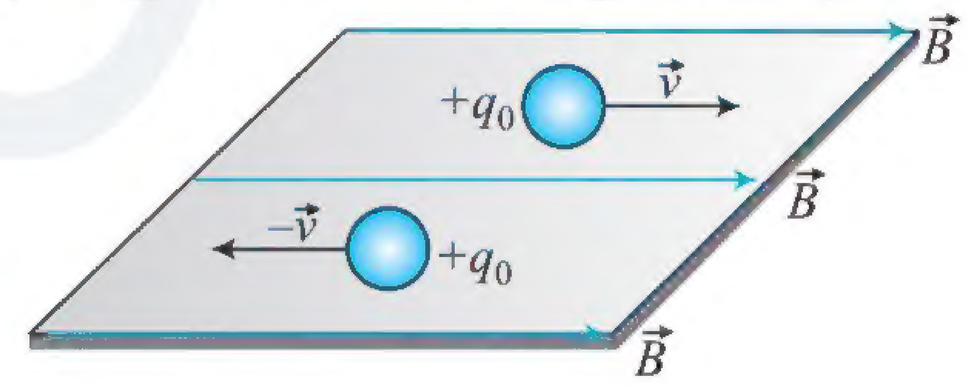
1. A current or a moving charge creates a magnetic field in the surrounding space (in addition to its electric field).
2. The magnetic field exerts a force on any other moving charge or current that is present in the field.

Like electric field, magnetic field is a vector field—that is, a vector quantity associated with each point in space. We will use the symbol $\vec{B}$ for magnetic field. At any position, the direction of $\vec{B}$ is defined as that in which the north pole of a compass needle tends to point.

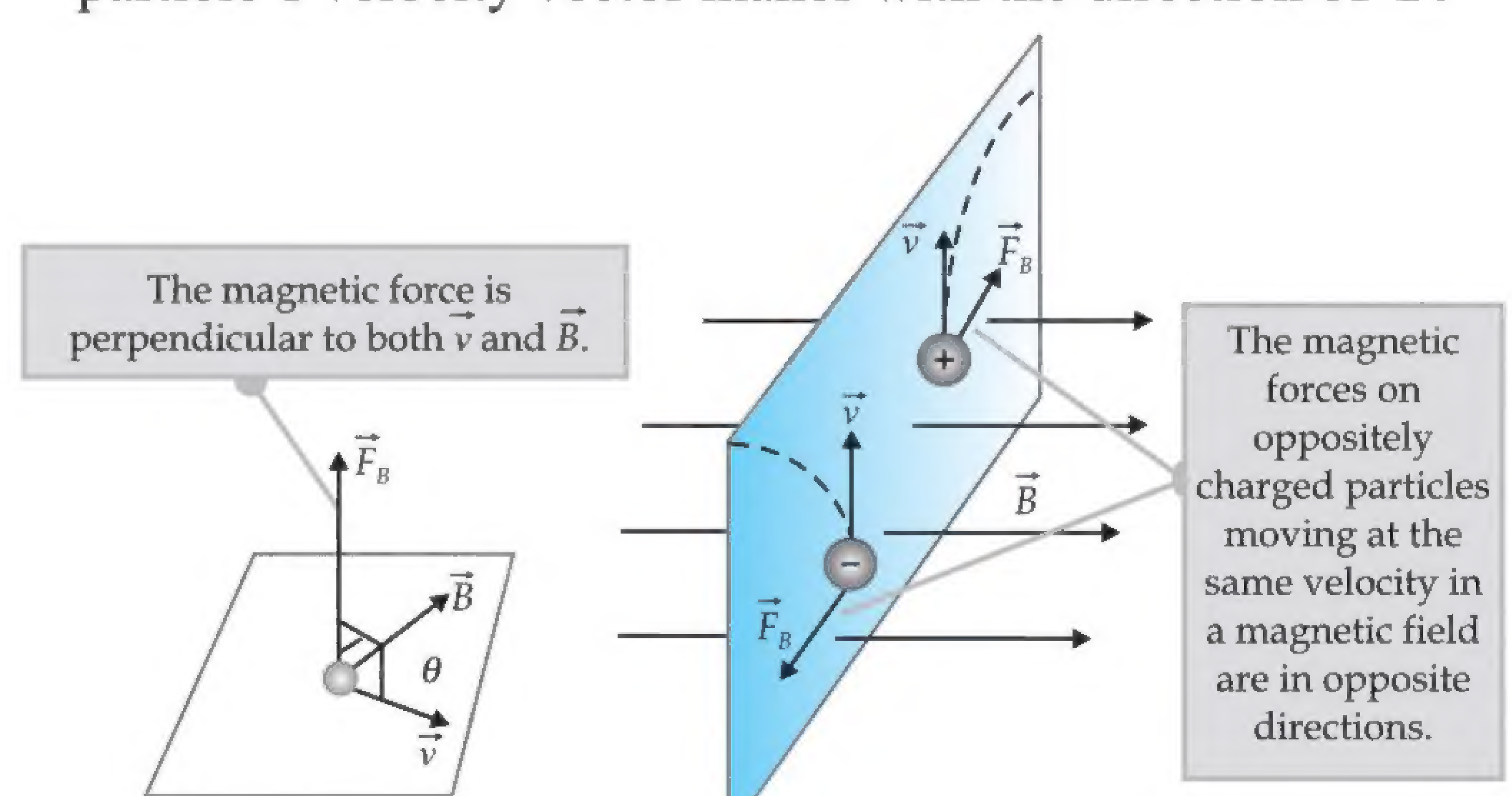
FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

We can define a magnetic field $\vec{B}$ at some point in space in terms of the magnetic force $\vec{F}_B$ the field exerts on a charged particle moving with a velocity $\vec{v}$, which we call the test object. For the time being, let's assume no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

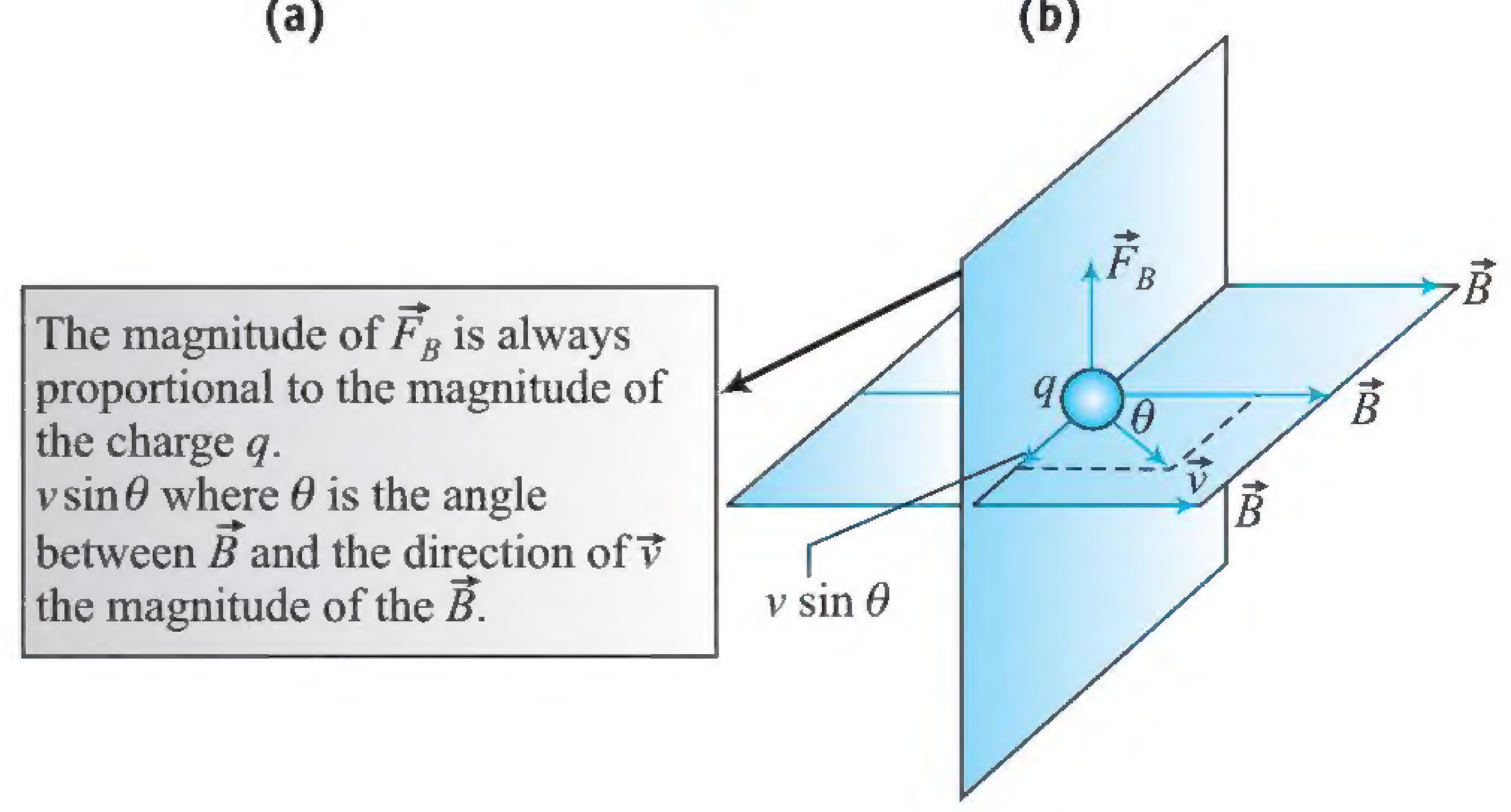
- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.



- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; i.e. $\vec{F}_B$ perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ [Fig. (a)].The magnetic force exerted on a positive charge is in the direction opposite to the direction of the magnetic force exerted on a negative charge moving in the same direction [Fig. (b)].
- The magnitude of the magnetic force exerted on the moving particle is proportional to sin θ, where θ is the angle the particle's velocity vector makes with the direction of $\vec{B}$.



(a) (b)



We can summarize all these results with the following equation:

$$|\vec{F}_B| = q(v\sin\theta)B$$

In vector form: $\vec{F}_B = q\vec{v} \times \vec{B}$

which by definition of the cross product is perpendicular to both $\vec{v}$ and $\vec{B}$. We can regard this equation as an operational definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle.

Units of Magnetic Field Strength

To discuss the motion of charges in magnetic fields, we need to know what units are used to measure the magnetic field strength. Solving above equation for the field strength and inserting the units of the other quantities gives

$$[F_B] = [q][v][B] \Rightarrow [B] = \frac{[F_B]}{[q][v]} = \frac{\text{N s}}{\text{C m}}$$

Because the ampere (A) is defined as 1 C/s, (N s)/(C m) = N/(A m). The unit of magnetic field strength has been named the **tesla** (T), in honor of Croatian-born American physicist and inventor Nikola Tesla:

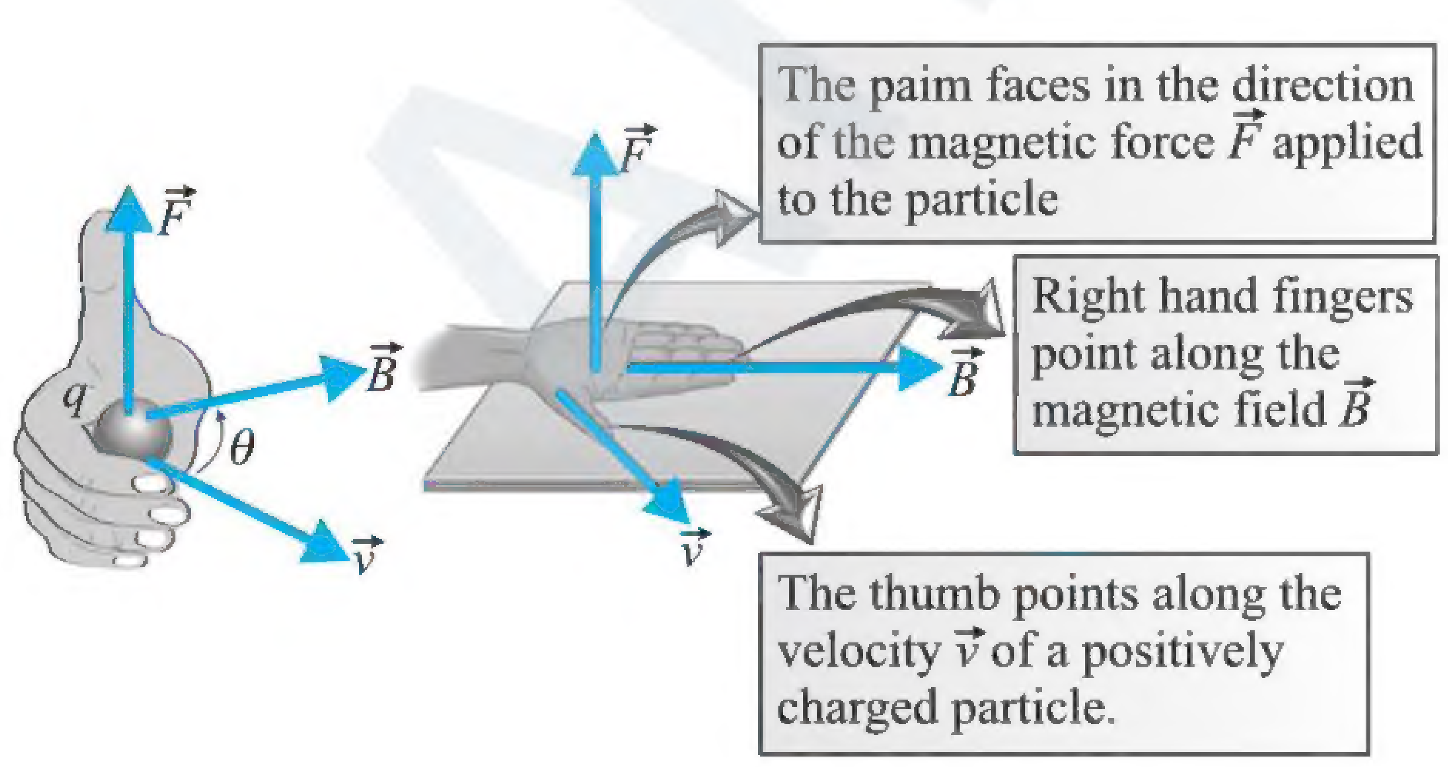
$$1\,\text{T} = 1\frac{\text{N s}}{\text{C m}} = 1\frac{\text{N}}{\text{A m}}$$

A tesla is a rather large amount of magnetic field strength. Sometimes magnetic field strength is given in gauss (G), which is not an SI unit:

$$1\,\text{G} = 10^{-4}\,\text{T}$$

Right Hand Rules for Determining the Direction of the Magnetic Force Acting on a Moving Charged Particle

The figure below reviews two right-hand rules for determining the direction of the cross product $\vec{v} \times \vec{B}$ and determining the direction of $\vec{F}_B$. The rule in Fig. (a) depends on our right-hand rule for the cross product in figure. Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. Your extended thumb, which is at a right angle to your fingers, points in the direction of $\vec{v} \times \vec{B}$. Because $\vec{F}_B = q\vec{v} \times \vec{B}$, $\vec{F}_B$ is in the direction of your thumb if q is positive and is opposite the direction of your thumb if q is negative.



(a) (b)

An alternative rule is shown in Fig. (b). Here the thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$. Now, the force $\vec{F}_B$ on a positive charge extends outward from the palm. The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand: outward from your palm. The force on a negative charge is in the opposite direction. You can use either of these two right-hand rules.

The magnitude of the magnetic force on a charged particle is $F_B = |q|\,vB\sin\theta$, where u is the smaller angle between $\vec{v}$ and $\vec{B}$. From this expression, we see that F_B is zero when $\vec{v}$ is parallel or antiparallel to $\vec{B}$ ($\theta = 0$ or 180°) and maximum when $\vec{v}$ is perpendicular to $\vec{B}$ ($\theta = 90°$).

There are important differences between electric and magnetic forces on charged particles:

- The electric force is always parallel or antiparallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a constant magnetic field does no work when a charged particle is displaced.

This last statement is true because when a charge moves in a constant magnetic field, the magnetic force is always perpendicular to the displacement. Hence, the work done by the magnetic force on the particle is zero.

Proof: We observed that the force acting on a particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\vec{F} = q\vec{v} \times \vec{B}$$

The force is always perpendicular to the particle's velocity and so does no work on the particle:

$$dW = \vec{F}\cdot d\vec{s} = \vec{F}\cdot\vec{v}\,dt = 0, \text{ since } \vec{F} \text{ is perpendicular to } \vec{v}.$$

Magnetic force can alter the direction of a particle's motion but cannot alter its energy. Since the force is also perpendicular to $\vec{B}$, it cannot change a particle's velocity component parallel to the field.

From the work–energy theorem, we conclude that the kinetic energy of a charged particle cannot be altered by a constant magnetic field alone. In other words, when a charge moves with a velocity, an applied magnetic field can alter the direction of the velocity vector, but it cannot change the speed of the particle.

Illustration 1.1

A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by $\vec{B} = -B_0\vec{j}$ and $\vec{E} = E_0\vec{k}$. Find the speed of the particle as a function of its z-coordinate.

Sol. Since the magnetic field does not perform any work, therefore, whatever has been the gain in kinetic energy it is only bec-ause of the work done by electric field. Applying work–energy theorem, $W_E = \Delta K$

$$qEz = \frac{1}{2}mv^2 - O \quad \text{or,} \quad v = \sqrt{\frac{2qEz}{m}}$$

LORENTZ FORCE

The combination of electric and magnetic forces is known as Lorentz forces. Let a charge particle moves in a magnetic field where both electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are present. Then net force acting on the charge particle is given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

ILLUSTRATION 1.2

A charge $q = -4\ \mu C$ has an instantaneous velocity $\vec{v} = (2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6\ ms^{-1}$ in a uniform magnetic field $\vec{B} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}$ T. What is the force on the charge?

Sol. $\vec{F} = q\vec{v} \times \vec{B}$

$$= (-4 \times 10^{-6})[(2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6]$$

$$\times (2\hat{i} + 5\hat{j} - 3k) \times 10^{-2}]$$

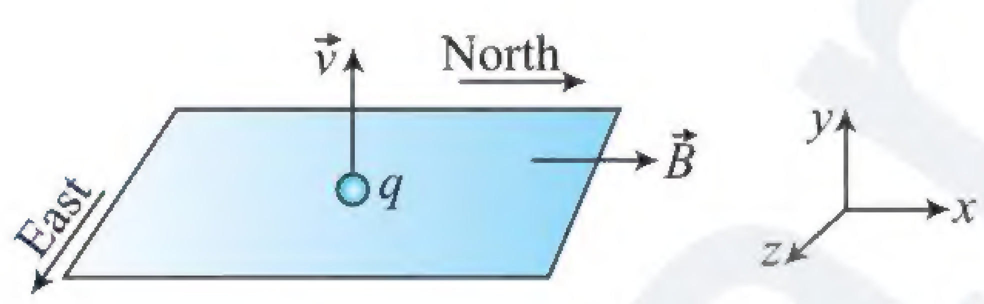
$$= -(-16\hat{i} + 32\hat{j} + 64\hat{k}) \times 10^{-2}\ \text{N}$$

$$= -16(\hat{i} + 2\hat{j} + 4\hat{k}) \times 10^{-2}\ \text{N}$$

ILLUSTRATION 1.3

A charged particle is having charge $q = 50\ \mu C$ is projected vertically upward with a speed of 5.0×10^4 km/s in a region where a magnetic field of magnitude 2.0 T exits in the direction south to north. Find the magnetic force that acts on the particle.

Sol. Given $\vec{B} = 2.0(\hat{i})$ T , $\vec{v} = 5.0 \times 10^7\ (\hat{j})$ m/s



The force acting on a charged particle moving in magnetic field is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$= 50 \times 10^{-6}[\{5.0 \times 10^7(\hat{j})\} \times \{2.0(\hat{i})\}]$$

$$= 5.0 \times 10^3(-\hat{k})\ \text{N}$$

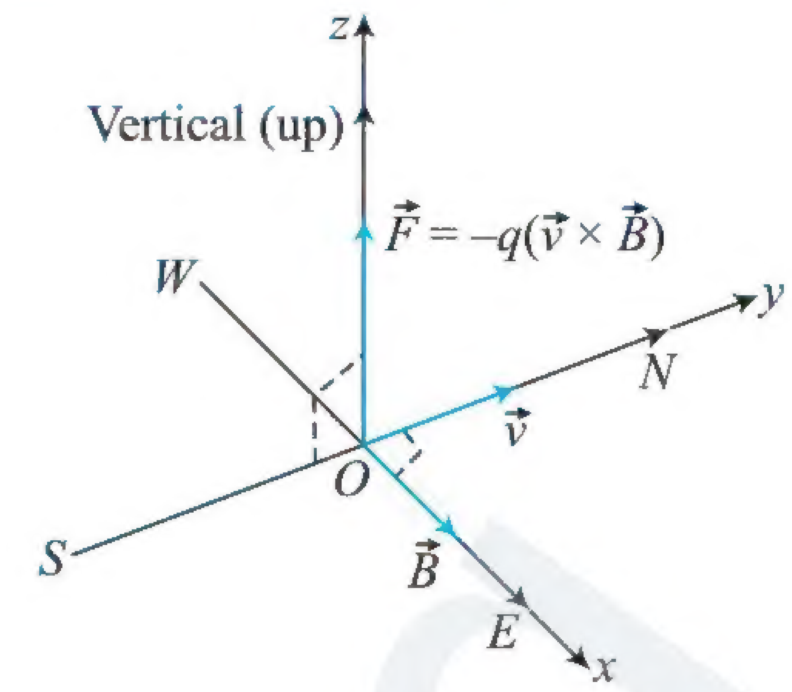
Hence the charged particle will experience a force 5 kN towards west direction.

ILLUSTRATION 1.4

An electron is moving northwards with a velocity $10^7\ ms^{-1}$ in a magnetic field of 3 T directed eastwards. Calculate the instantaneous force on the electron. Given that charge on electron $= -1.6 \times 10^{-19}$ C.

Sol. Here, $q = 1.6 \times 10^{-19}$ C , $v = 10^7\ ms^{-1}$; $B = 3$ T

As electron is moving northward and the magnetic field is eastwards, the angle between $\vec{v}$ and $\vec{B}$ is 90°.



Now, $F = qvB \sin 90°$

$$= 1.6 \times 10^{-19} \times 10^7 \times 3 \times 1 = 4.8 \times 10^{-12}\ \text{N}$$

Since the charge on electron is negative, force on electron,

$$\vec{F} = -q(\vec{v} \times \vec{B})$$

The direction of force on electron will be that of vector $-q(\vec{v} \times \vec{B})$ i.e., along vertically upwards.

Alternate: Velocity of the electron $\vec{v} = 10^7(\hat{j})$ m/s

Magnetic field $\vec{B} = 3(\hat{i})T$

Force on electron $\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{F} = (-1.6 \times 10^{-19})[10^7\ \hat{j} \times 3\hat{i}]$$

$$= -4.8 \times 10^{-12}[\hat{j} \times \hat{i}]$$

$$= -4.8 \times 10^{-12}[-\hat{k}] = 4.8 \times 10^{-12}(\hat{k})\ \text{N}$$

ILLUSTRATION 1.5

A magnetic field of $(5.0 \times 10^{-3}\hat{k})$ T exerts a force of $(2.0\hat{i} + 1.5\hat{j}) \times 10^{-10}$ N on a particle having a charge of 1.0×10^{-9} C and going in the x-y plane. Find the velocity of the particle.

Sol. Let velocity of the charged particle be $(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$

Force on the charged particle $F = q(\vec{v} \times \vec{B})$

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$$(2.0\hat{i} + 1.5\hat{j}) \times 10^{-10} = 1.0 \times 10^{-9}[(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times 5.0 \times 10^{-3}(\hat{k})]$$

$$(2.0\hat{i} + 1.5\hat{j}) \times 10^{-10} = 5.0 \times 10^{-12}[v_x(\hat{i} \times \hat{k}) + v_y(\hat{j} \times \hat{k})]$$

$$(200\hat{i} + 150\hat{j}) = 5(-v_x\hat{j} + v_y\hat{i}) \qquad \text{...(i)}$$

From Eq. (i), we get $v_y = \frac{200}{5}\text{m/s} = 40\,\text{m/s}$

and $v_x = -\frac{150}{5}\text{m/s} = -30$ m/s

Hence velocity of the charged particle $\vec{v} = (-30\hat{i} + 40\hat{j})$ m/s

ILLUSTRATION 1.6

A charged particle is projected in a magnetic field of $(5.0\hat{i} - 2.0\hat{j}) \times 10^{-3}$ T. If the acceleration of the particle is found to be $(x\hat{i} + 5.0\,\hat{j}) \times 10^{-6}\ \text{m/s}^2$. What should be the value of x?

Sol. Force acting on a charged particle in magnetic field is given by $\vec{F} = q\,\vec{v} \times \vec{B}$

It means the force vector (or acceleration vector) and magnetic field vector should be perpendicular to each other.

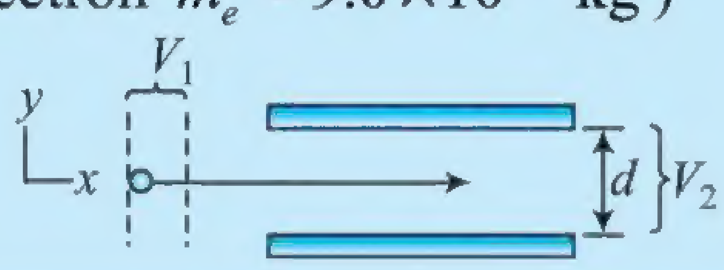
Hence, $\vec{a}.\vec{B} = 0 \Rightarrow ((x\hat{i} + 5.0\hat{j}) \times 10^{-6}).((5.0\hat{i} - 2.0\hat{j}) \times 10^{-3}) = 0$

$\Rightarrow \quad [5.0x + 5.0 \times (-2.0)] \times 10^{-9} = 0$

$\Rightarrow \quad 5.0x + 5.0 \times (-2.0) = 0$ or $x = 2$

ILLUSTRATION 1.7

In figure given, an electron gets accelerated from rest through potential difference $V_1 = 500\,\text{V}$. It enters the gap between two parallel plates having separation $d = 20.0$ mm and potential difference $V_2 = 100\,\text{V}$. The upper plate is at the higher potential. Neglecting fringing and assuming that the electron's velocity vector is perpendicular to the electric field vector between the plates, find the magnitude and direction of uniform magnetic field which allows the electron to travel in a straight line in the gap?(Mass of electron $m_e = 9.0 \times 10^{-31}$ kg)



Sol. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Due to electric field, the electron experiences force in upward direction, hence due to magnetic field it will experience force in down wards direction. Thus,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0. \text{ Note that } \vec{v} \perp \vec{B} \text{ so } |\vec{v} \times \vec{B}| = vB.$$

Hence, $\vec{E} = -\vec{v} \times \vec{B} \Rightarrow |E| = |vB|$

The electric field between the plates,

$$E = \frac{V_2}{d} = \frac{100}{20 \times 10^{-3}} = 5.0 \times 10^4 \text{ V/m}$$

The electron gets accelerated through potential difference

$V_1 = 500\,\text{V}$

Hence its kinetic energy, $K = eV_1 = \frac{1}{2}m_e v^2$

The speed of the electron,

$$v = \sqrt{\frac{2eV_1}{m_e}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 500}{9.0 \times 10^{-31}}} = \frac{4}{3} \times 10^7 \text{ m/s}$$

Hence magnetic field, $B = \frac{E}{v} = \frac{5.0 \times 10^4}{4/3 \times 10^7} = 3.75 \times 10^{-3}$ T

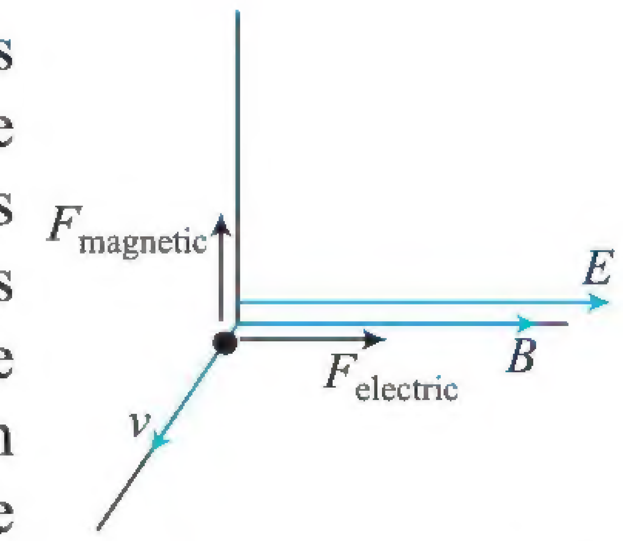
Since the velocity of the electron is in the $+x$ direction, and the force due to magnetic field acts in downwards direction then using the right-hand rule we conclude that the magnetic field must point in the $-z$ direction. In unit-vector notation, we have

$$\vec{B} = -(3.75 \times 10^{-3} \text{ T})\hat{k}$$

ILLUSTRATION 1.8

A magnetic field has a magnitude of 1.5×10^{-3} T, and an electric field has a magnitude of 4.0×10^3 N/C. Both fields point in the same direction. A positive 1.8 μC charge moves at a speed of 2.0×10^6 m/s in a direction that is perpendicular to both fields. Determine the magnitude of the net force that acts on the charge.

Sol. The magnetic field applies the maximum magnetic force to the moving charge, because the motion is perpendicular to the field. This force is perpendicular to both the field and the velocity. The electric field applies an electric force to the charge that is in the same direction as the field, since the charge is positive. These two forces are shown in the drawing, and they are perpendicular to one another. Therefore, the magnitude of the net force acting on the charge, $F = \sqrt{F_{\text{magnetic}}^2 + F_{\text{electric}}^2}$



The magnetic force $\vec{F}_{\text{mag}} = q\,\vec{v} \times \vec{B}$ has a magnitude of $F_{\text{magnetic}} = |q|vB \sin\theta$, where $|q|$ is the magnitude of the charge, B is the magnitude of the magnetic field, v is the speed, and $\theta = 90°$ is the angle of the velocity with respect to the field. Thus,

$$F_{\text{magnetic}} = |q|vB$$

Andthe electric force has a magnitude of $F_{\text{electric}} = |q|E$

Hence,

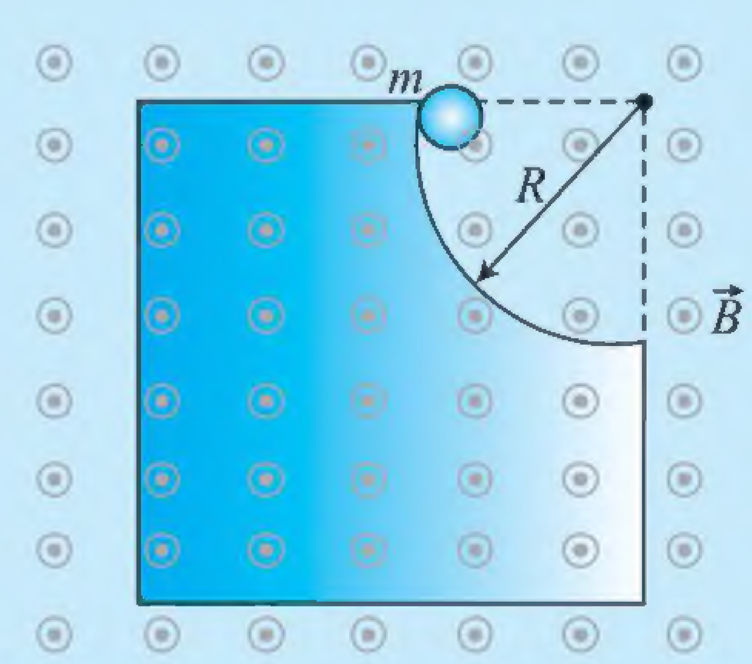
$$F = \sqrt{F_{\text{magnetic}}^2 + F_{\text{electric}}^2} = \sqrt{(|q|vB)^2 + (|q|E)^2} = |q|\sqrt{(vB)^2 + E^2}$$

$$= (1.8 \times 10^{-6})\sqrt{[(2.0 \times 10^6)(1.5 \times 10^{-3})]^2 + (4.0 \times 10^3)^2}$$

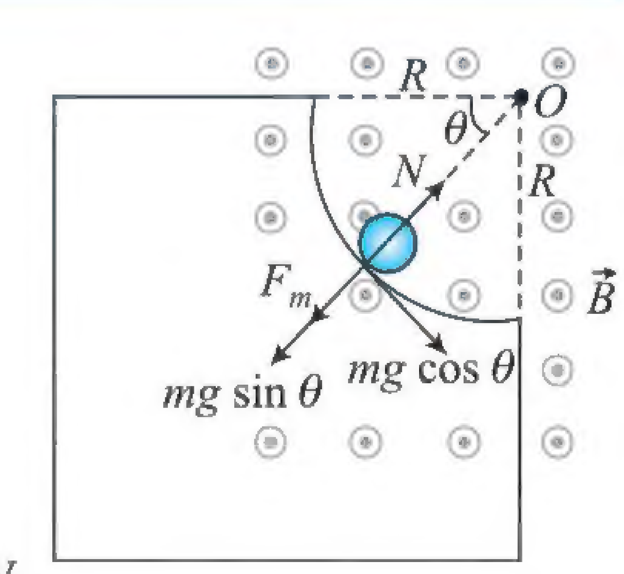
$$= 9.0 \times 10^{-3} \text{ N}$$

ILLUSTRATION 1.9

A charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position as shown in figure. There exists a uniform and constant horizontal magnetic field of induction B. Find the maximum force exerted by the track on the sphere.



Sol. Magnetic force on sphere



$F_m = qvB$ (directed radially outward)

$$\because \quad N - mg\sin\theta - qvB = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{mv^2}{R} + mg\sin\theta + qvB$$

From conservation of mechanical energy,

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + (-mgR) = 0 \text{ or, } v = \sqrt{2gR}$$

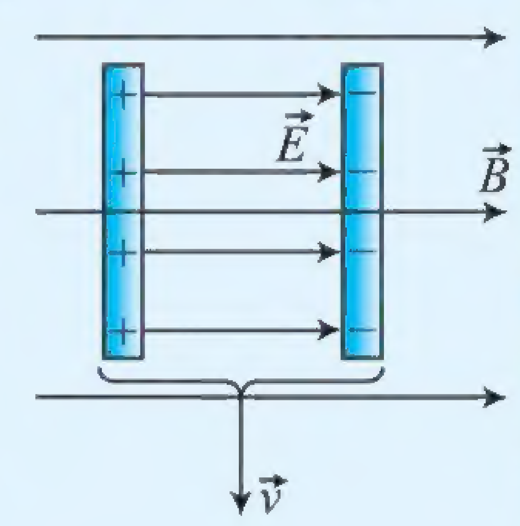
N will be maximum, when v is maximum, $\theta = 90°$

$$N_{\max} = \frac{m(2gR)}{R} + mg\sin 90° + q(\sqrt{2gR})B$$

$$= 3mg + q(\sqrt{2gR})B$$

Concept Application Exercise 1.1

1. A charged particle of mass 5 mg and charge $q = +2\ \mu C$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m s^{-1} and Wb m^{-2}, respectively.
2. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.
3. A particle with charge –5.60 nC is moving in a uniform magnetic field $\vec{B} = -(1.25\,T)\hat{k}$. The magnetic force on the particle is measured to be
$$\vec{F} = -(3.36 \times 10^{-7}\ N)\hat{i} + (7.42 \times 10^{-7}\ N)\hat{j}.$$
 (a) Calculate all components of velocity of the particle from this information.
 (b) Calculate the scalar product $\vec{v} \cdot \vec{F}$. What is the angle between v and F?
4. A particle with charge 7.00 μC is moving with velocity $\vec{v} = -(4 \times 10^3\ m\ s^{-1})\hat{j}$. The magnetic force on the particle is measured to be $\vec{F} = +(8.4 \times 10^{-2}\ N)\hat{i} - (5.60 \times 10^{-2}\ N)\hat{k}$.
 (a) Calculate all the components of the magnetic field you can from this information.
 (b) Calculate the scalar product $\vec{B} \cdot \vec{F}$. What is the angle between $\vec{B}$ and $\vec{F}$?
5. When a proton has a velocity $\vec{v} = (2\hat{i} + 3\hat{j}) \times 10^6\ ms^{-1}$, it experiences a force $\vec{F} = -(1.28 \times 10^{-13}\ \hat{k})$ N. When its velocity is along the z-axis, it experiences a force along the x-axis. What is the magnetic field?
6. The force on a charged particle moving in a magnetic field can be computed as the vector sum of the force due to each separate component of the magnetic field. As an example, a particle with charge q is moving with speed v in the $-y$ direction. It is moving in a uniform magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.
 (a) What is the force $\vec{F}$ exerted on the particle by the magnetic field?
 (b) If $q > 0$, what must the signs of the components of $\vec{B}$ be if the components of $\vec{F}$ are all non-negative?
 (c) If $q < 0$ and $B_x = B_y = B_z > 0$, find $\vec{F}$ in terms of $|q|$, v and B_x.
7. A particle of charge $q > 0$ is moving at speed v in the $+z$ direction through a region of uniform magnetic field. The magnetic force on the particle $\vec{F} = F_0(3\hat{i} + 4\hat{j})$, where F_0 is a positive constant.
 (a) Determine the components B_x, B_y and B_z or at least as many of the three components as is possible from the information given.
 (b) If it is given in addition that the magnetic field has magnitude $6F_0/qv$, determine the magnitude of B_z.
8. A charged particle of mass 10 g and charge 50 μC moves through a uniform magnetic field, in a region where the free-fall acceleration is $-10\hat{j}$ m/s^2. The velocity of the particle is a constant $20\hat{i}$ km/s, which is perpendicular to the magnetic field. What, then, is the magnetic field?
9. An electron is moving through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (3.0B_x)\hat{j}$
 At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19}\ N)\hat{k}$. Find B_x.
10. Two charged particles move in the same direction with respect to the same magnetic field. Particle 1 travels three times faster than particle 2. However, each particle experiences a magnetic force of the same magnitude. Find the ratio $|q_1|/|q_2|$ of the magnitudes of the charges.
11. The drawing shows a parallel plate capacitor that is moving with a speed of 20 m/s through a 5.0-T magnetic field. The velocity v is perpendicular to the magnetic field. The electric field within the capacitor has a value of 200 N/C, and each plate has an area of 5.0 × 10^{-4} m^2. What is the magnetic force (magnitude and direction) exerted on the positive plate of the capacitor?



12. The electrons in the beam of a television tube have a kinetic energy of 4.5 × 10^{-15} J. Initially, the electrons move horizontally from west to east. The vertical component of the earth's magnetic field points down, toward the surface of the earth, and has a magnitude of 2.25 × 10^{-5} T.
 (a) In what direction are the electrons deflected by this field component?
 (b) What is the acceleration of an electron in part (a)? (Mass of electron $m_e = 9.0 \times 10^{-31}$ kg)

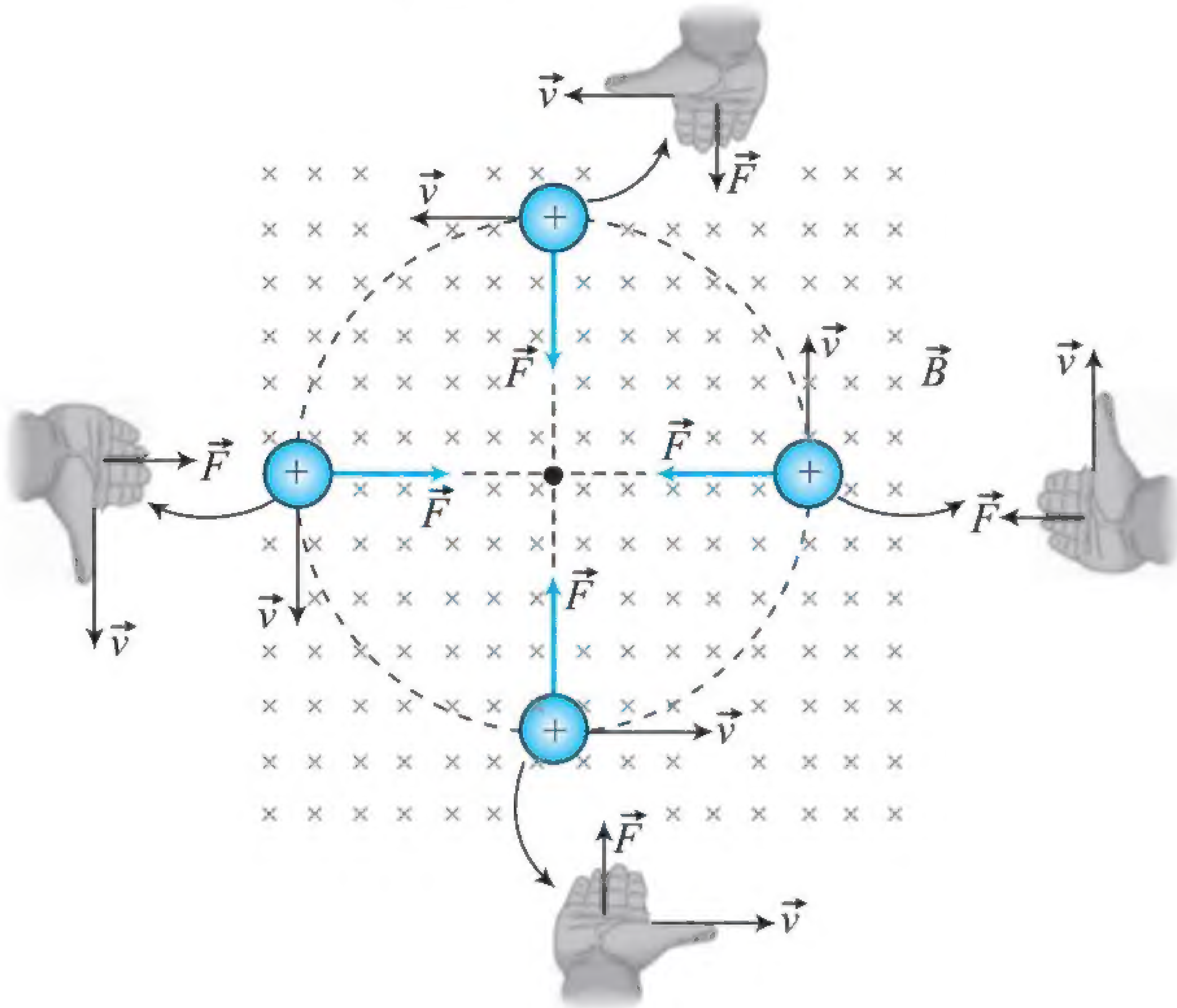
Answers

1. $2 \times 10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]$ N; $0.8[-3\hat{i} + 2\hat{j} + 3\hat{k}]$ ms^{-2}
2. 3
3. (a) $v_x = -106$ ms^{-1}; $v_y = -48$ ms^{-1} (b) $\vec{v} \cdot \vec{F} = 0$; 90°
4. (a) $B_x = -2$ T, $B_z = -3$ T (b) $\vec{B} \cdot \vec{F} = 0$; 90° **5.** $-(0.4\hat{j})$ T
6. (a) $qvB_x\hat{k} - qvB_z\hat{i}$
(b) $B_x > 0$, $B_z < 0$, sign of B_y does not matter
(c) $|q|vB_z\hat{i} - |q|vB_x\hat{k}$
7. (a) $B_x = \dfrac{4F_0}{qv}$, $B_y = \dfrac{-3F_0}{qv}$, B_z is arbitrary (b) $\pm\dfrac{11F_0}{qv}$
8. $(-0.10\ T)\hat{k}$ **9.** –2.0 T **10.** $\dfrac{1}{3}$
11. 8.85×10^{13} N, directed out of the page
12. (a) Due south (b) 4.0×10^{14} m/s^2

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Consider a charged particle of mass m projected in a uniform magnetic field $\vec{B}$ with an initial velocity vector $\vec{v}$ perpendicular to the field (figure below). B points away from the reader.

A force will act on the particle as shown which is perpendicular to the velocity at any instant. And also magnitude of the velocity will remain same, only direction will change. Finally the particle will move in a circular path.



In this situation, the particle moves in a circle with constant speed v and the magnetic force of magnitude $F = |q|vB$ supplies the centripetal force that keeps the particle moving in a circle. So we can write

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB}$$

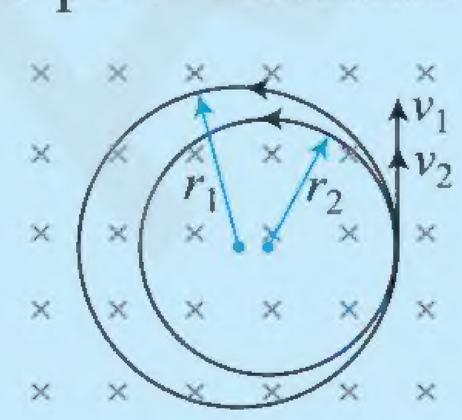
The above equation gives the radius of the circular path.

The angular speed ω of the particle is given by $\omega = \frac{v}{r} = \frac{qB}{m}$.

- The time period T of the motion is given by $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$
- The time taken by charged particle to rotate an angle θ in magnetic field, $t = \left(\frac{\theta}{2\pi}\right)T = \frac{m\theta}{qB}$
- Frequency [or the number of revolutions completed in unit time] is given by $f = \frac{1}{T} = \frac{qB}{2\pi m}$.

Important Point:

We see that time period is independent of velocity given to the charge particle. If two identical charge particles are given different speeds from a point in the same direction, then they will return to the initial point simultaneously.



Charged Particle Entering into Magnetic Field Region from Outside

If a charged particle enters into an uniform magnetic field region, the particle starts moving in circular arc, but it can not complete a full circle.

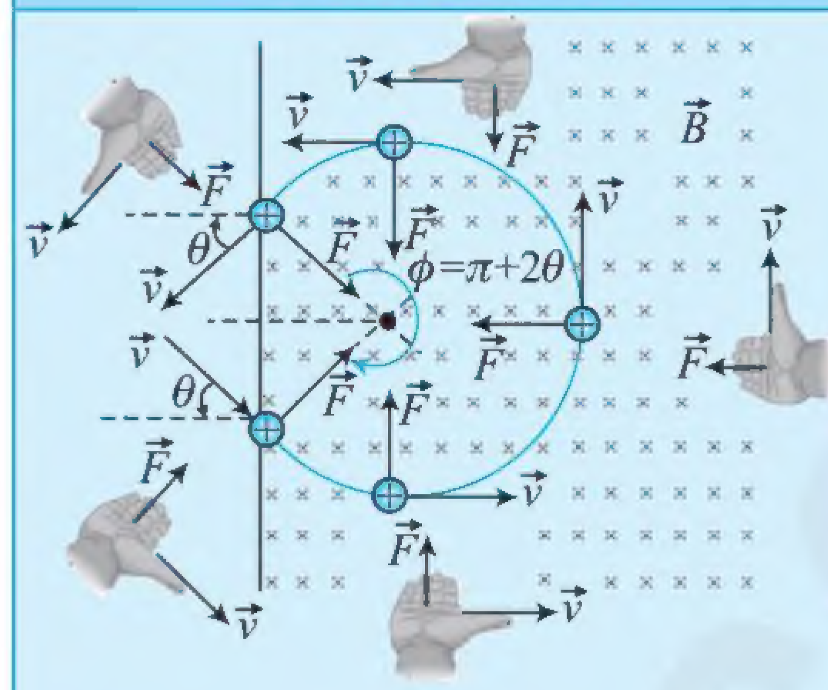
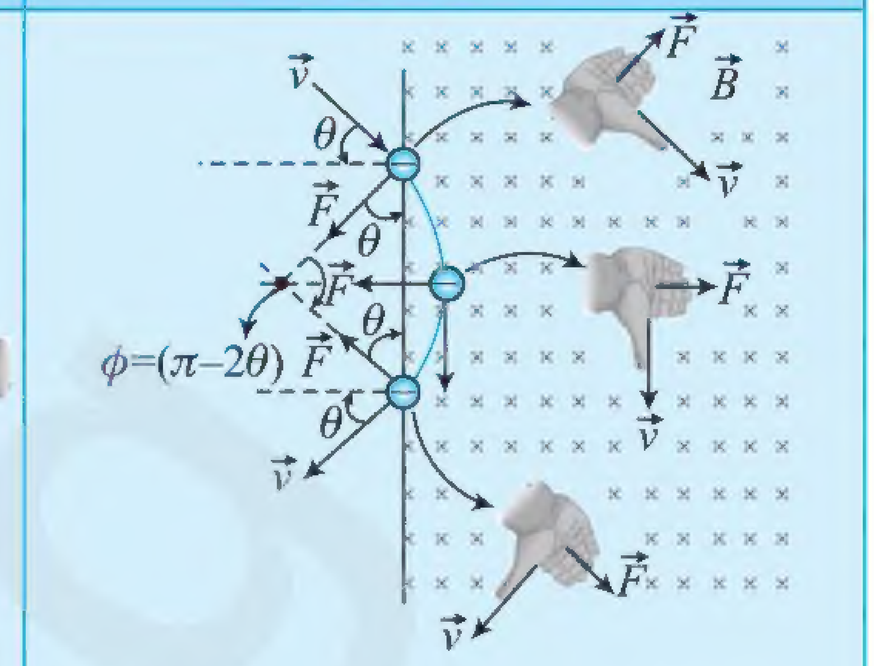
Motion of a positively charged particle	Motion of a negatively charged particle
 Time taken by charged particle in magnetic field $t = \left(\frac{\phi}{2\pi}\right)T = \left(\frac{\phi}{2\pi}\right)\frac{2\pi m}{qB} = \frac{m\phi}{qB}$ $\Rightarrow t = \frac{m(\pi + 2\theta)}{qB}$	 Time taken by charged particle in magnetic field $t = \left(\frac{\phi}{2\pi}\right)T = \left(\frac{\phi}{2\pi}\right)\frac{2\pi m}{qB} = \frac{m\phi}{qB}$ $\Rightarrow t = \frac{m(\pi - 2\theta)}{qB}$

Illustration 1.10

Doubly-ionized helium ions are projected with a speed of 10 km s^{-1} in a direction perpendicular to a uniform magnetic field of magnitude 1.0 T. Find **(a)** the force acting on an ion, **(b)** the radius of the circle in which it circulates and **(c)** the time taken by an ion to complete the circle.

Sol.

(a) $F = qvB\sin\theta$

$$= (2 \times 1.6 \times 10^{-19}) \times (10 \times 10^3) \times 1.0 \times \sin 90°$$

$$= 3.2 \times 10^{-15}\text{ N}$$

(b) $r = \frac{mv}{qB} = \frac{(4 \times 1.67 \times 10^{-27}) \times 10 \times 10^3}{(2 \times 1.6 \times 10^{-19}) \times 1.0} = 2.1 \times 10^{-4}\text{ m}$

(c) $T = \frac{2\pi m}{qB} = \frac{2\pi \times (4 \times 1.67 \times 10^{-27})}{(2 \times 1.6 \times 10^{-19}) \times 1.0} = 1.32 \times 10^{-7}\text{ s}$

Illustration 1.11

A stream of protons and deuterons in a vacuum chamber enters a uniform magnetic field. Both protons and deuterons have been subjected to same accelerating potential, hence the kinetic energies of the particles are the same. If the ion-stream is perpendicular to the magnetic field and the protons move in a circular path of radius 15 cm, find the radius of the path traversed by the deuterons. Given that mass of deuteron is twice that of a proton.

Sol. The radius r of the path is given by

$$\frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

For proton, $r_p = \frac{m_p v_p}{q_p B}$

For deuteron, $r_d = \frac{m_d v_d}{q_d B}$

$$\therefore \quad \frac{r_d}{r_p} = \frac{m_d v_d}{m_p v_p} \times \frac{q_p}{q_d} \qquad \text{...(i)}$$

Given that $\frac{1}{2} m_p v_p^2 = \frac{1}{2} m_d v_d^2$ also $q_p = q_d$

$$\text{Now} \quad \frac{v_d}{v_p} = \left(\frac{m_p}{m_d}\right)^{1/2} \qquad \text{...(ii)}$$

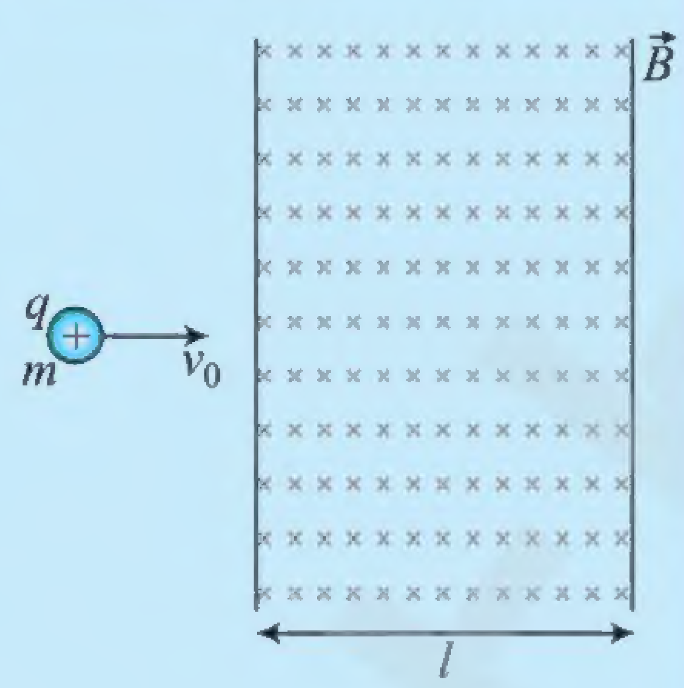
Subsituting the value of v_d/v_p from Eq. (ii) in Eq. (i) we get

$$\frac{r_d}{r_p} = \frac{m_d}{m_p}\left(\frac{m_p}{m_d}\right)^{1/2} = \left(\frac{m_d}{m_p}\right)^{1/2} = \sqrt{2} \qquad (\because \; m_d = 2m_p)$$

or $\quad r_d = \sqrt{2}.r_p = 1.414 \times 0.15 = 0.212$ m

ILLUSTRATION 1.12

A positive charged particle having charge q and mass m is projected into a region having perpendicular uniform magnetic field B. The particle enter into a magnetic field region with speed v_0 normal to the boundary of magnetic field as shown in the figure. Find the angle of deviation of the particle from its path when it comes out of the magnetic field region, if the width of the region



(i) l is slightly greater than $\frac{mv_0}{qB}$

(ii) $l = \frac{mv_0}{qB}$

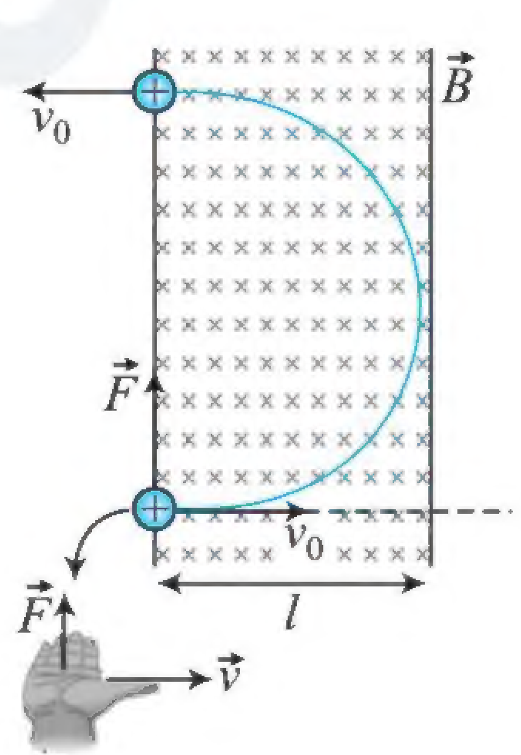
(iii) $l = \frac{mv_0}{2qB}$

Sol. When the particle enters into uniform magnetic field region, its path becomes circular. The radius of the circular path,

$$R = \frac{mv_0}{qB}$$

Case I: l is slightly greater than $\frac{mv_0}{qB}$

In this case the radius of the circular path is less than the width of the magnetic field region. Hence the particle will return to the same side of the magnetic field region.

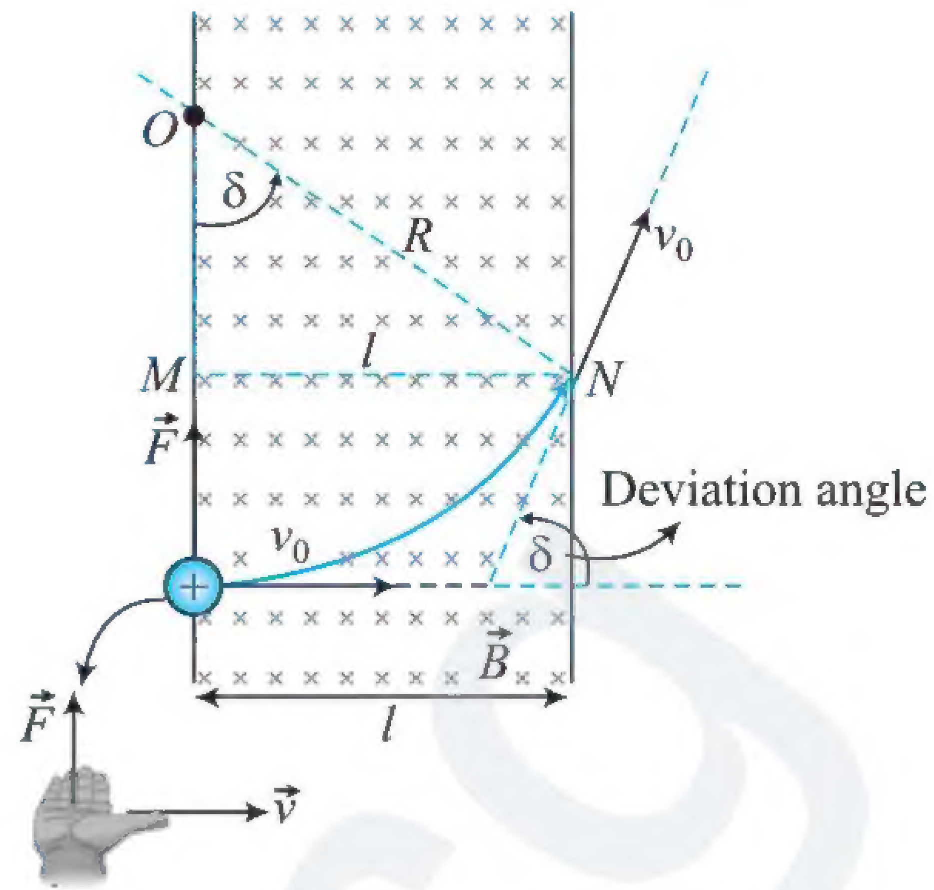


Hence, the angle of deviation should be 180°.

If $l < \frac{mv_0}{qB}$, the particle will came out from other side of magnetic field region as shown in figure.

In ΔOMN

$$\sin\delta = \frac{l}{R} = \frac{qBl}{mv_0} \qquad \text{...(i)}$$



Here δ is the angle of deviation.

Case II: If $l = \frac{mv_0}{qB}$

$$\sin\delta = 1 \quad \text{or} \quad \delta = \frac{\pi}{2}$$

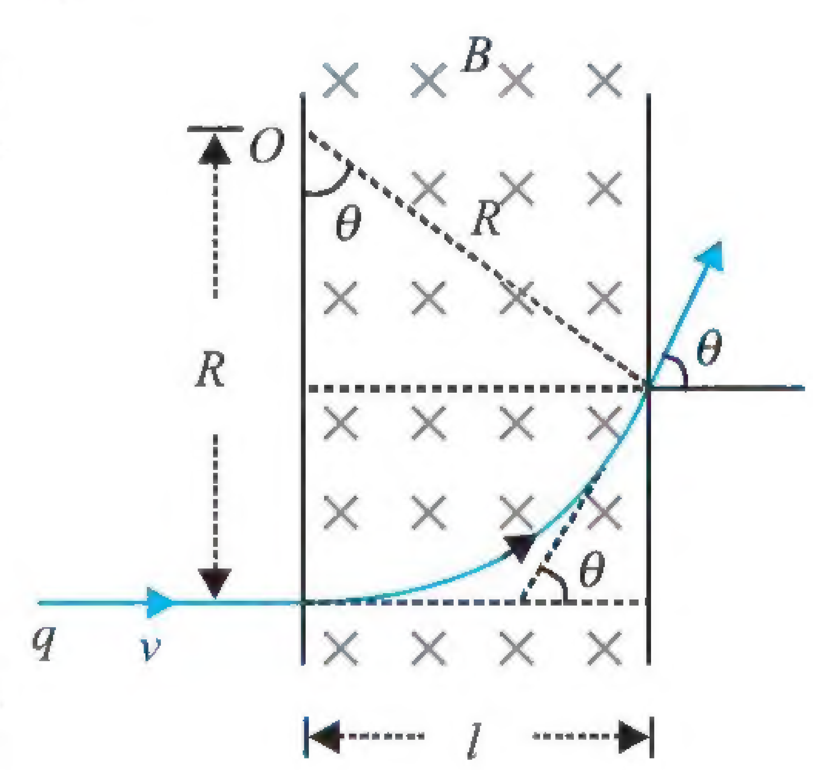
Case III: If $l = \frac{mv_0}{2qB}$

From (i), $\sin\delta = \frac{1}{2}$ or $\delta = \frac{\pi}{6}$

ILLUSTRATION 1.13

An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion, if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 tesla. (Given: mass of α-particle 6.4×10^{-27} kg).

Sol. The situation is shown in figure.



When a charged particle with charge q is accelerated through a potential difference V volt, then

$$\frac{1}{2}mv^2 = qV$$

$$\text{or} \quad v = \sqrt{\left(\frac{2qV}{m}\right)} \qquad \text{...(i)}$$

α-particle in magnetic field moves in a circle of radius R which is given by

$$R = \frac{mv}{qB} \quad \text{or} \quad R = \frac{1}{B}\sqrt{\left(\frac{2mV}{q}\right)} \qquad \text{...(ii)}$$

The change in direction of α-particle (θ) from the figure is given by

$$\sin\theta = \frac{l}{R} = lB\sqrt{\left(\frac{q}{2mV}\right)}$$

Here $l = 0.1$ m, $B = 0.1$ tesla, $V = 10^4$ volt

$q = 2e = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ C

and $m = 6.4 \times 10^{-27}$ kg

$$\therefore \quad \sin\theta = 0.1 \times 0.1 \times \sqrt{\left(\frac{3.2 \times 10^{-19}}{2 \times 6.4 \times 10^{-27} \times 10^4}\right)} = \frac{1}{2}$$

or $\quad \theta = 30°$

ILLUSTRATION 1.14

A uniform magnetic field of magnitude B_0 is present in a region along z-direction in a given coordinate system. A particle having specific charge σ is projected in xy plane with a speed v. The direction of magnetic field is made to reverse after every interval of $\frac{2\pi}{\sigma B}$. Find the maximum separation between two positions of the particle during its course of motion.

Sol. The charged particle moves in uniform magnetic field hence it will move in circular path.

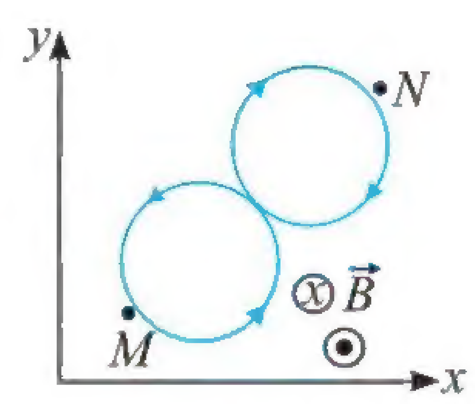
The time period of circular motion of the particle,

$$T = \frac{2\pi m}{qB} = \frac{2\pi}{\left(q/m\right)B} = \frac{2\pi}{\sigma B}$$

The radius of circular path of the particle,

$$r = \frac{mv}{qB} = \frac{v}{\left(q/m\right)B} = \frac{v}{\sigma B}$$

It means the direction of magnetic field changes in every one time period of circular motion; hence, the rotation sense of the particle also changes in every one time period. If the particle moves clockwise in one circular path then in next circular path it should move in counter clockwise sense. The path of the motion of the charged particle will be as shown in figure,



Hence the maximum separation between two positions of the particle during its course of motion should be MN,

$$\therefore \quad MN = 4r = \frac{4v}{\sigma B_0}$$

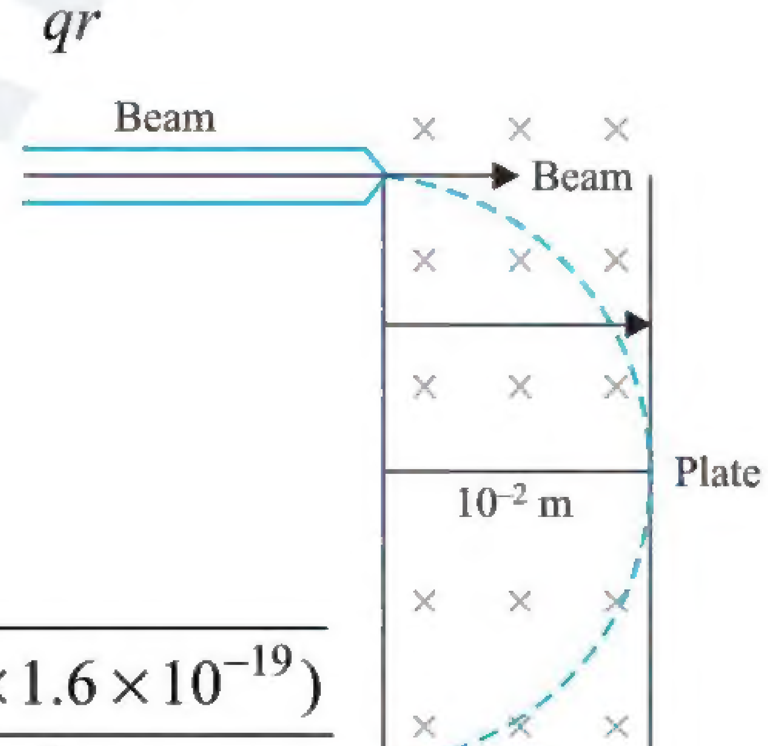
ILLUSTRATION 1.15

A beam of charged particles, having kinetic energy 10^3 eV, contains masses 8×10^{-27} kg and 1.6×10^{-26} kg emerge from the end of an accelerator tube. There is a plate at distance 10^{-2} m from the end of the tube and placed perpendicular to the beam. Calculate the magnitude of the smallest magnetic field which can prevent the beam from striking the plate. (The particles in the beam has identical charge $q = 1.6 \times 10^{-19}$ C)

Sol. Let $\vec{B}$ be required magnetic field and E_k the kinetic energy. Maximum radius of circular path for the beam not to strike the plane

$$r = \frac{mv}{qB} = \frac{\sqrt{2mE_k}}{qB} \Rightarrow B = \frac{\sqrt{2mE_K}}{qr}$$

Here $r = 10^{-2}$ m is same for both types of particles. For the particle of larger mass, B required will be more. This B will be minimum value required to prevent the beam from striking the plate.

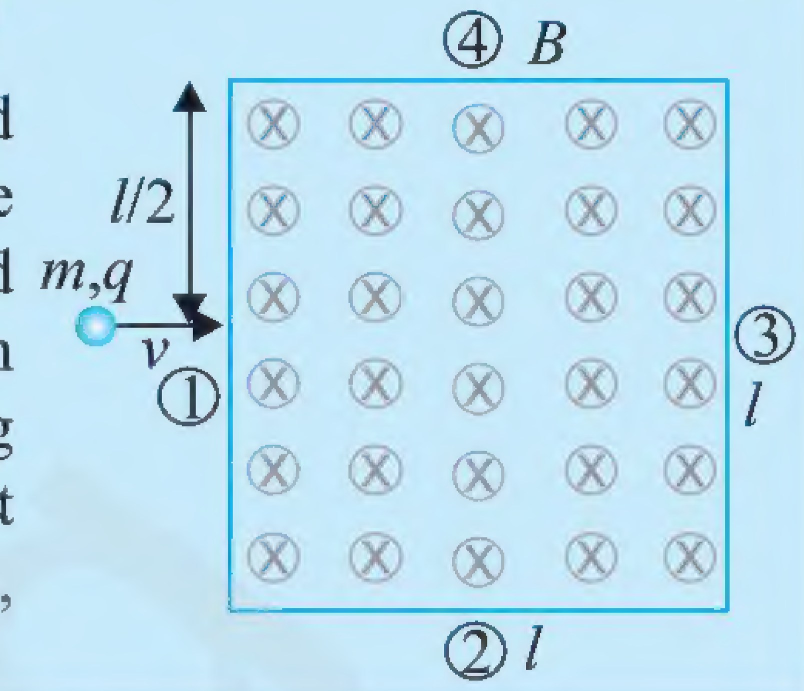


$$B_{\min} = \frac{\sqrt{2 \times 1.6 \times 10^{-26} \times (10^3 \times 1.6 \times 10^{-19})}}{1.6 \times 10^{-19} \times 10^{-2}}$$

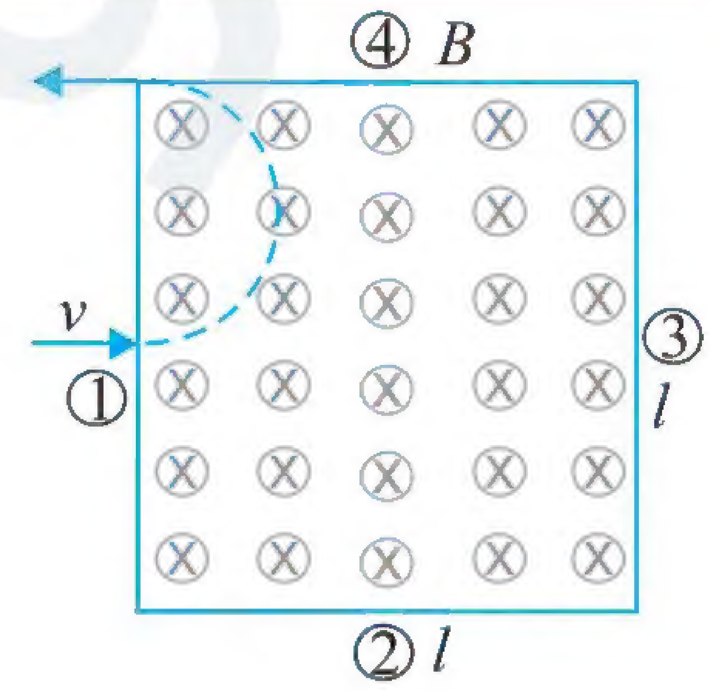
$$= \frac{1.6\sqrt{2} \times 10^{-21}}{1.6 \times 10^{-21}} = \sqrt{2}\text{ T} = 1.414\text{ T}$$

ILLUSTRATION 1.16

The magnetic field is confined in a square region. A positive charged particle of charge q and mass m is projected as shown in figure. Find the limiting velocities of the particle so that it may come out of face (1), (2), (3), and (4).



Sol. For the positive charge coming out from face (1), the radius of the path in magnetic field should be less than or equal to $l/4$. For limiting case



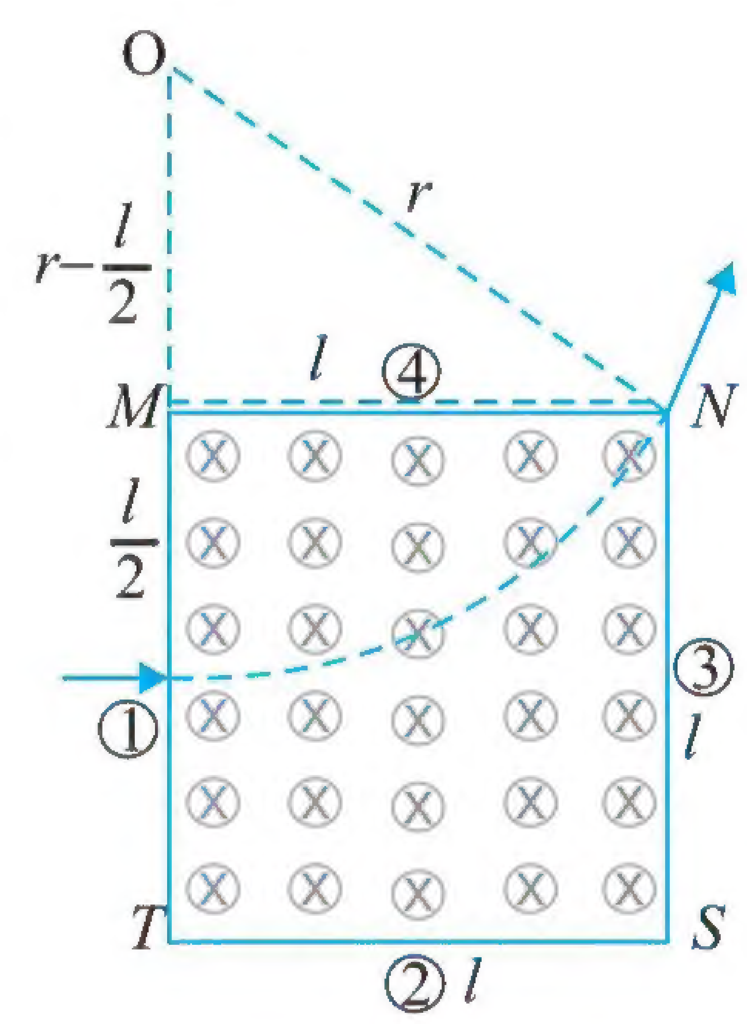
$$r_{\max} = \frac{l}{4} = \frac{mv}{qB}$$

$$\Rightarrow \quad v_{\max} = \frac{qBl}{4m}$$

Hence, if the velocity is $< \frac{qBl}{4m}$, the charge particle comes out of face (1).

We can observe from right palm rule that the particle cannot come out from face (2).

For a positive charge coming out of face (4), let particle come out at point N from ΔOMN.



$$(OM)^2 = (ON)^2 + (MN)^2$$

$$r^2 = \left(r - \frac{l}{2}\right)^2 + l^2$$

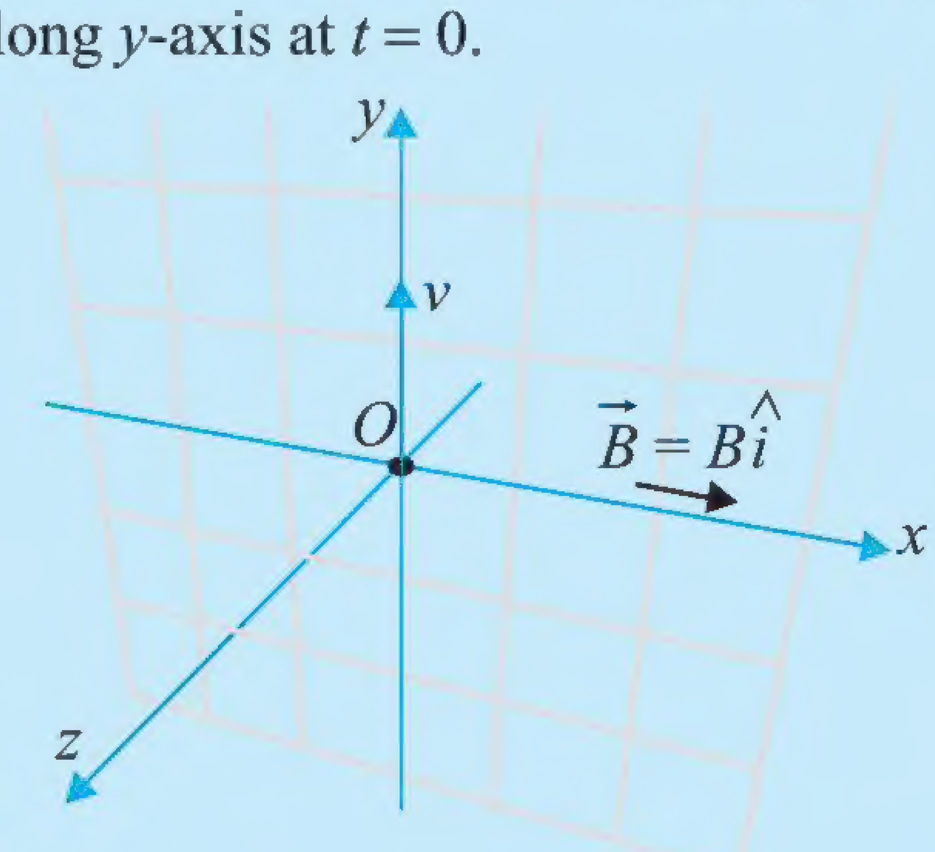
$$\Rightarrow \quad r = \frac{5}{4}l$$

If the particle comes out from face (4), $r < \frac{5}{4}l \Rightarrow \frac{mv}{qB} < \frac{5}{4}l$

or $v < \frac{5}{4}\frac{qBl}{m}$. If velocity $v > \frac{5}{4}\frac{qBl}{m}$, the particle will come out from face (3).

ILLUSTRATION 1.17

A charge particle of mass m and charge q is projected with velocity v along y-axis at $t = 0$.



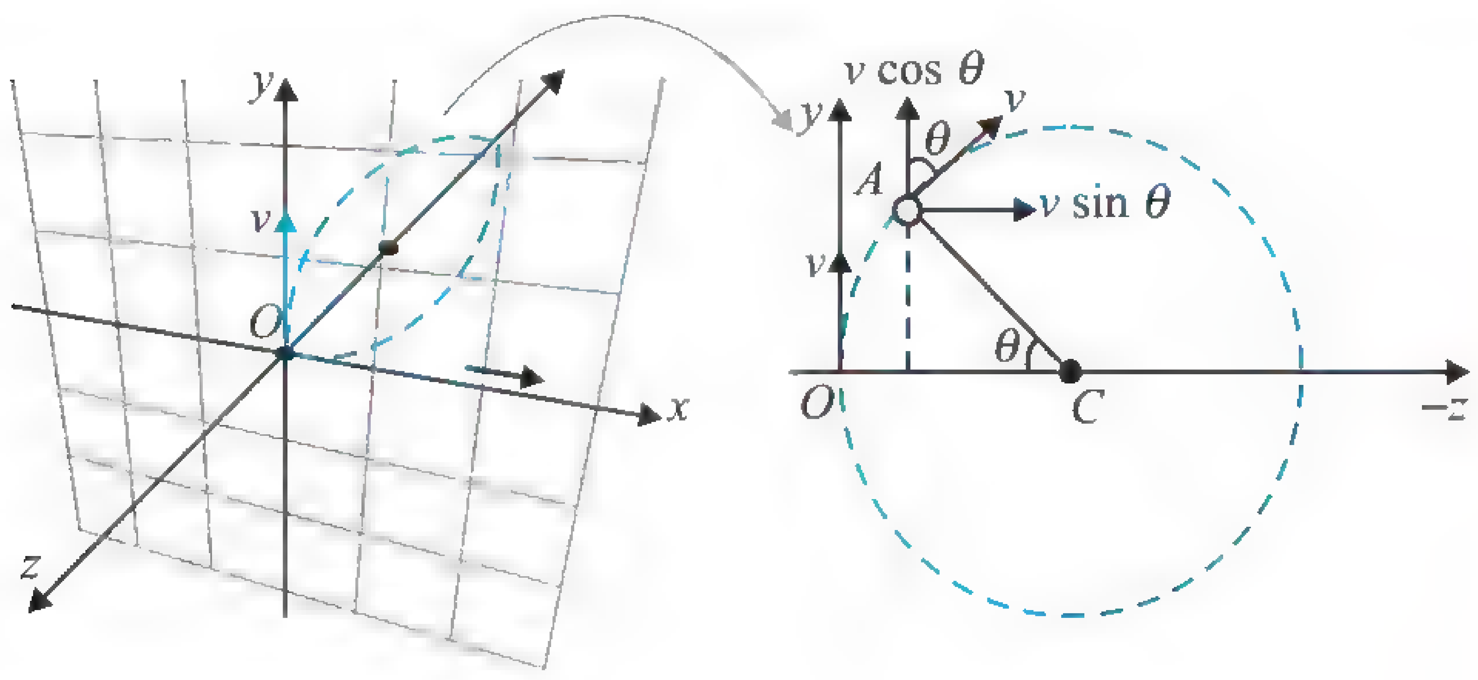
Find the velocity vector and position vector of the particle $\vec{v}(t)$ and $\vec{r}(t)$ in relation with time.

Sol. Radius of the circular path $R = \dfrac{mv}{qB}$

Angular velocity of the particle $\omega = \dfrac{v}{R} = \dfrac{qB}{m}$

The speed of the particle will remain unchanged. The center of circular path will be located on $-z$-axis at point C. Let us take time interval t. during this time interval the particle will rotate an angle $\theta = \omega t = \dfrac{qBt}{m}$.

Let us draw the front view of the circular path and locate the instantaneous position of the particle.



Writing velocity vector at time t.

$$\vec{v} = v\cos\theta\hat{j} - v\sin\theta\hat{k}$$

$$\vec{v}(t) = v\cos\left(\frac{qBt}{m}\right)\hat{j} - v\sin\left(\frac{qBt}{m}\right)\hat{k}$$

Now writing y and z coordinates of the particle.

y-coordinate of $P = R\sin\theta$

z-coordinate of $P = -(R - R\cos\theta)$

$$\vec{r}(t) = R\sin\frac{qBt}{m}\,j - R\left(1 - \cos\frac{qBt}{m}\right)\hat{k}$$

$$\left[\text{where}\quad R = \frac{mv}{qB}\right]$$

If Particle is Projected at Some Angle with Magnetic Field (Helical Paths)

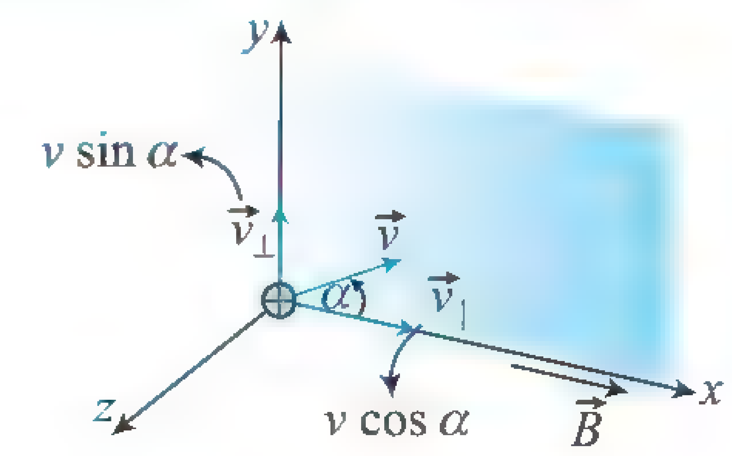
If a charged particle is projected into uniform magnetic field in such a way that the velocity of a charged particle has a component parallel to the magnetic field, the particle will move in a helical path about the direction of the field vector. Figure, for example, shows the velocity vector $\vec{v}$ of such a particle resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it:

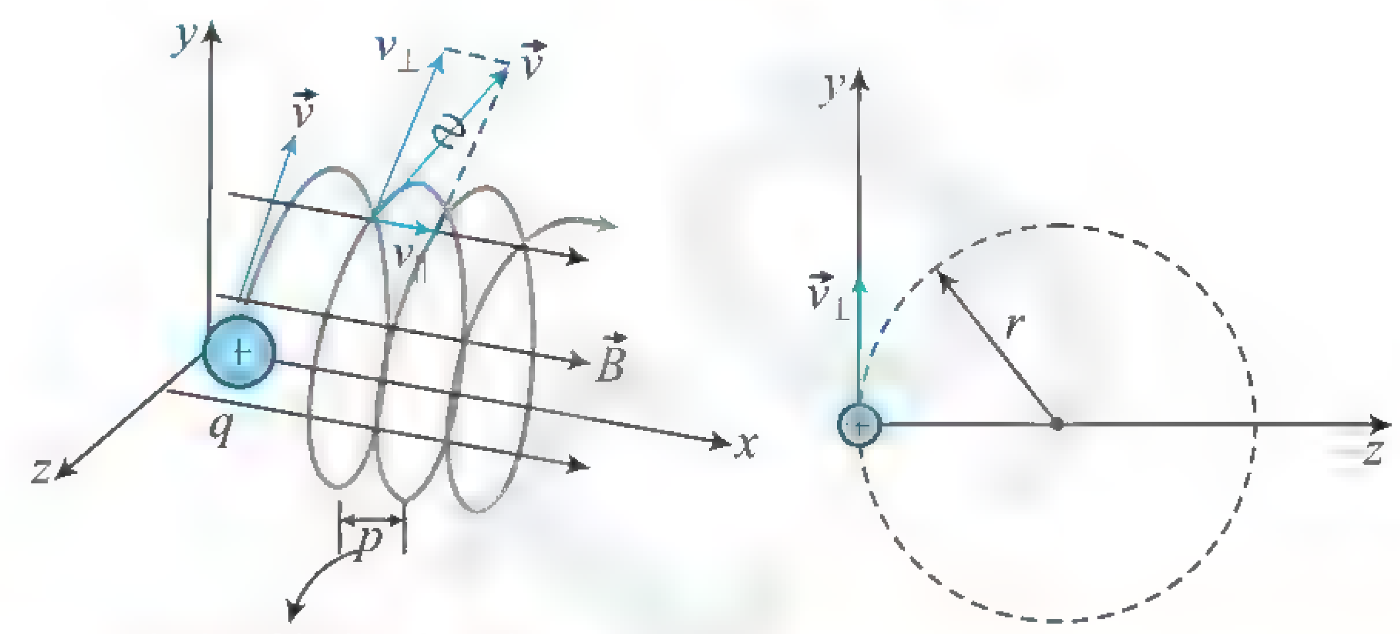
$$v_{\parallel} = v\cos\alpha \quad \text{and} \quad v_{\perp} = v\sin\alpha \qquad \text{...(i)}$$

With perpendicular component of velocity, it moves in a circular path of radius $r = \dfrac{mv_{\perp}}{qB} = \dfrac{mv\sin\alpha}{qB}$ and, with parallel component of velocity it moves along the field lines.

The linear distance travelled (along the field line) in one revolution (time period) is called pitch (p) (figure). The parallel component determines the pitch p of the helix—that is, the distance between adjacent turns.

$$\text{Pitch: } p = v_{\parallel}T = \frac{2\pi m v_{\parallel}}{qB} = \frac{2\pi m v\cos\alpha}{qB}$$



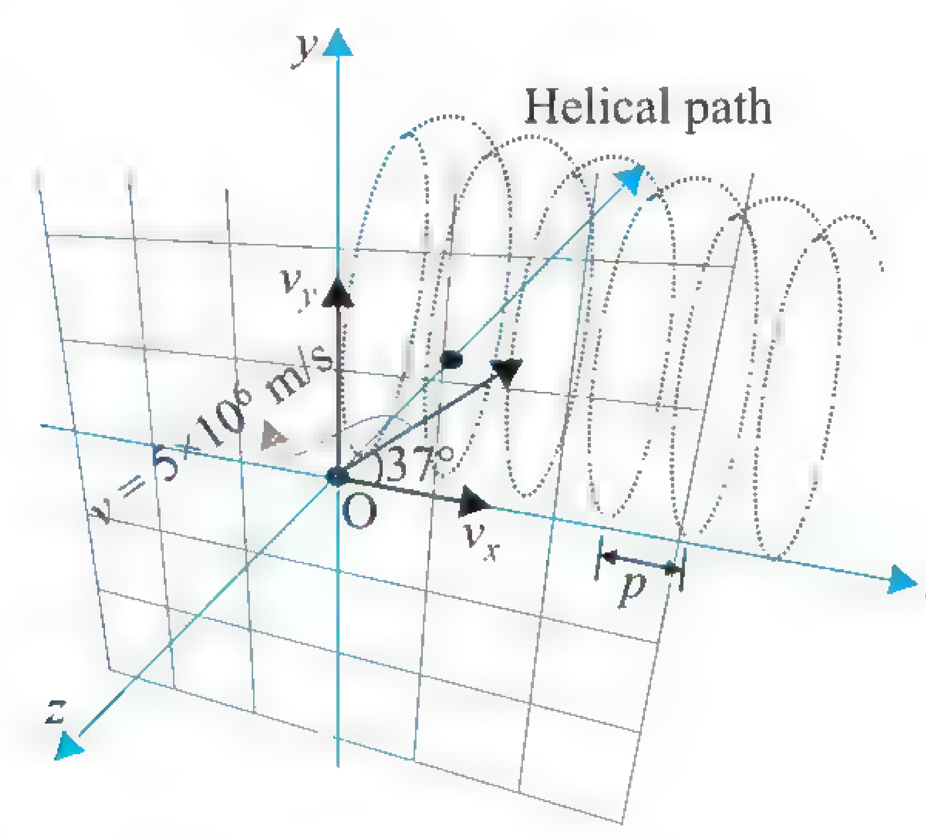


Pitch of helical path

ILLUSTRATION 1.18

A proton (charge 1.6×10^{-19} C, mass $= 1.60 \times 10^{-27}$ kg) is shot with a speed 5×10^6 m s^{-1} at an angle of 37° with the X-axis. A uniform magnetic field $B = 0.30$ T exists along the X-axis. Show that path of the proton is a helix. Find the radius and pitch of the helix.

Sol. The situation is shown in figure.



$$v_x = v\cos 37° = (5\times10^6)\left(\frac{4}{5}\right) = 4.0\times10^6\ \text{ms}^{-1}$$

$$v_y = v\sin 37° = (5\times10^6)\left(\frac{3}{5}\right) = 3.0\times10^6\ \text{ms}^{-1}$$

Since the velocity has both components, parallel and transverse to magnetic field, so the resulting path will be helix. Due to combined action of v_x, v_y and B, the proton moves in a helical path.

The radius of helix is

$$r = \frac{mv_y}{qB} = \frac{(1.60\times10^{-27})(3\times10^6)}{(1.6\times10^{-19})\times0.3} = 0.10\ \text{m}$$

Time taken to complete one circle:

$$T = \frac{2\pi m}{qB} = \frac{2\pi\times1.6\times10^{-27}}{1.6\times10^{-19}\times0.3} = \frac{2\pi}{3}\times10^{-7}\ \text{s}$$

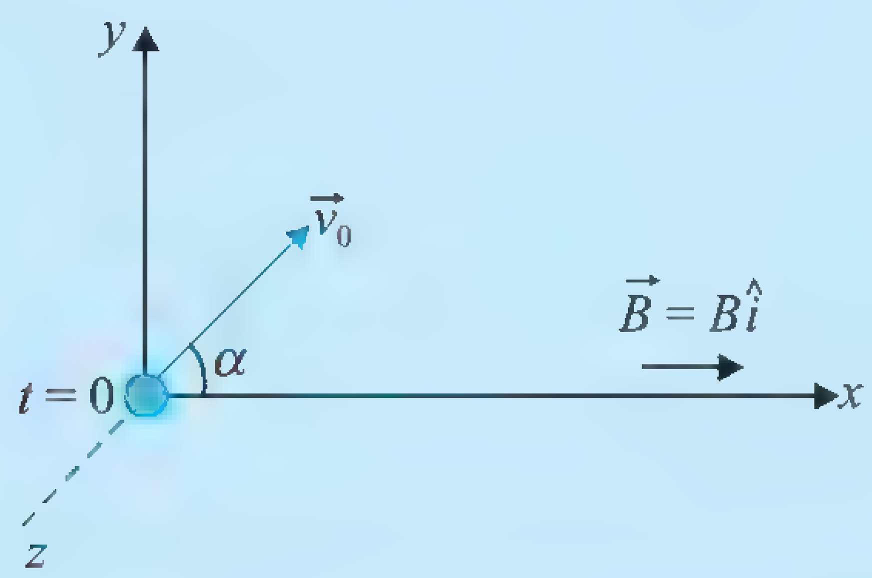
The pitch of the helix

= Distance travelled by the proton along x-axis in time T.

$$p = v_x\times T = (4.0\times10^6)\times\left(\frac{2\pi}{3}\times10^{-7}\right) = \frac{8\pi}{30}\text{m}$$

ILLUSTRATION 1.19

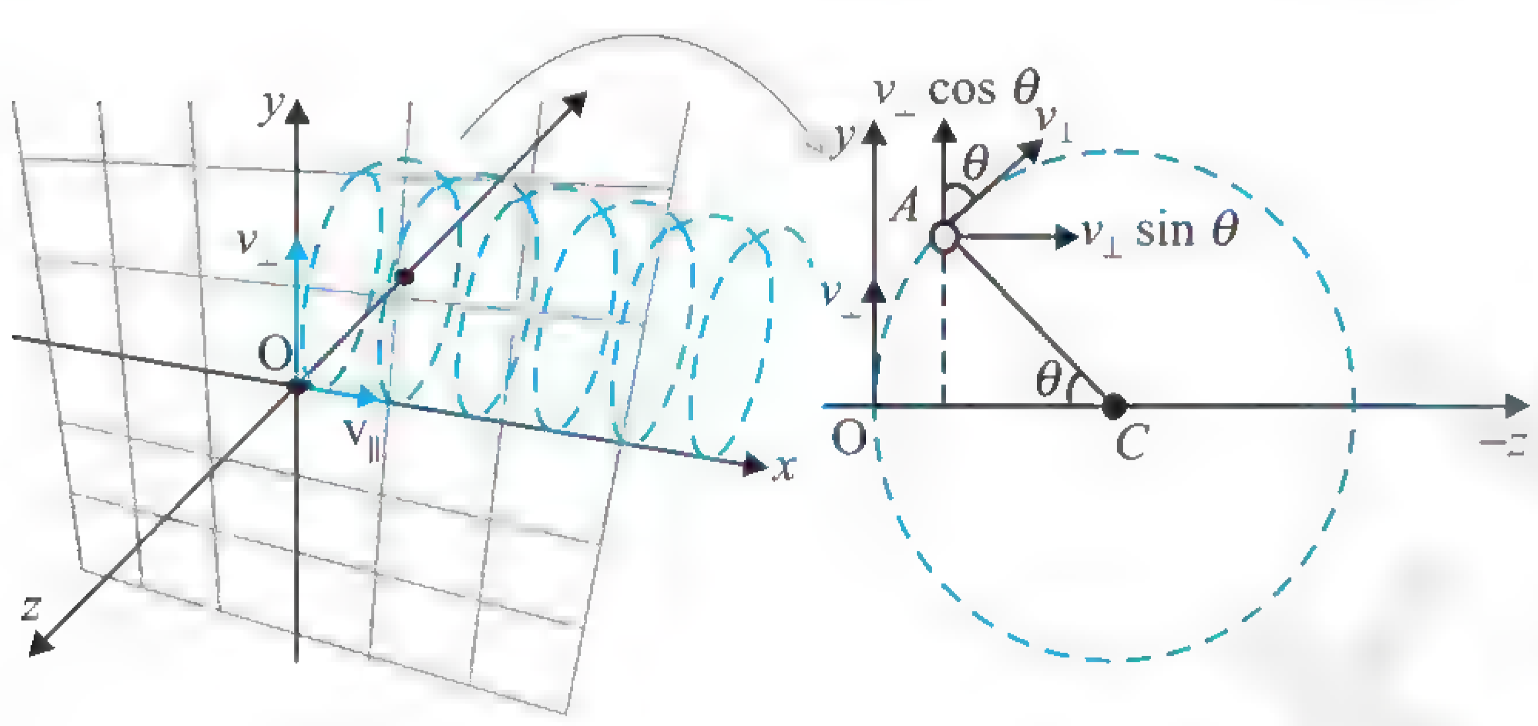
A charged particle having charge q and mass m is projected at angle α with x – axis with magnitude of velocity v_0. The magnetic field having magnitude B is along x-axis as shown in figure. Find



(a) velocity vector in function of time $\vec{v}(t)$.

(b) position vector in function of time $\vec{r}(t)$.

Sol. We can make components of the velocity of the particle along x-axis and y-axis. The component which is along the magnetic field will remain unchanged. But the component perpendicular to the magnetic field will provide circular motion. In a combined way, the path of the particle will be helical.



The component of velocity parallel to magnetic field

$$v_{\parallel} = v\cos\alpha$$

The component of velocity perpendicular to magnetic field

$$v_{\perp} = v\sin\alpha$$

The y and z components of velocity and position vector can be calculated in the same way as in illustration 1.17. The only difference is in place of v we substitute $v_{\perp}$. Hence, velocity vector $[\vec{v}_{\perp}(t)]$ in function of time can be written as

$$\vec{v}_{\perp}(t) = v_{\perp}\cos\frac{qB}{m}t\hat{j} - v_{\perp}\sin\frac{qB}{m}t\hat{k}$$

Hence net velocity

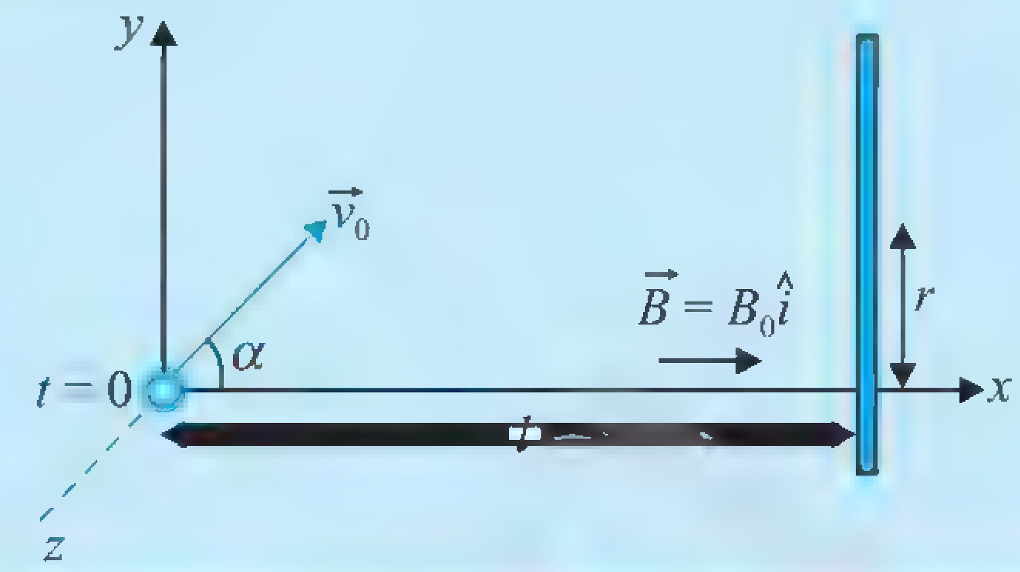
$$\vec{v} = v_{\parallel}\hat{i} + v_{\perp}\cos\frac{qB}{m}t\hat{j} - v_{\perp}\sin\frac{qB}{m}t\hat{k}$$

And position vector in function of time

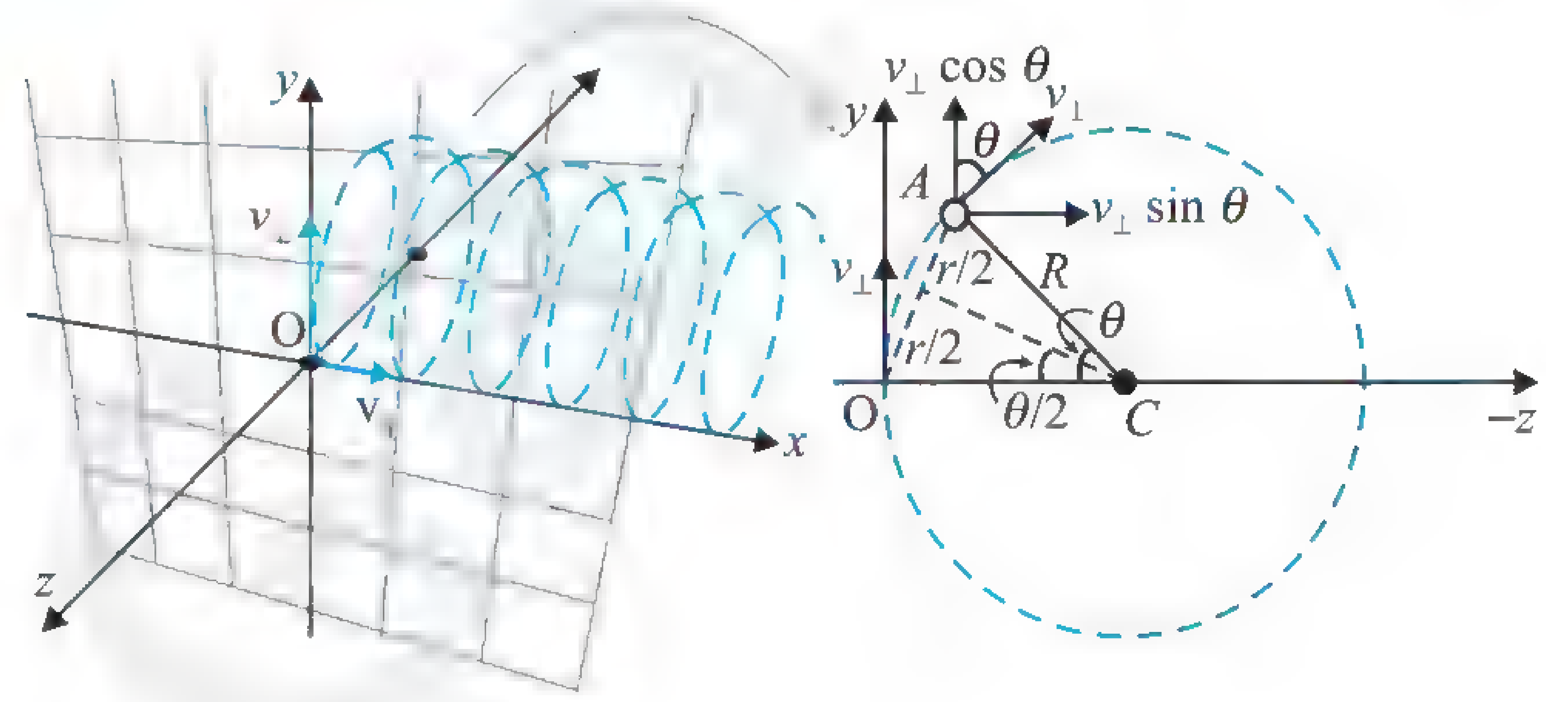
$$\vec{r}(t) = v_{\parallel}t\hat{i} + R\sin\frac{qBt}{m}\hat{j} - R\left(1 - \cos\frac{qBt}{m}\right)\hat{k}$$

ILLUSTRATION 1.20

A non-relativistic charge q of mass m originates at a point A lying on x-axis and moves with velocity v at an angle α to the x-axis. A screen is located at a distance l from A. Find the distance r from the x-axis to the point on the screen into which the charge strikes.



Sol. Time taken by particle to reach the screen: $t = \dfrac{l}{v\cos\alpha}$



Radius of helical path: $R = \dfrac{mv\sin\alpha}{qB}$

angle rotated during this time $\theta = \omega t = \dfrac{qB}{m}t$

Required distance:

$$r = 2R\sin\left(\frac{\theta}{2}\right) = \frac{2mv\sin\alpha}{qB}\sin\left(\frac{qB}{2m}\frac{l}{v\cos\alpha}\right)$$

ILLUSTRATION 1.21

An electron accelerated by a potential difference $V = 1.0$ kV moves in a uniform magnetic field at an angle $\alpha = 30°$ to the vector B whose modulus is $B = 29$ mT. Find the pitch of the helical trajectory of the electron.

Sol. Time period of the electron moving in helical path

$$T = \frac{2\pi m}{eB} = \frac{2 \times 3.14 \times (9.1 \times 10^{-31})}{(1.6 \times 10^{-19})(29 \times 10^{-3})} = 1.232 \times 10^{-9}\ \text{s}$$

The KE acquired by the electron is given by

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

$$\therefore \quad v = \sqrt{\left[\frac{2 \times (1.6 \times 10^{-19}) \times 10^3}{9.1 \times 10^{-31}}\right]} = 1.875 \times 10^7\ \text{m s}^{-1}$$

Now, if $v_{\parallel}$ be the velocity of electron parallel to magnetic field, then

$$v_{\parallel} = v\cos\alpha = 1.875 \times 10^7 \cos 30 = 1.624 \times 10^7\ \text{m s}^{-1}$$

$$\therefore \quad \text{Pitch} = v_{\parallel} \times T = (1.624 \times 10^7)(1.232 \times 10^{-9})$$

$$= 2 \times 10^{-2}\ \text{m} = 2\ \text{cm}$$

Path of a Charged Particle in Both Electric and Magnetic Fields

Consider a particle of charge q and mass m released from the origin with velocity $\vec{v} = v_0\hat{i}$ into a region of uniform electric and magnetic fields parallel to y-axis, *i.e.*, $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. The electric field accelerates the particle in y-direction, i.e., y component of velocity goes on increasing with acceleration,

$$a_y = \frac{F_y}{m} = \frac{F_e}{m} = \frac{qE_0}{m}.$$

The magnetic field rotates the particle in a circle in x–z plane (perpendicular to magnetic field). The resultant path of the particle is a helix with increasing pitch. The axis of the helix is parallel to y-axis. Velocity of the particle at time t would be,

$$\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

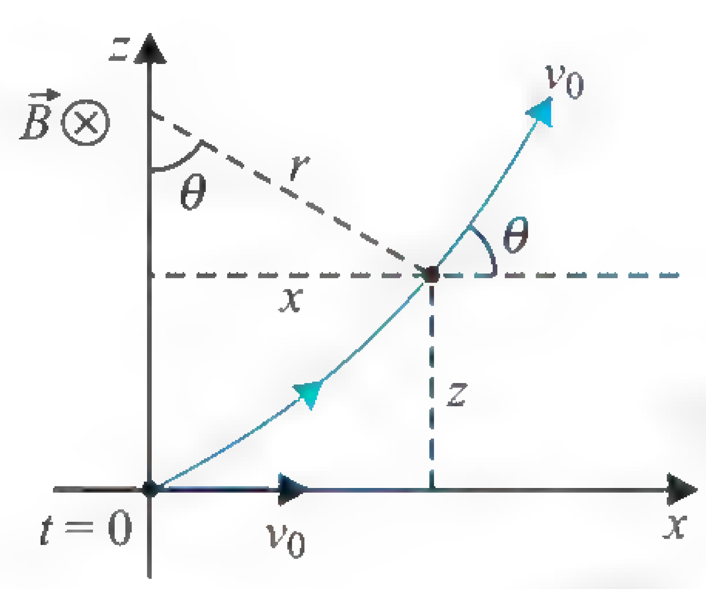
Here $v_y = a_y t = \dfrac{qE_0}{m}t$

and $v_x^2 + v_z^2 = \text{constant} = v_0^2; \quad \theta = \omega t = \dfrac{Bq}{m}t$

$$v_x = v_0 \cos\theta = v_0 \cos\left(\frac{Bqt}{m}\right)$$

and $v_z = v_0 \sin\theta = v_0 \sin\left(\dfrac{Bqt}{m}\right)$

$$\vec{v}(t) = v_0\cos\left(\frac{Bqt}{m}\right)\hat{i} + \left(\frac{qE_0}{m}t\right)\hat{j} + v_0\sin\left(\frac{Bqt}{m}\right)\hat{k}$$



Similarly, position vector of particle at time t can be given by,

$$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$$

Here, $y = \dfrac{1}{2}a_y t^2 = \dfrac{1}{2}\left(\dfrac{qE_0}{m}\right)t^2$

$$x = r\sin\theta = \left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)$$

and $z = r(1-\cos\theta) = \left(\dfrac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\dfrac{Bqt}{m}\right)\right\}\right]$

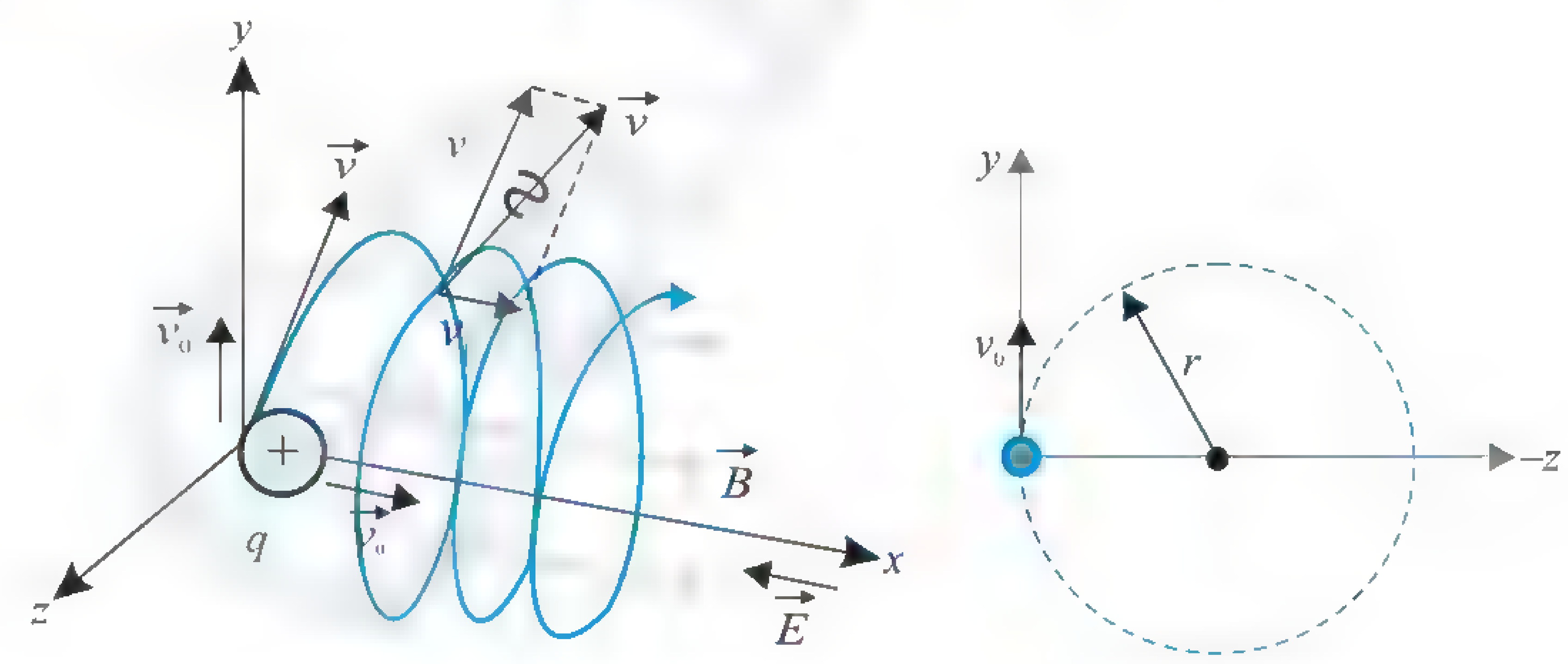
$$\vec{r}(t) = \left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)\hat{i} + \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2\hat{j}$$

$$+\left(\frac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\frac{Bqt}{m}\right)\right\}\right]\hat{k}$$

Illustration 1.22

In a region where both magnetic field $\vec{B} = B_0\hat{i}$ and electric field $\vec{E} = -E_0\hat{i}$ are present. A positive charge particle is projected from origin at $t = 0$, with a velocity $\vec{v} = v_0(\hat{i}+\hat{j})$. After some time, the charged particle is observed moving with velocity $v_0\hat{j}$ and tangential to positive x-axis. Find all possible values $\dfrac{E_0}{B_0}$.

Sol. The components of the velocities of the particle is parallel and perpendicular to magnetic field, hence the particle will move in helical path.



If an observer looks at the motion of the particle from a point on positive x-axis, he will find the particle rotating in yz plane with speed v_0 as shown in figure.

While rotating in yz plane the particle also advances in x direction. Due to electric force, x component of velocity decreases.

Retardation of the particle, $a_x = \dfrac{qE_0}{m}$

x-component of the velocity at time t, $v_x = v_0 - a_x t$

v_x becomes zero at time: $t_0 = \dfrac{v_0}{a_x} = \dfrac{mv_0}{qE_0}$

The particle will be exactly at x-axis travelling in y-direction if time period of circular motion multiplied by an integer is equal to t_0.

$$\Rightarrow n\cdot\frac{2\pi m}{qB} = t_0 = \frac{mv_0}{qE_0}$$

$$\Rightarrow E_0 = \frac{v_0 B_0}{2\pi n}, \; n = 1, 2, 3.....$$

$$\Rightarrow \frac{E_0}{B_0} = \frac{v_0}{2\pi n}$$

Illustration 1.23

A positively charged particle with specific charge σ starts moving from the origin with a velocity u directed along positive x-direction. The entire space has a uniform electric field (E) and magnetic field (B) directed along the positive y-direction. Find the angle that the velocity of the particle makes with y-direction at the instant it crosses the y-axis for n^{th} time.

Sol. The charged particle will move in helical path as velocity provided is perpendicular to magnetic field and the particle moves due to electric field which is parallel to magnetic field. The pitch of the helical path will keep on increasing and after

every revolution the helical trajectory will be touching the y-axis when the particle crosses it.

There will be an acceleration a_y on the particle due to electric force which acts along positive y-direction which is given as

Acceleration of the particle along y-direction, $a_y = \dfrac{qE}{m}$

If n be the number of revolution, then we have

$$t = n \times T = \frac{2\pi mn}{qB}$$

In y-direction the velocity of the particle is only due to electric field which is given as

$$v_y = a_y \times t$$

$$\Rightarrow \quad v_y = \left(\frac{qE}{m}\right) \times \left(\frac{2\pi mn}{qB}\right) = \frac{2\pi nE}{B}$$

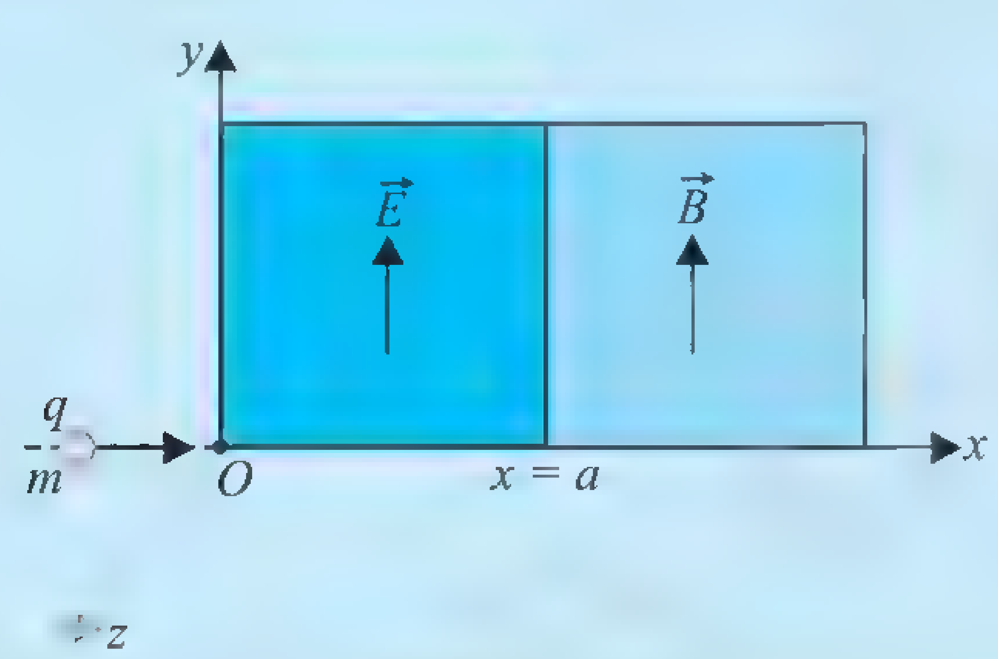
As along xz plane the particle velocity component will remain constant at v_0, the angle which the velocity vector makes with y-axis is given as

$$\tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi nE}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$$

ILLUSTRATION 1.24

A positively charged particle, having charge q, is accelerated by a potential difference V. This particle moving along the x-axis enters a region where an electric field E exists. The direction of the electric field is along positive y-axis. The electric field exists in the region bounded by the lines $x = 0$ and $x = a$. Beyond the line $x = a$ (i.e., in the region $x > a$), there exists a magnetic field of strength B, directed along the positive y-axis. Find



(a) at which point does the particle meet the line $x = a$?

(b) the pitch of the helix formed after the particle enters the region $x \geq a$. (Mass of the particle is m.)

Sol. The work done by the potential difference gets stored as its kinetic energy.

$$\therefore \quad \frac{1}{2}mv^2 = qV \Rightarrow v = \left(\frac{2qV}{m}\right)^{1/2} \quad \text{...(i)}$$

(a) When it enters the region $x \in [0, a)$, it experiences an electric field $\vec{E} = E\hat{j}$

Time taken to cross the region:

$$\Rightarrow \quad t = \frac{a}{v} = a\left(\frac{m}{2qV}\right)^{1/2} \quad \text{...(ii)}$$

The distance travelled in y-direction during this time is

$$y = \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2} \times \frac{qE}{m} \times a^2 \times \frac{m}{2qV}$$

$$\Rightarrow \quad y = \frac{1}{4}\frac{Ea^2}{V}$$

Hence, the particle meets the line $x = a$ at point

$$(x, y) = \left(a, \frac{1}{4}\frac{Ea^2}{V}\right)$$

(b) Now, velocity of the particle as it crosses the line $x = a$

$$\vec{v} = \left(\frac{2qV}{m}\right)^{1/2}\hat{i} + \frac{qE}{m}a\left(\frac{m}{2qV}\right)^{1/2}\hat{j}$$

Magnetic field in this region, $\vec{B} = B\hat{j}$

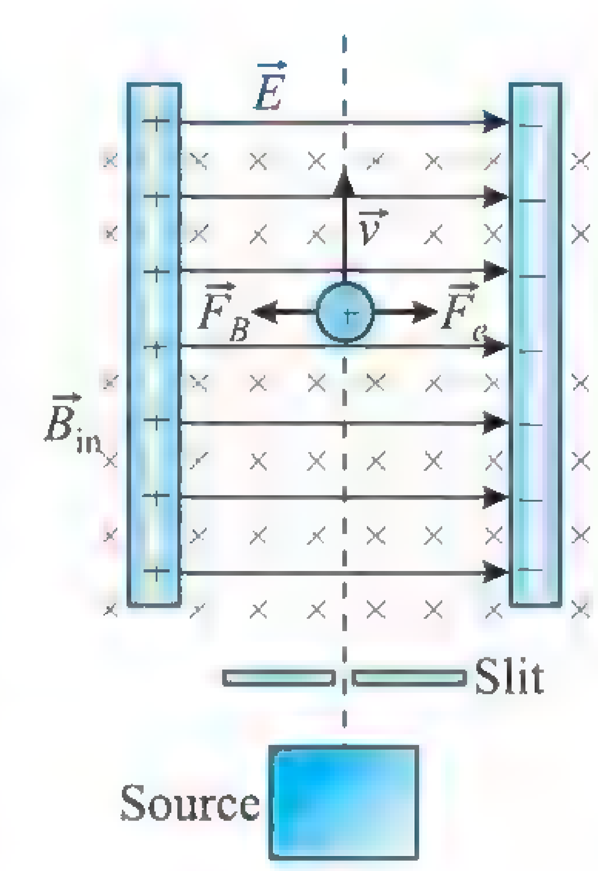
Hence, the time period of revolution, $t = \dfrac{2\pi m}{qB}$

$$\text{Pitch } p = v_{\parallel}t = \frac{2\pi m}{qB} \times \frac{qE}{m}.a\left(\frac{m}{2qV}\right)^{1/2}$$

$$p = \frac{\pi aE}{B}\left(\frac{2m}{Vq}\right)^{1/2}$$

APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

VELOCITY SELECTOR



In many experiments involving moving charged particles, it is important that all particles move with essentially the same velocity, which can be achieved by applying a combination of an electric field and a magnetic field oriented as shown in figure. A uniform electric field is directed to the right (in the plane of the page as shown in the figure), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page in the figure). If q is positive and the velocity $\vec{v}$ is upward, the magnetic force $q\vec{v} \times \vec{B}$ is to the left and the electric force $q\vec{E}$ is to the right. When the magnitudes of the two fields are chosen so that $qE = qvB$, the charged particle is modeled as a particle in equilibrium and moves in a straight vertical line through the region of the fields. From the expression $qE = qvB$, we find that

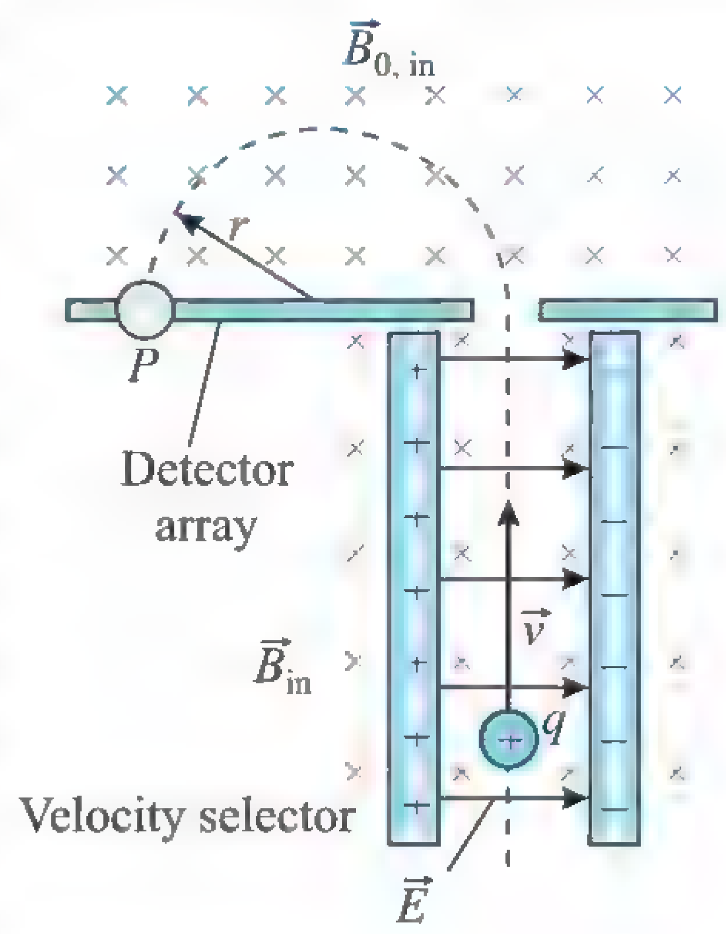
$$v = \frac{E}{B} \quad \text{...(i)}$$

Only those particles having this speed pass undeflected through the mutually perpendicular electric and magnetic fields. The magnetic force exerted on particles moving at speeds greater than that is stronger than the electric force, and the particles are deflected to the left. Those moving at slower speeds are deflected to the right.

MASS SPECTROMETER

A mass spectrometer separates ions according to their mass-to-charge ratio. In one version of this device, known as the Bainbridge mass spectrometer, a beam of ions first passes

through a velocity selector and then enters a second uniform magnetic field $\vec{B}_0$ that has the same direction as the magnetic field in the selector. Upon entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector array at P. If the ions are positively charged, the beam deflects to the left as shown in figure. If the ions are negatively charged, the beam deflects to the right. From equation, we can express the ratio m/q as



$$\frac{m}{q} = \frac{rB_0}{v}$$

Using Eq. (i) gives

$$\frac{m}{q} = \frac{rB_0 B}{E}$$

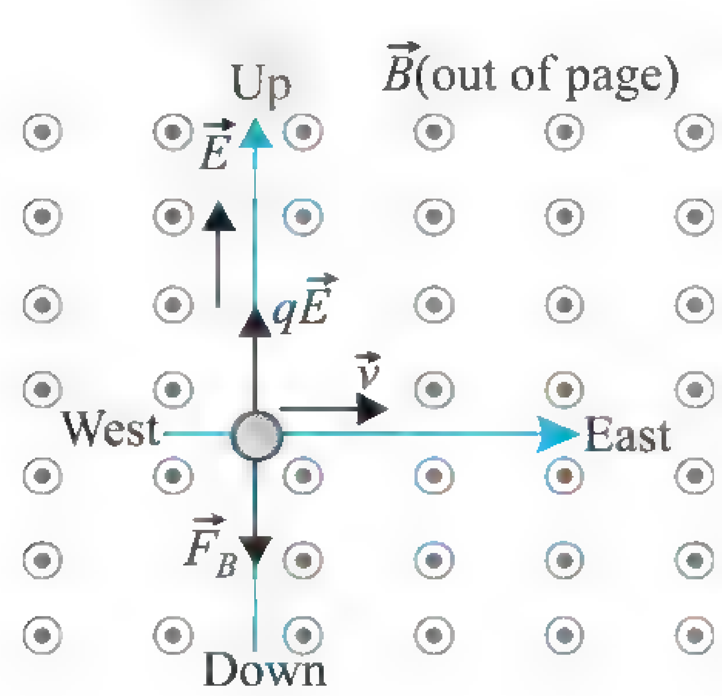
Therefore, we can determine m/q by measuring the radius of curvature and knowing the field magnitudes B, B_0, and E. In practice, one usually measures the masses of various isotopes of a given ion, with the ions all carrying the same charge q. In this way, the mass ratios can be determined even if q is unknown.

ILLUSTRATION 1.25

A velocity selector has an electric field of magnitude 2500 N/C, directed vertically upward, and a horizontal magnetic field that is directed south. Charged particles, traveling east at a speed of 6.0×10^3 m/s, enter the velocity selector and are able to pass completely through without being deflected. When a different particle with an electric charge of $+4.00 \times 10^{-12}$ C enters the velocity selector traveling east, the net force (due to the electric and magnetic fields) acting on it is 2.0×10^{-9} N, pointing directly upward. What is the speed of this particle?

Sol. The magnitude F_B of the magnetic force acting on the particle is related to its speed v by $F_B = |q_0|vB \sin\theta$, where B is the magnitude of the magnetic field, q_0 is the particle's charge, and θ is the angle between the magnetic field B and the particle's velocity v. As the drawing shows, the vector v (east, to the right) is perpendicular to the vector B (south, out of the page). Therefore, $\theta = 90°$, then

$$F_B = |q_0|vB\sin 90° = |q_0|vB \qquad \text{...(i)}$$



In addition to the magnetic force, there is also an electric force of magnitude F_E acting on the particle. This force magnitude does not depend upon the speed v of the particle, as we see from $F_E = |q_0|E$. The particle is positively charged, so the electric force acting on it points upward in the same direction as the electric field. By right-hand, the magnetic force acting on the positively charged particle points down, and is therefore opposite to the electric force. The net force on the particle points upward, so we conclude that the electric force is greater than the magnetic force. Thus, the magnitude F of the net force acting on the particle is equal to the magnitude of the electric force minus the magnitude of the magnetic force:

$$F = F_E - F_B \qquad \text{...(ii)}$$

We also note that particles traveling at a speed $v_0 = 6.0 \times 10^3$ m/s experience no net force. Therefore, $F_E = F_B$ for particles moving at the speed v_0.

Substituting Eq. (i) and $F_E = |q_0|E$ into Eq. (ii) yields

$$F = |q_0|E - |q_0|vB \qquad \text{...(iii)}$$

The magnetic field magnitude B is not given, but, as noted above, for particles with speed $v_0 = 6.0 \times 10^3$ m/s, the magnetic force of Eq. (i), $F_B = |q_0|v_0 B$, is equal to the electric force $F_E = |q_0|E$. Therefore, we have that

$$|q_0|v_0 B = |q_0|E \quad \text{or} \quad B = \frac{E}{v_0} \qquad \text{...(iv)}$$

Substituting Eq. (iv) into Eq. (iii) yields

$$F = |q_0|E - |q_0|vB = |q_0|E - \frac{|q_0|vE}{v_0} \qquad \text{...(v)}$$

Solving Eq. (v) for v, we obtain

$$\frac{|q_0|vE}{v_0} = |q_0|E - F \quad \text{or} \quad \frac{v}{v_0} = 1 - \frac{F}{|q_0|E}$$

or
$$v = v_0\left(1 - \frac{F}{|q_0|E}\right) \qquad \text{...(vi)}$$

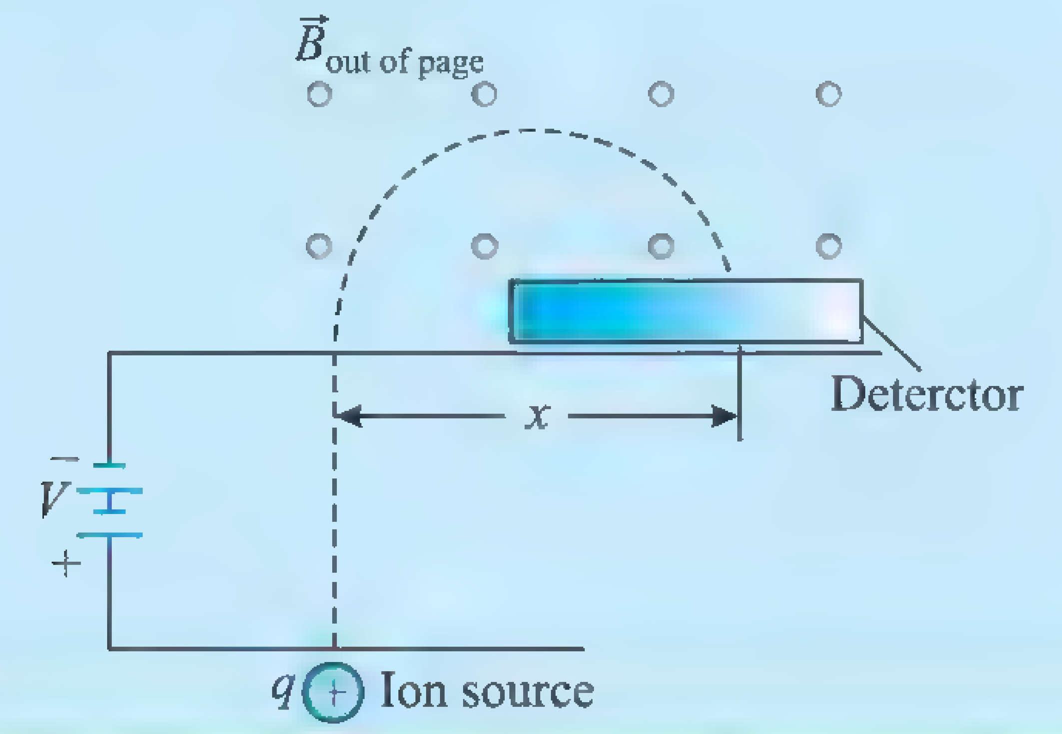
Substituting the given values into Eq. (vi), we find that

$$v = (6.0\times10^3)\left[1 - \frac{2.0\times10^{-9}}{(4.00\times10^{-12})(2500)}\right] = 4.8\times10^3 \text{ m/s}$$

ILLUSTRATION 1.26

A mass spectrometer separates ions according to their ratio of charge to mass. Such devices are widely used in science and engineering to analyze unknown mixtures and to separate isotopes of chemical elements. Figure shows ions of charge q and mass m first being accelerated from rest through a potential difference V and then entering a region of uniform magnetic field B pointing out of the page. Only the magnetic force acts on the ions in this region, so they undergo circular motion and, after half an orbit, land on a detector. Find an expression for

the horizontal distance x from the entrance slit to the point where an ion lands on the detector.



Sol. This problem is about charged particle undergoing circular motion in a uniform magnetic field. The distance we are asked for is the diameter of the particle's circular path.

Equation, $r = mv/qB$, shows that the path radius depends on the field and on the particle's mass, charge, and speed. We know everything but the speed, so this becomes a two-step problem in which we will first find the speed. We are given the potential difference—energy per unit charge—so we can use energy conservation to find the kinetic energy and hence the ion's speed in the magnetic field region. Then we will use equation to find the radius of the ion's circular path.

A charge q gains kinetic energy qV in "falling" through a potential difference V, so an ion's kinetic energy once it enters the magnetic field is

$$\frac{1}{2}mv^2 = qV$$

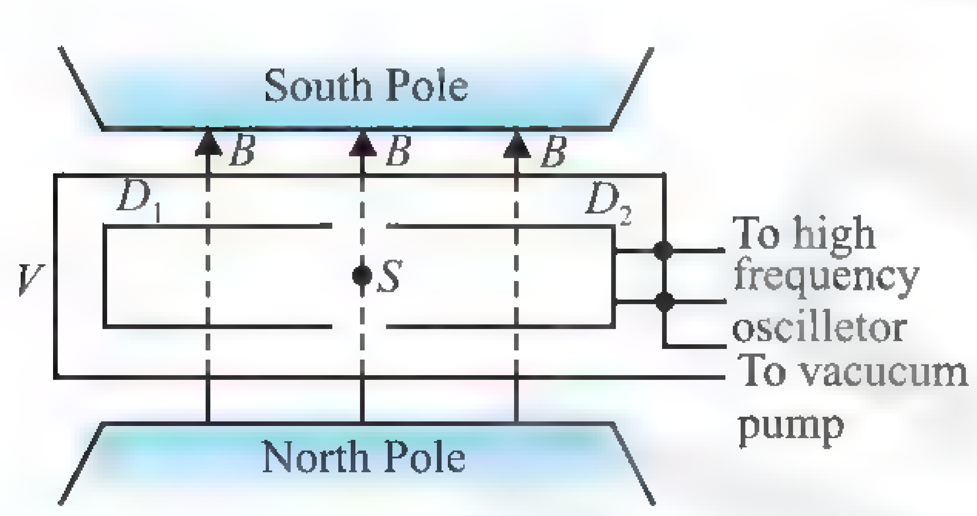
Solving for v gives $v = \sqrt{2qV/m}$.

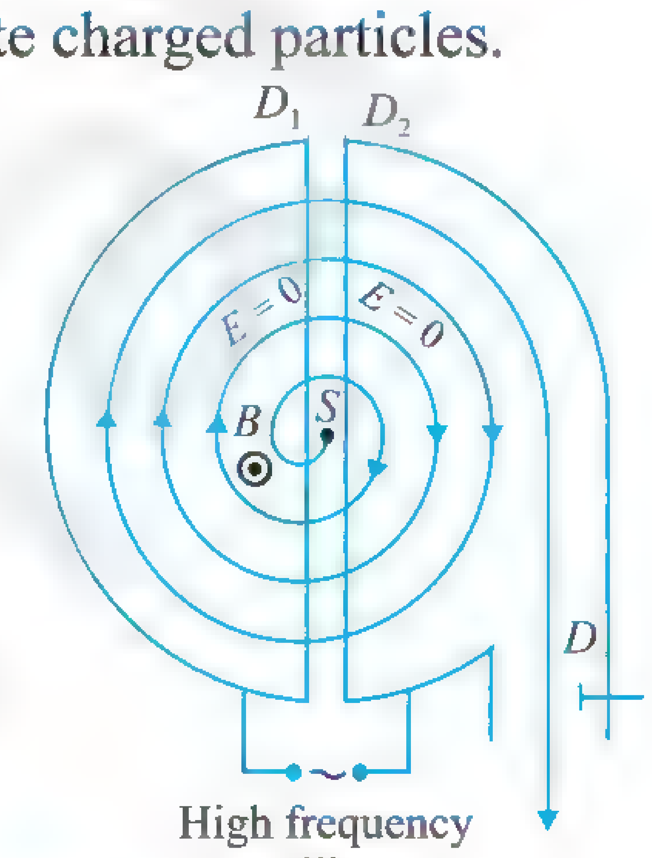
Our answer, the path diameter x, is then twice the radius given in equation.

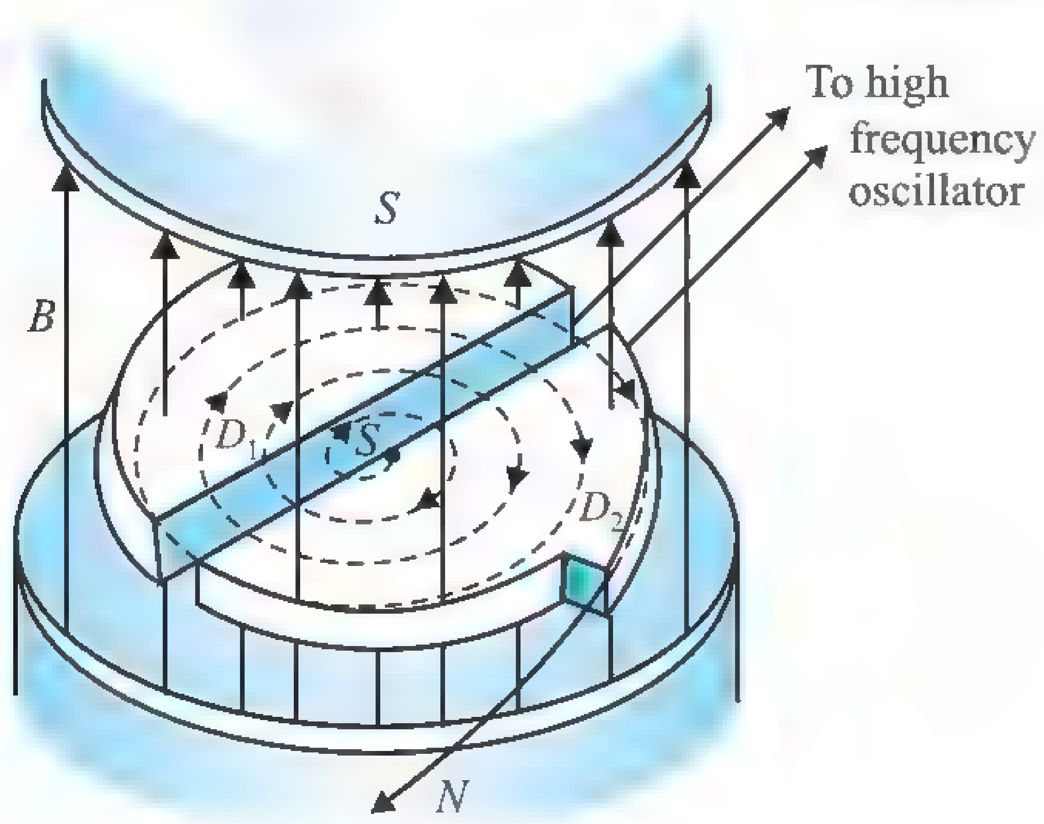
$$x = 2r = \frac{2mv}{qB} = \frac{2m\sqrt{2qV/m}}{qB} = \frac{2}{B}\sqrt{\frac{2mV}{q}}$$

CYCLOTRON

It is a device which is used to accelerate charged particles.







Construction

It consists of two hollow flat semicircular metal boxes D_1 and D_2 called the "dees" on account of their shape like the letter D. The two dees are separated by narrow parallel gap. A high frequency oscillator, which provides an alternating potential of the order of 10^4 to 10^5 volt at a frequency of 10^7 cycles is connected between the two dees. The oscillator establishes an alternating electric field in the air gap, i.e., the electric field is once directed toward D_1 and then toward D_2. Thus D_1 and D_2 become alternately positive and negative at the same rate as the frequency of the oscillator. A source S is placed at the center of the dees which supplies the positive ions to be accelerated. These dees are mounted inside a vacuum chamber. The chamber is mounted horizontally between the pole pieces NS of a huge electromagnet capable of producing a vertical field of about 1.6 T.

Working

The positive ions emitted from the source will be acceleraed in the gap towards the dees which is negative at that time. Let it be D_2.Since there is no electric field inside the dees, the positive ions move with constant speed along circles of constant radius under the influence of magnetic field which is perpendicular to the dees. If by the time the ions emerge from D_2, the polarity of the applied potential is reversed (i.e., the dee D_1 now becomes negative), the positive ions will again face the negative dee and thus will be again accelerated by the field in the gap. Since their velocity is increased, they will now move through D_1 along circular arc of greater radius as shown in figure. Here the time of passage to complete the semi circle in the dee remains the same as in D_2. If the time of travel in D_1 is equal to half the time period of the oscillator voltage, the positive ions after coming from D_1 will find the reversed field and hence they are accelerated again in the gap D_1D_2. In this way, the positive ions move faster and faster in ever-expanding circles until they reach the outer edge of the dees where they are deflected by deflector plate and strike the target. Here it should be remembered that the time required for the positive ions to make one complete turn within dees is the same for all speeds and is equal to the time period of the oscillator.

Theory

When a particle of mass m and charge q moves with a velocity v in the magnetic field of flux density B, then the radius r of the circular path is given by

$$r = \frac{mv}{qB} \qquad \text{...(i)}$$

As the speed v increases every time the particle passes through the gap, so radius r also increases.

For resonance between the applied ac voltage and the moving particle, the time taken by the particle to travel the circular arc within any of the dees must be equal to half the time period of the applied oscillator voltage. Or the frequency of oscillator voltage should be same as the frequency of revolution of charge particle. So the frequency of oscillator is given by

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

The value of q/m being fixed for an ion hence the value of f is adjusted corresponding to B or vice-versa.

Energy of a Particle Accelerated by a Cyclotron

The ion will have maximum energy when it will travel at the boundary of dee. If the outside radius of dee is R then according to Eq. (i), the maximum velocity v_m of the ion may be written as

$$v_m = \frac{RqB}{m} \quad ...(ii)$$

and so the maximum kinetic energy of the ion will be given by

$$E_m = \frac{1}{2}mv_m^2 = \frac{R^2q^2B^2}{2m} \quad ...(iii)$$

Limitations of Cyclotron

The maximum available particle energy is limited due to the following factors:

1. due to the limited power and frequency of the oscillator.
2. due to the maximum strength of the magnetic field which can be produced, and
3. the energy of charged particle emerging from cyclotron is limited due to variation of mass with velocity, i.e.,

$$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

where m_0 is the rest mass, m, the mass in motion when velocity is v and c, the velocity of light.

The frequency of rotation of charged particle becomes

$$f = \frac{qB}{2\pi m} = \frac{qB\sqrt{\left(1-\frac{v^2}{c^2}\right)}}{2\pi m_0}$$

Thus, the frequency of rotation of charged particle decreases with increase in velocity. Consequently, the charged particle takes a longer time to complete semicircular path. Now the particle continuously goes on lagging behind the alternating potential difference till a stage is reached when it is no longer accelerated further.

The frequency of the charged particle can be kept constant by making $B\sqrt{\left(\frac{1-v^2}{c^2}\right)}$ constant. To achieve this, the magnetic field B is increased as the velocity of particle increases. Alternatively, the frequency of applied alternating potential difference may be varied so that it is always equal to the frequency of rotation of charged particle.

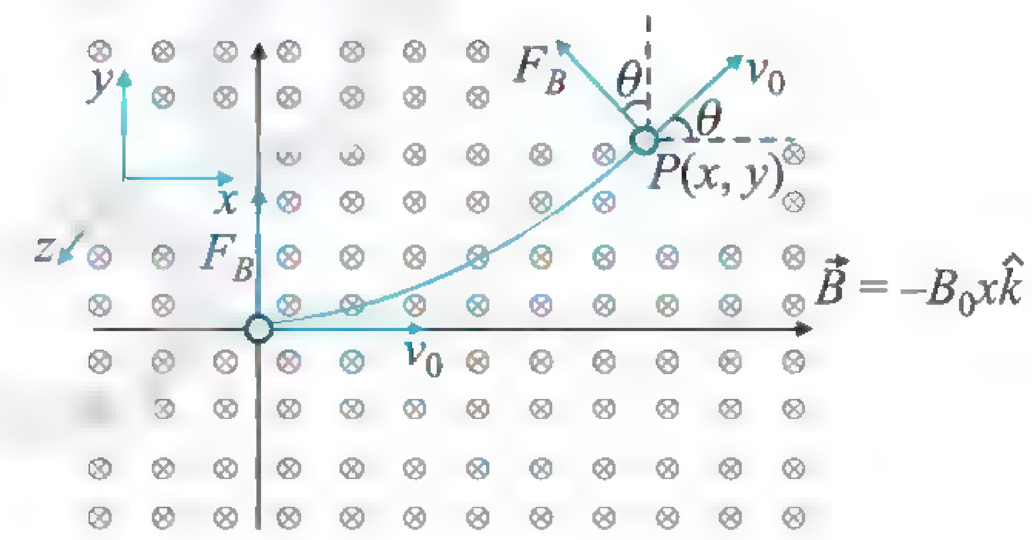
Motion of a Charged Particle in Non-Uniform Magnetic Field

It is known that a charged particle in a uniform magnetic field exhibits circular motion with specified frequency of revolution and specified radius which depends on the charge and mass of particle and magnitude magnetic field. But when a charged particle moves in non-uniform magnetic field, the path of the charged particle no longer remains circular. The magnetic field always applies force on the charged particle perpendicular to its motion, and the magnitude of this force is not constant. Hence, the particle moves in a curved path. In illustration given below, we will learn to analyze this type of motion.

ILLUSTRATION 1.27

A particle of charge q and mass m is projected from the origin with velocity $\vec{v} = v_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = -B_0x\hat{k}$. Here v_0 and B_0 are positive constants of proper dimensions. Find the maximum positive x-coordinate of the particle during its motion.

Sol. As the magnetic field is not constant, the particle will move in a curve path. Let at point $P(x, y)$ its velocity vector make an angle θ with positive x-axis.



Then the magnetic force acting on the particle will be at an angle θ with the positive y-direction.

So, $$a_y = \left(\frac{F_B}{m}\right)\cos\theta$$

$$a_y = \left(\frac{F_B}{m}\right)\cos\theta \text{ where } F_B = Bqv_0\sin 90° = (B_0x)qv_0$$

$$\frac{dv_y}{dt} = \frac{(B_0x)qv_0}{m}\cos\theta$$

or $$\left(\frac{dv_y}{dt}\right)\left(\frac{dx}{dx}\right) = \left(\frac{B_0qx}{m}\right)(v_0\cos\theta)$$

or $$\left(\frac{dv_y}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{B_0qx}{m}\right)(v_x)$$

Thus $$\frac{dv_y}{dx} = \left(\frac{B_0q}{m}\right)x \quad \left(\text{As } v_x = \frac{dx}{dt}\right)$$

$$\frac{dv_y}{dx} = \left(\frac{B_0q}{m}\right)x \Rightarrow dv_y = \left(\frac{B_0q}{m}\right)x\,dx \quad ...(i)$$

At maximum x-displacement, velocity is along $+y$-direction. As magnetic field do no work on moving charge hence its speed remains unchanged. Now integrating equation (i) both sides

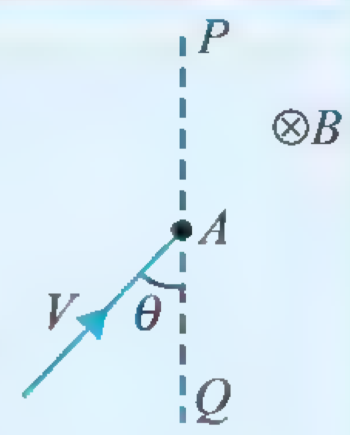
$$\int_0^{v_0} dv_y = \left(\frac{B_0q}{m}\right)\int_0^{x_{max}} x\,dx$$

or $$v_0 = \left(\frac{B_0q}{m}\right)\left(\frac{x_{max}^2}{2}\right)$$

or $$x_{max} = \sqrt{\frac{2mv_0}{B_0q}}$$

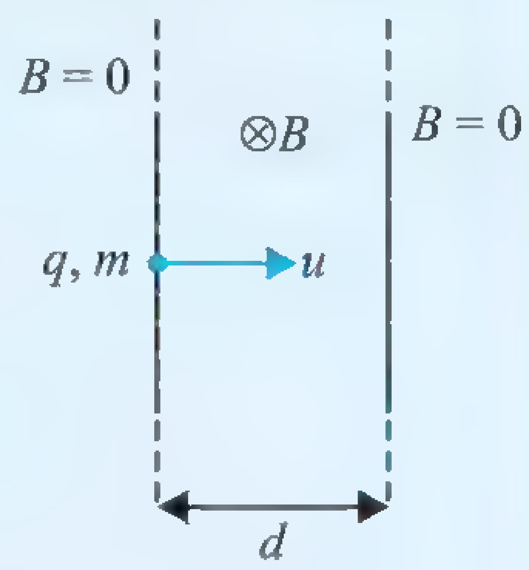
Concept Application Exercise 1.2

1. A positive charge particle of charge q and mass m enters into a uniform magnetic field with velocity v as shown in figure. There is no magnetic field to the left of PQ. Find **(a)** time spent, **(b)** distance travelled in the magnetic field, **(c)** impulse of magnetic force.



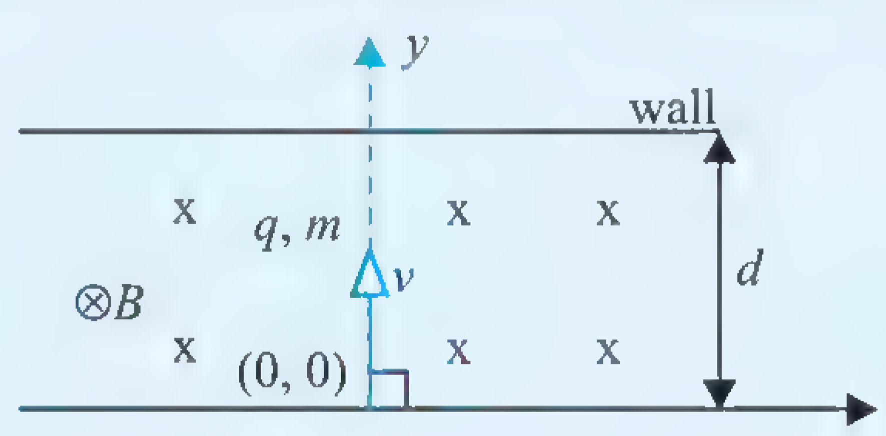
2. Repeat above Question 1, if the charge is negative and the angle made by the boundary with the velocity is $\theta = \pi/6$.

3. A uniform magnetic field of strength B exists in a region of width d. A particle of charge q and mass m is shot perpendicularly (as shown in figure) into the magnetic field. Find the time spent by the particle in the magnetic field if



(a) $d > \dfrac{mu}{qB}$ **(b)** $d < \dfrac{mu}{qB}$

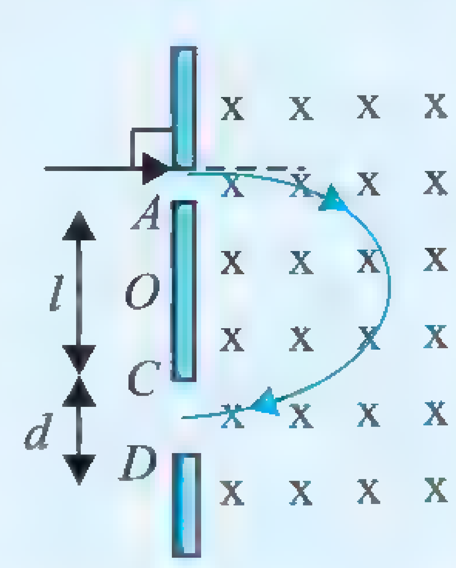
4. In figure, what should be the speed of the charged particle so that it cannot collide with the upper wall? Also, find the coordinates of the point where the particle strikes the lower plate in the limiting case of velocity.



5. A particle with charge 6.0×10^{-19} C travels in a circular orbit with radius 5.0 mm due to the force exerted on it by a magnetic field of magnitude 2.0 T and perpendicular to the orbit.
   (a) What is the magnitude of the linear momentum $\vec{p}$ of the particle?
   (b) What is the magnitude of the angular momentum $\vec{L}$ of the particle?

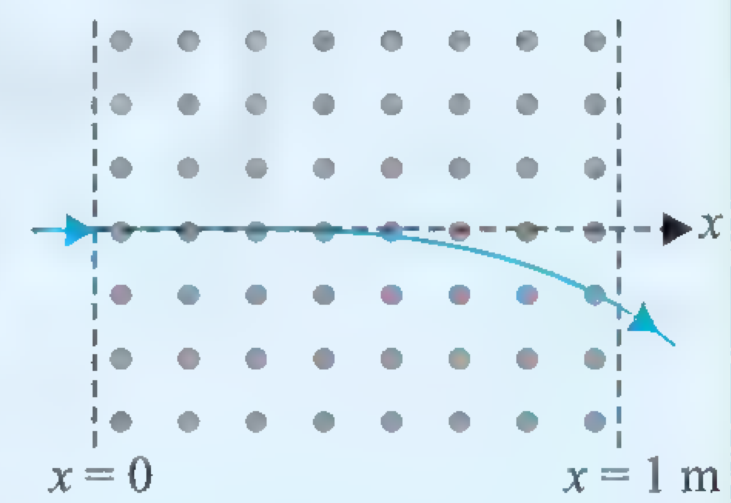
6. A particle of mass m and charge q is accelerated by a potential difference V volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field induction is B. What would be the speed of second particle so that both particles meet again and again after regular interval of time. Also, find the time interval after which they meet and the distance travelled by the second particle during that interval.

7. A beam of equally charged particles after being accelerated through a voltage V enters into a magnetic field B as shown in figure. It is found that all the particles hit the plate between C and D. Find the ratio between the masses of the heaviest and lightest particles of the beam.



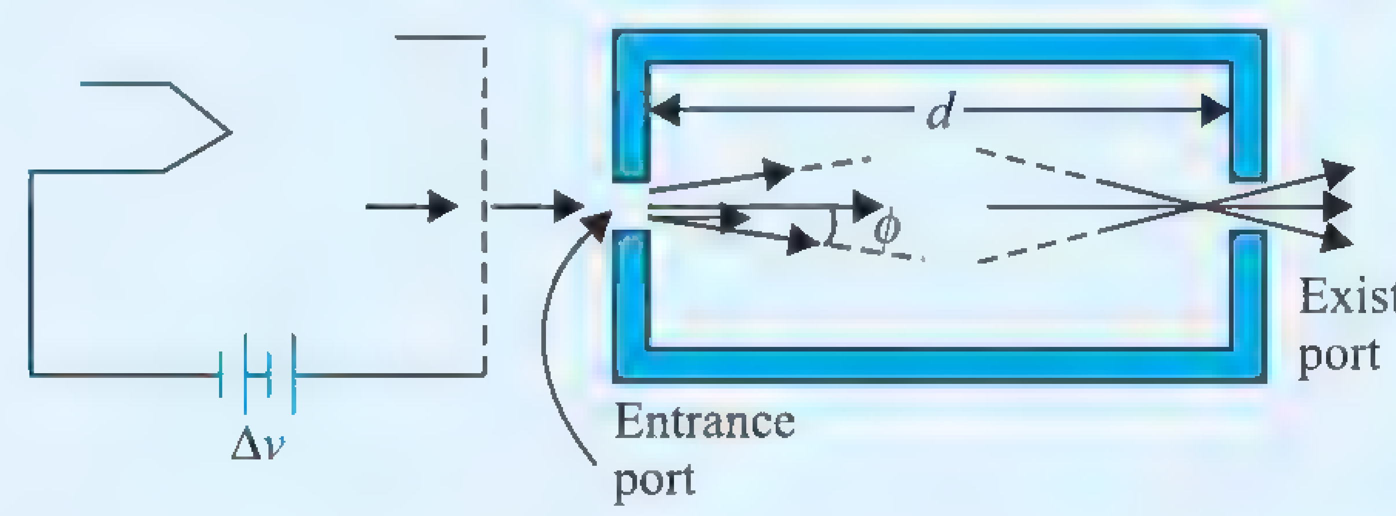
8. A proton and an alpha particle are projected in a magnetic field which exists in the width of region d. Compare the angles of deviation suffered by the proton and the alpha particle if before entering the magnetic field both the particles.
   (a) have the same momentum,
   (b) have the same kinetic energy, and
   (c) are accelerated through the same potential difference.
   Take $m_\alpha = 4m_p$, $q_\alpha = 2q_p$.

9. Protons having a kinetic energy of 50 eV are moving in the positive x-direction and enter a magnetic field $B = 0.5\ \text{mT}(\hat{k})$ directed out of the plane of the page and extending from $x = 0$ to $x = 1$ m as shown in figure.

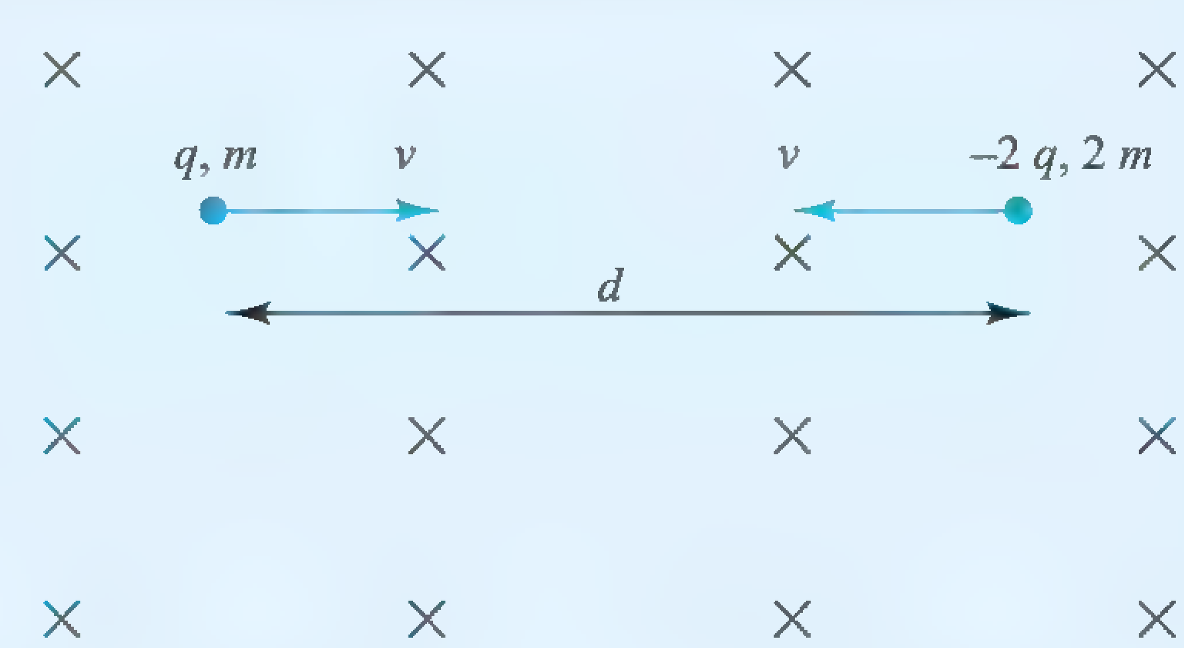


   (a) Calculate the y-component of the protons' momentum as they leave the magnetic field.
   (b) Find the angle ϕ between the initial velocity vector of the proton beam and the velocity vector after the beam emerges from the field. Ignore relativistic effects and note that 1 eV = 1.60×10^{-19} J and mass of proton $m_p = 1.6 \times 10^{-27}$ kg.

10. Electrons in a beam are accelerated from rest through a potential difference ΔV. The beam enters an experimental chamber through a small hole. As shown in figure, the electron velocity vector lie within a narrow cone of half angle ϕ oriented along the beam axis. We wish to use a uniform magnetic field directed parallel to the axis to focus the beam, so that all of the electrons can pass through a small exit port on the opposite side of the chamber after they travel the length d of the chamber. What is the required magnitude of the magnetic field?



11. A charged particle $+q$ of mass m is placed at a distance d from another charged particle $-2q$ of mass $2m$ in a uniform magnetic field B as shown in figure. If the particles are projected towards each other with same speed v,



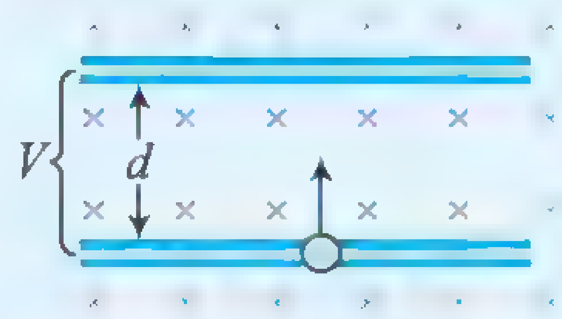
   (a) find the maximum value of projection speed v_m so that the two particles do not collide.

(b) find the time after which collision occurs between the particles if projection speed equals $2v_m$.

(c) Assuming the collision to be perfectly inelastic, find the radius of the particle in subsequent motion.

(Neglect the electric force between the charges.)

12. In the given figure, a charged particle of mass m, charge $-q$, and having low (negligible) speed enters the region between two plates of potential difference V and plate separation d, initially headed directly toward the top plate. A uniform magnetic field of magnitude B is normal to the plane of the paper. Find the minimum value of magnetic field (B) such that the electron will not strike the top plate.



Answers

1. (a) $\dfrac{m2\theta}{qB}$ (b) $\dfrac{mv}{qB}\times 2\theta$ (c) $-2mv\sin\theta\hat{i}$

2. (a) $\dfrac{5\pi m}{3qB}$ (b) $\dfrac{5\pi r}{3}$ (c) $-mv\hat{j}$

3. (a) $\dfrac{\pi m}{qB}$ (b) $\dfrac{m}{qB}\sin^{-1}\left(\dfrac{d}{R}\right)$

4. $\dfrac{qBd}{m}$; $(-2d, 0, 0)$

5. (a) 6.0×10^{-21} kgms^{-1} (b) 3.0×10^{-23} kgm^{2}s^{-1}

6. $\sqrt{\dfrac{2qV}{m}}\cos\theta$; $\dfrac{2\pi m}{qB}$; $\pi\sqrt{\dfrac{8Vm}{qB^2}}\cos\theta$

7. $\left(\dfrac{l+d}{l}\right)^2$

8. (a) $\dfrac{1}{2}$ (b) 1 (c) $\sqrt{2}$

9. (a) 8.0×10^{-23} kg m/s (b) 30°

10. $\dfrac{2\pi m}{qd}\left(\sqrt{\dfrac{2\Delta Ve}{m}}\right)\cos\phi$

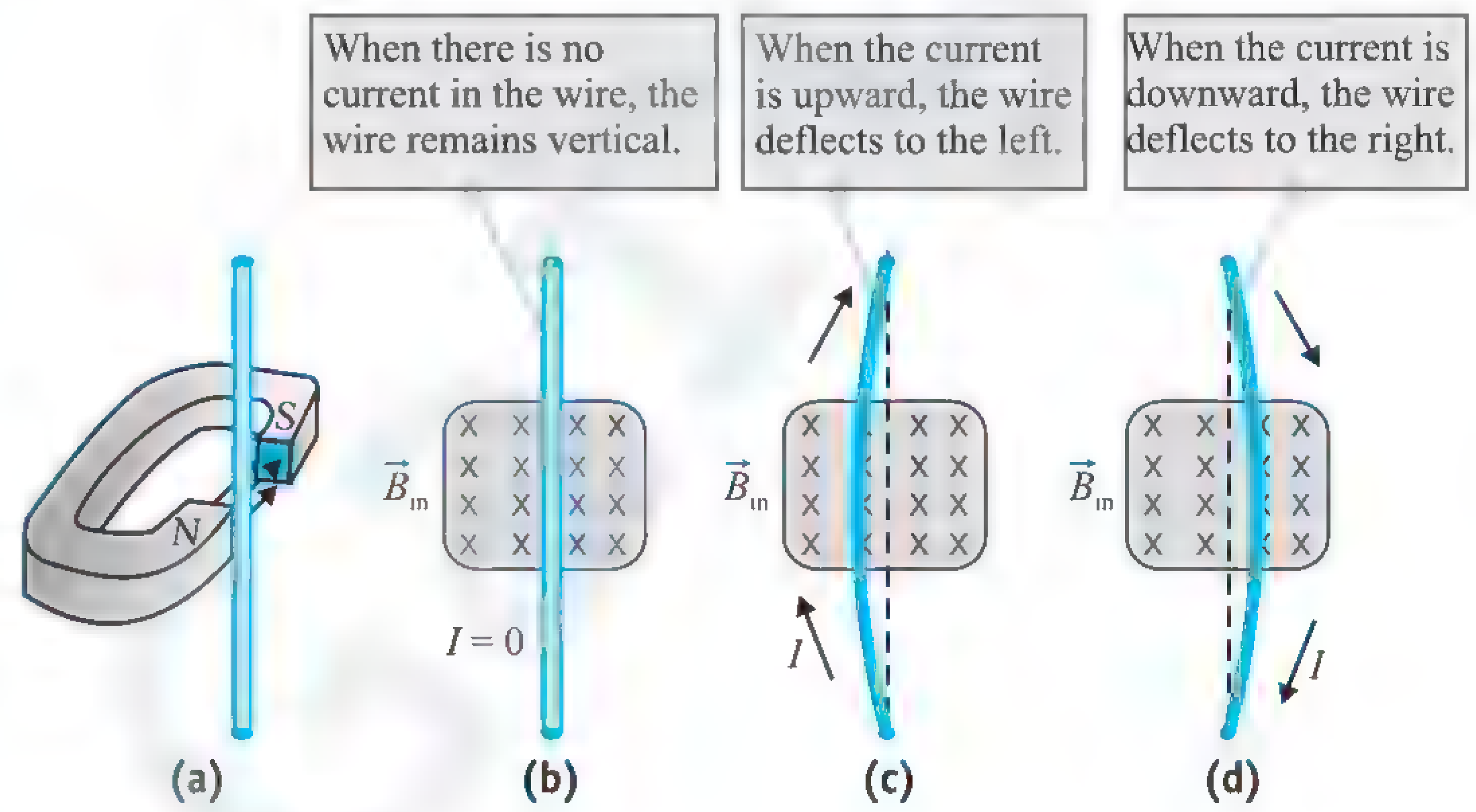
11. (a) $\dfrac{Bqd}{2m}$ (b) $\dfrac{\pi m}{6qB}$ (c) $\sqrt{3}d$

12. $\sqrt{\dfrac{mV}{2qd^2}}$

FORCE ON A CURRENT-CARRYING WIRE

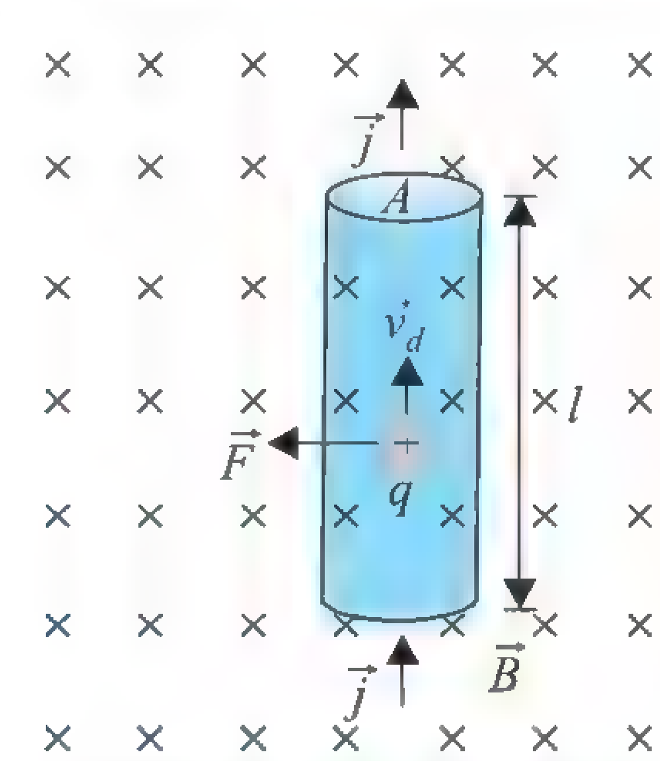
Magnetic Force Acting on a Current Carrying Conductor

If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field. The current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire. One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet as shown in figure. For ease in visualization, part of the horseshoe magnet in part (a) is removed to show the end face of the south pole in parts (b) through (d) of figure. The magnetic field is directed into the page and covers the region within the shaded squares. When the current in the wire is zero, the wire remains vertical as in Fig. (b). When the wire carries a current directed upward as in Fig. (c), however, the wire deflects to the left. If the current is reversed as in Fig. (d), the wire deflects to the right.



Let us quantify this discussion by considering a straight segment of wire of length l and cross-sectional area A carrying a current I is placed uniform magnetic field $\vec{B}$.

The figure below shows a straight segment of a conducting wire, with length l and cross-sectional area A; the current is from bottom to top. The wire is in a uniform magnetic field $\vec{B}$, perpendicular to the plane of the diagram and directed into the plane. Let us assume first that the moving charges are positive.

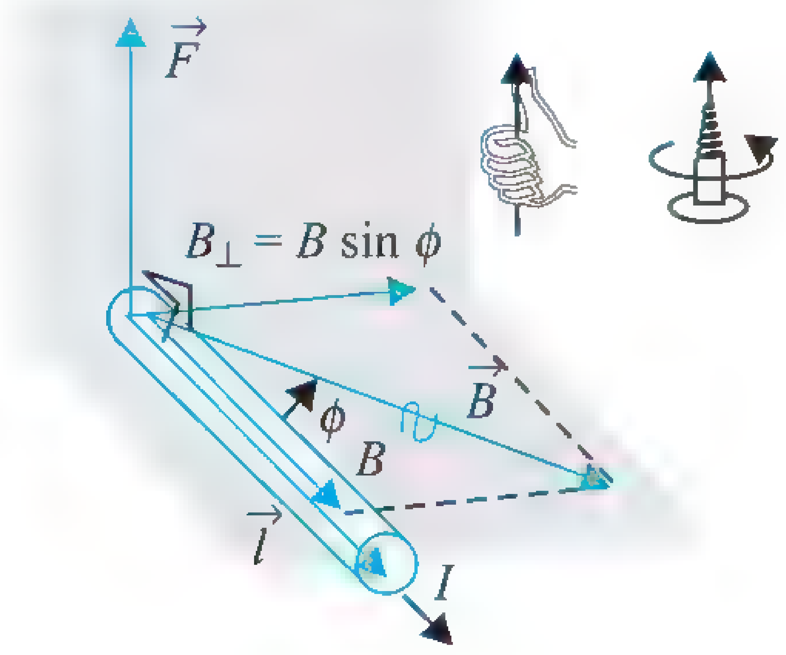


Forces on a moving positive charge in a current carrying conductor.

The drift velocity $\vec{v}_d$ is upward, perpendicular to $\vec{B}$. The average force on each charge is $\vec{F} = q\vec{v}_d \times \vec{B}$, directed to the left as shown in the figure; since $\vec{v}_d$ and $\vec{B}$ are perpendicular, the magnitude of the force is $F = qv_dB$.

We can derive an expression for the total force on all the moving charges in a length l of a conductor with cross-sectional area A. The number of charges per unit volume is n; a segment of conductor with length l has volume Al and contains a number of charges equal to nAl. The total force $\vec{F}$ on all the moving charges in this segment has magnitude

$$F = (nAl)(qv_dB) = (nqv_dA)(lB) \qquad \text{...(i)}$$



A straight wire segment of length $\vec{l}$ carries a current I in the direction of $\vec{l}$. The magnetic force on this segment is perpendicular to both $\vec{l}$ and the magnetic field $\vec{B}$.

The current density is $J = nqv_d$. The product JA is the total current I, so we can rewrite Eq. (i) as,

$$F = IlB \quad \text{...(ii)}$$

If the field $\vec{B}$ is not perpendicular to the wire but makes an angle ϕ with it, we handle the situation the same way we did for a single charge. Only the component of $\vec{B}$ perpendicular to the wire (and to the drift velocities of the charges) exerts a force; this component is $B_{\perp} = B \sin \phi$. The magnetic force on the wire segment is then

$$F = IlB_{\perp} = IlB \sin \phi \quad \text{...(iii)}$$

The force is always perpendicular to both the conductor and the field, with the direction determined by the same right hand rule we used for a moving positive charge (as shown in figure). Hence, this force can be expressed as a vector product, just like the force on a single moving charge. We represent the segment of wire with a vector $\vec{l}$ along the wire in the direction of the current; then the force $\vec{F}$ on this segment is

$$\vec{F} = I\vec{l} \times \vec{B} \text{ (magnetic force on a straight wire segment)} \quad \text{...(iv)}$$

If the conductor is not straight, we can divide it into infinitesimal segments $d\vec{l}$. The force $d\vec{F}$ on each segment is

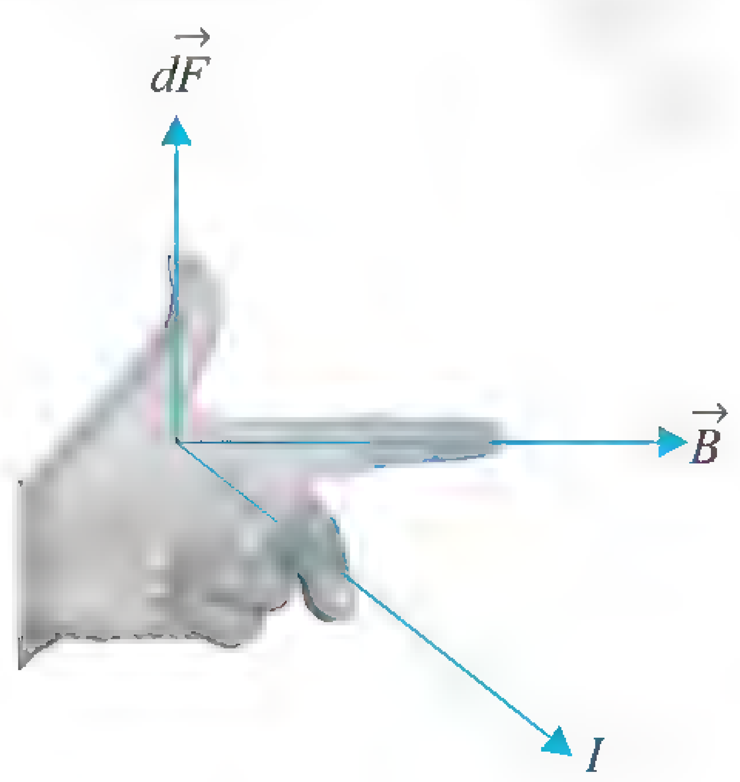
$$d\vec{F} = I\vec{l} \times \vec{B} \text{ (magnetic force on an infinitesimal wire section)}$$

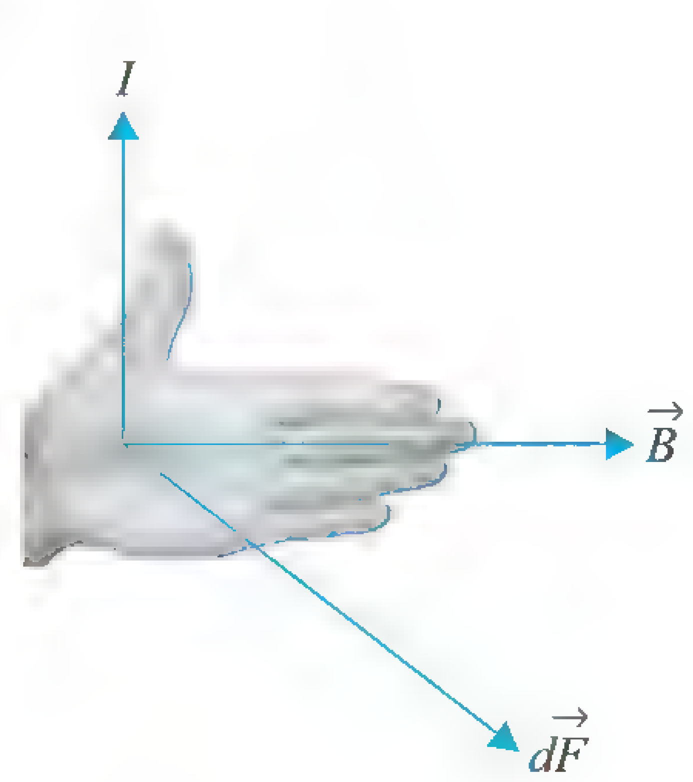
Integrating this, we get total force $\vec{F} = \int I d\vec{l} \times \vec{B}$ ...(v)

Direction of Force on a Current-Carrying Wire in Magnetic Field

Left Hand Rule

If the thumb and first two fingers of the left hand are held each at right angles to the other, with the first finger pointing in the direction of the field and the second finger in the direction of the current, then the thumb predicts the direction of the thrust or force (figure).





Right Hand Palm Rule

Stretch the fingers and thumb of right hand at right angles to each other. If the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current I, the normal to palm will point in the direction of force (or motion).

Regarding the force on a current carrying conductor in a magnetic field, it is worth mentioning that:

As the force $BI\,dL \sin \theta$ is not a function of position r, the magnetic force on a current element is non-central [a central force is of the form $F = Kf(r)\vec{n}_r$]

The force $d\vec{F}$ is always perpendicular to both $\vec{B}$ and $I\,d\vec{L}$ though $\vec{B}$ and $I\,d\vec{L}$ may or may not be perpendicular to each other.

ILLUSTRATION 1.28

A wire of length l carries a current i along the X-axis. A magnetic field exists which is given as

$$\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})\text{ T}$$

Find the magnitude of the magnetic force acting on the wire.

Sol. Here current is passing along X-axis

So $\vec{F} = i\vec{l} \times \vec{B} = i\{(l\hat{i}) \times (B_0\hat{i} + B_0\hat{j} + B_0\hat{k})\}$

$$= i\{lB_0\hat{k} - lB_0\hat{j}\} = ilB_0\{-\hat{j} + \hat{k}\}$$

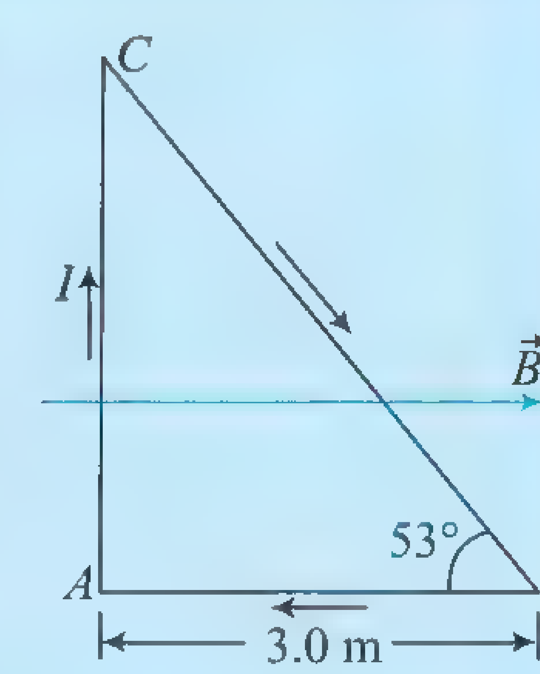
$$|\vec{F}| = ilB_0\sqrt{1+1} = \sqrt{2}\, ilB_0$$

ILLUSTRATION 1.29

A loop of wire has the shape of a right triangle (see the drawing) and carries a current of I = 5.0 A. A uniform magnetic field is directed parallel to side AB and has a magnitude of 2.0 T.

(a) Find the magnitude and direction of the magnetic force exerted on each side of the triangle;

(b) Determine the magnitude of the net force exerted on the triangle.



Sol. The magnitude of the magnetic force exerted on a long straight wire is given by equation, $F = ILB \sin \theta$. The direction of the magnetic force is predicted by right-hand rule. The net force on the triangular loop is the vector sum of the forces on the three sides.

(a) The direction of the current inside AB is opposite to the direction of the magnetic field, so the angle θ between them is $\theta = 180°$.

The magnitude of the magnetic force,

$$F_{AB} = ILB\sin\theta = ILB\sin 180° = 0 \text{ N}$$

For the side BC, the angle is $\theta = 53°$, and the length of the side,

$$L = \frac{3.0 \text{ m}}{\cos 53°} = 5.0\,\text{m}$$

The magnetic force is

$$F_{BC} = ILB\sin\theta = (5.0)(5.0)(2.0)\sin 53° = 40.0\,\text{N}$$

An application of right-hand rule shows that the magnetic force on side BC is directed perpendicularly out of the paper, toward $+z$ direction.

For the side AC, the angle is $\theta = 90°$. We see that the length of the side is $L = (3.0)\tan 53° = 4.0$ m

The magnetic force is

$$F_{AC} = ILB\sin\theta$$
$$= (5.0)(4)(2.0)\sin 90° = 40.0\,\text{N}$$

An application of right-hand no. 1 shows that the magnetic force on side AC is directed perpendicularly into the paper, toward $-z$ direction.

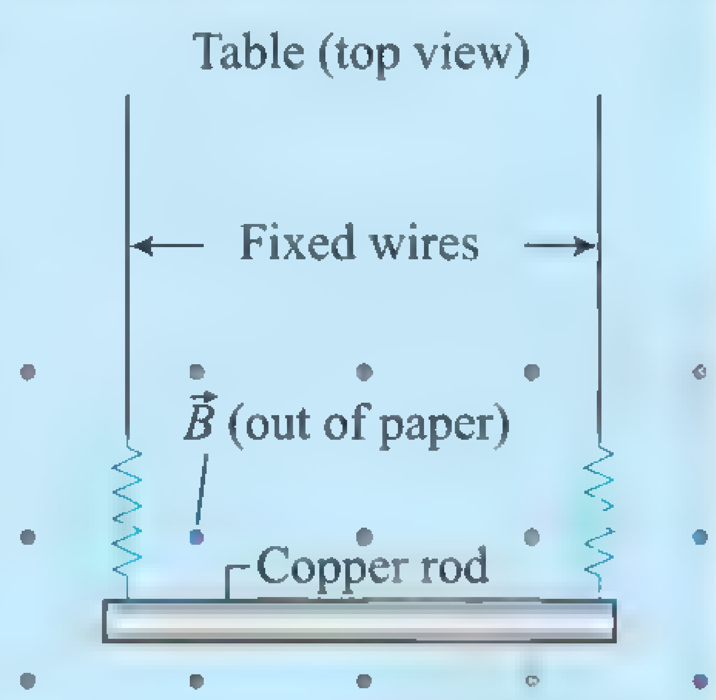
(b) The net force is the vector sum of the forces on the three sides. Taking the positive direction as being out of the paper,

The net force is

$$\Sigma F = 0\text{ N} + 40.0\text{ N} + (-40.0\text{ N}) = 0\text{ N}$$

ILLUSTRATION 1.30

A copper rod of length 1.0 m is lying on a frictionless table (see the drawing). Each end of the rod is attached to a fixed wire by an unstretched spring that has a spring constant of k =50 N/m. A magnetic field with a strength of 0.20 T is oriented perpendicular to the surface of the table. **(a)** What must be the direction of the current in the copper rod that causes the springs to stretch? **(b)** If the current is 10 A, by how much does each spring stretch?



Sol.

(a) From right-hand rule, if we extend the right hand so that the fingers point in the direction of the magnetic field, and the thumb points in the direction of the current, the palm of the hand faces the direction of the magnetic force on the current.

The springs will stretch when the magnetic force exerted on the copper rod is downward, toward the bottom of the page. Therefore, if you extend your right hand with your fingers pointing out of the page and the palm of your hand facing the bottom of the page, your thumb points left-to-right along the copper rod. Thus, the current flows left-to-right in the copper rod.

(b) The downward magnetic force exerted on the copper rod is,

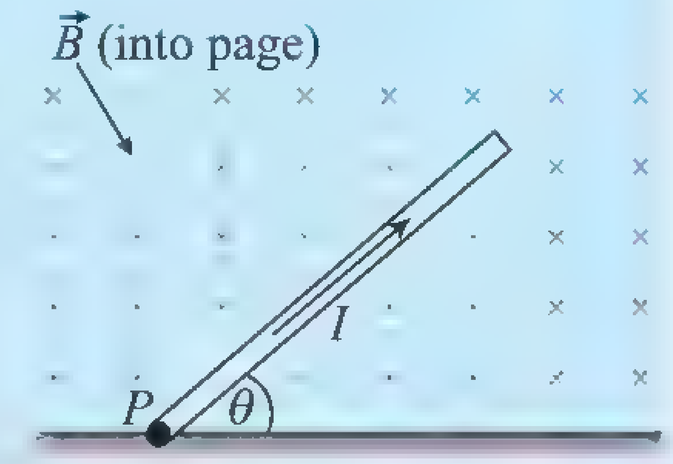
$$F = ilB\sin\theta$$
$$= (10)(1.0)(0.20)\sin 90° = 2.0\text{ N}$$

The force required to stretch each spring is $F = kx$,where k is the spring constant. Since there are two springs, the magnetic force F exerted on the rod must equal $F = 2kx$. Solving for x, we find that

$$x = \frac{F}{2k} = \frac{2.0\text{ N}}{2(50\text{ N/m})} = 2.0\times10^{-2}\text{ m}$$

ILLUSTRATION 1.31

The drawing shows a thin, uniform rod that has a length of 0.50 m and a mass of 0.20 kg. This rod lies in the plane of the paper and is attached to the floor by a hinge at point P.A uniform magnetic field of 0.50 T is directed perpendicularly into the plane of the paper. There is a current I = 4.0 A in the rod, which does not rotate clockwise or counter clockwise. Find the angle.



Sol. Since the rod does not rotate about the axis at P, the net torque relative to that axis must be zero; $\Sigma\tau = 0$. There are two torques that must be considered, one due to the magnetic force and another due to the weight of the rod. We consider both of these to act at the rod's centre of gravity, which is at the geometrical center of the rod (length = L), because the rod is uniform. According to right-hand rule, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Therefore, the magnetic torque is a counter clockwise(positive) torque. The magnitude F of the magnetic force as $F = ILB$ sin 90.0°, since the current is perpendicular to the magnetic field. The weight is mg and acts downward, producing a clockwise (negative) torque.

Setting the sum of these torques equal to zero will enable us to find the angle θ that the rod makes with the ground.

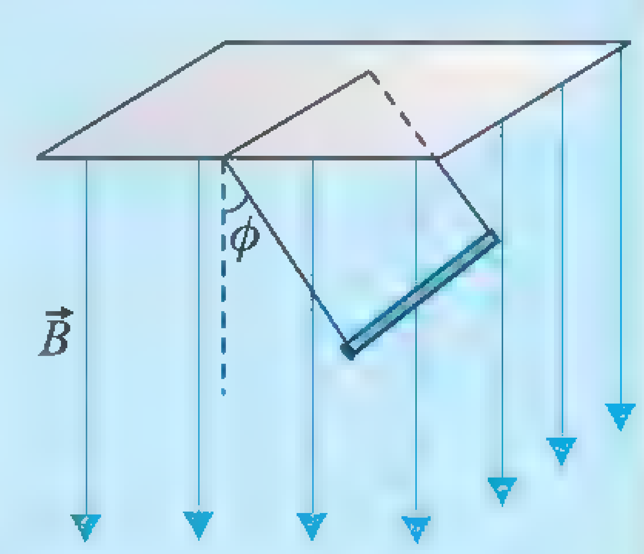
$$\Sigma\tau = \tau_{\text{magnetic}} + \tau_{\text{weight}} = 0$$

$$(ILB)(L/2) - (mg)[(L/2)\cos\theta] = 0 \text{ or } \cos\theta = \frac{ILB}{mg}$$

$$\cos\theta = \frac{(4.0)(0.50)(0.50)}{(0.20)(10)} = \frac{1}{2} \Rightarrow \theta = 60°.$$

ILLUSTRATION 1.32

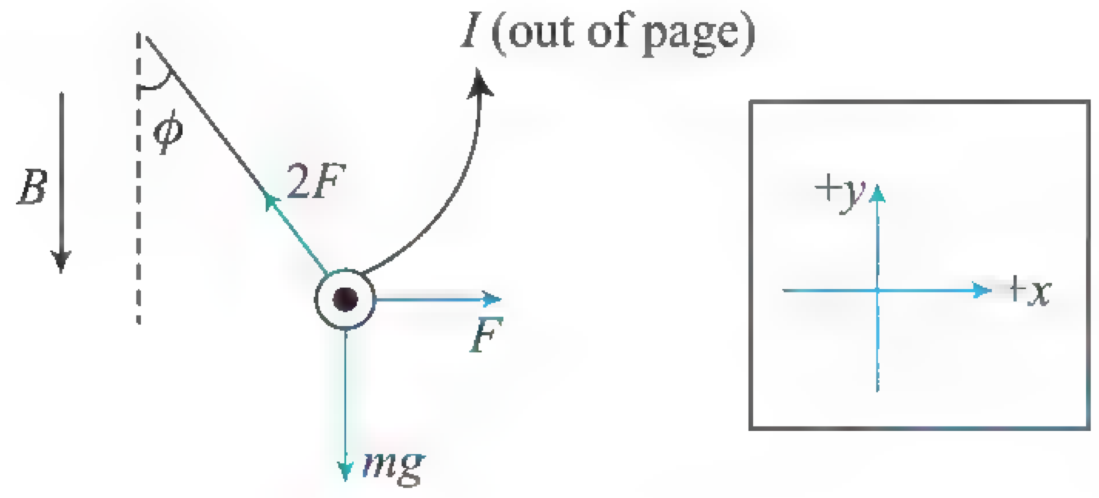
A horizontal wire is hung from the ceiling of a room by two massless strings. The wire has a length of 0.20 m and a mass of 0.080 kg. A uniform magnetic field of magnitude 0.075 T is directed from the ceiling to the floor. When a current of I = 40 A exists in the wire, the wire swings upward and, at equilibrium, makes an angle ϕ with respect to the vertical, as the drawing shows. Find **(a)** the angle ϕ and **(b)** the tension in each of the two strings.



Sol. There are four forces that act on the wire: the magnetic force (magnitude F), the weight mg of the wire, and the tension in each of the two strings (magnitude T in each string). Since there are two strings, the following drawing shows the total tension as $2T$. The magnitude F of the magnetic force is given by $F = ILB$ sin θ, where I is the current, L is the length of the wire, B is the magnitude of the magnetic field, and θ is the angle between the wire and the magnetic field. In this problem $\theta = 90°$.

(a) The direction of the magnetic force is given by right-hand rule. The drawing shows an end view of the wire, where it

can be seen that the magnetic force (magnitude = F) points to the right, in the $+x$ direction.



In order for the wire to be in equilibrium, the net force ΣF_x and ΣF_y must be zero.

$$\Sigma F_x = 0 \Rightarrow 2T\sin\phi = F_B = ILB\sin 90° \qquad \text{...(i)}$$

$$\Sigma F_y = 0 \Rightarrow 2T\cos\phi = mg \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\sin\phi}{\cos\phi} = \tan\phi = \frac{ILB}{mg} = \frac{(40)(0.20)(0.075)}{(0.080)(10)} = \frac{3}{4}$$

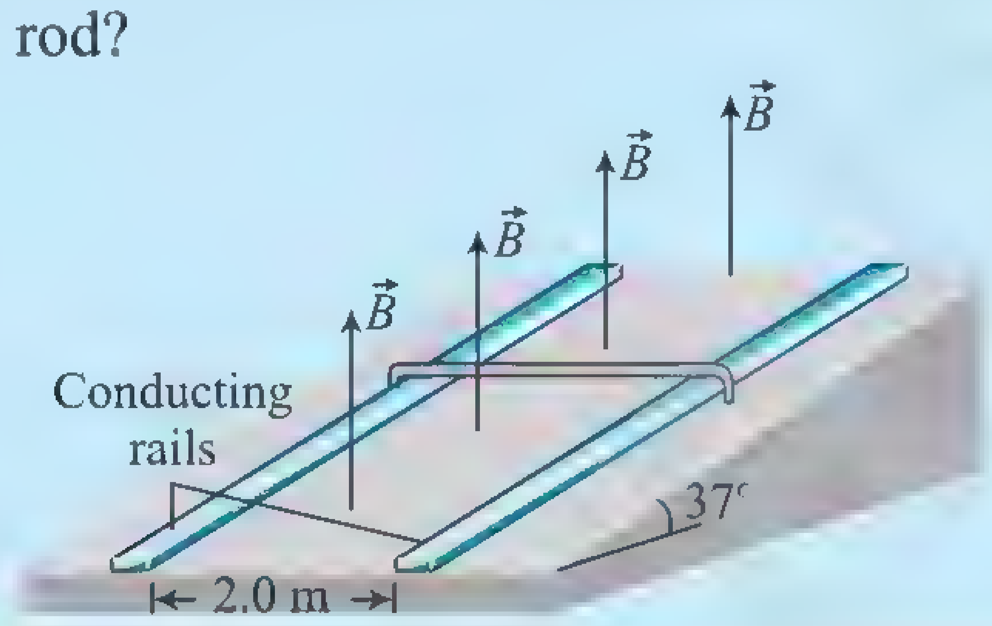
Hence, $\phi = 37°$

(b) The tension in each wire can be found directly from Eq. (ii):

$$T = \frac{mg}{2\cos\phi} = \frac{(0.080\text{ kg})(10\text{ m/s}^2)}{2\cos 37°} = 0.50\text{ N}$$

ILLUSTRATION 1.33

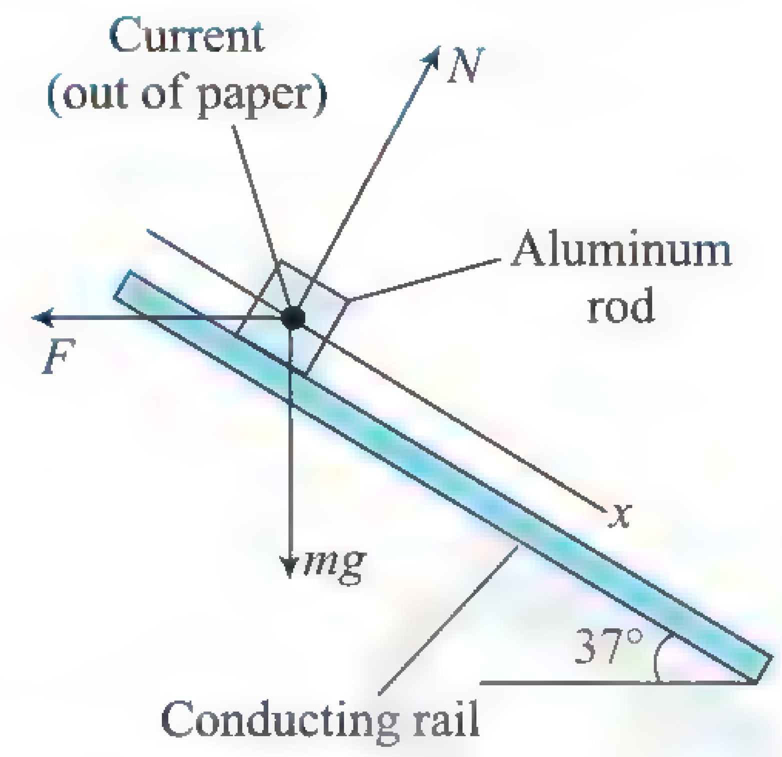
Two conducting rails in the drawing are tilted upward so they each make an angle of 37° with respect to the ground. The vertical magnetic field has a magnitude of 0.050 T. The 0.20-kg aluminium rod (length = 2.0 m) slides without friction down the rails at a constant velocity. How much current flows through the rod?



Sol. The following drawing shows a side view of the conducting rails and the aluminum rod. Three forces act on the rod: (1) its weight mg, (2) the magnetic force F, and the normal force N. An application of right-hand rule shows that the magnetic force is directed to the left, as shown in the drawing. Since the rod slides down the rails at a constant velocity, its acceleration is zero. If we choose the x-axis to be along the rails, the net force along the x-direction is zero: $\Sigma F_x = ma_x = 0$. Using the components of F and mg that are along the x-axis,

$$F\cos 37° = mg\sin 37° \qquad \text{...(i)}$$

The magnetic force is given by Eq. (i) as $F = ILB\sin\theta$, where $\theta = 90°$ is the angle between the magnetic field and the current. We can use these two relations to find the current in the rod.



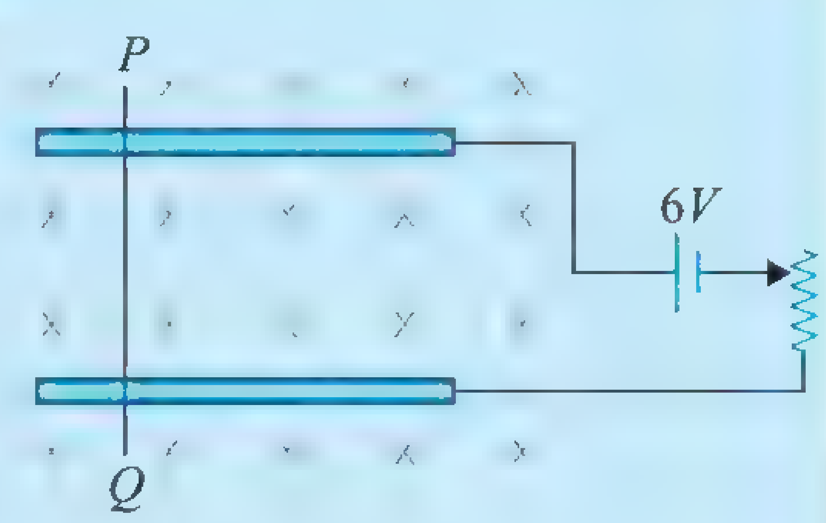
Substituting the expression $F = ILB\sin 90°$ into Newton's second law and solving for the current I, we obtain

$$I = \frac{mg\sin 37°}{(LB\sin 90°)\cos 37°}$$

$$= \frac{(0.20\text{ kg})(10\text{ m/s}^2)\sin 37°}{(2.0\text{ m})(0.050\text{ T})\sin 90°\cos 37°} = 15\text{ A}$$

ILLUSTRATION 1.34

A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 5 cm. A vertically downward magnetic field of magnitude 0.800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction.



Sol. The wire starts sliding when magnetic force on the wire overcomes friction force.

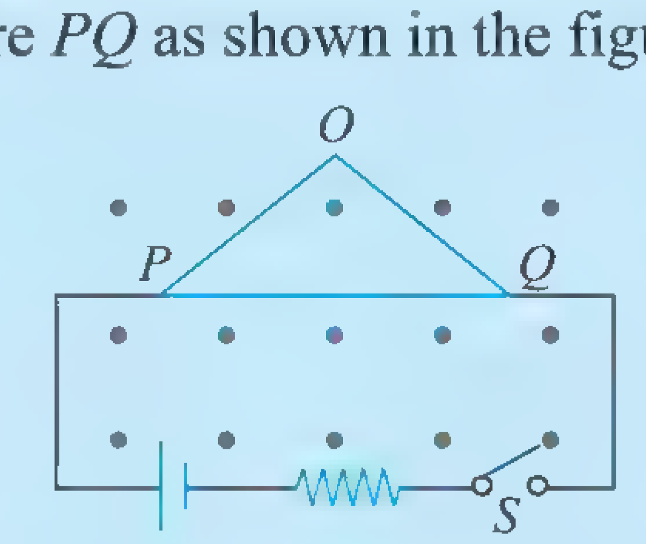
$$\mu N = F \quad \Rightarrow \quad \mu mg = ilB$$

$$\mu \times 10 \times 10^{-3} \times 10 = \frac{6}{20} \times 5 \times 10^{-2} \times 0.8$$

$$\mu = \frac{6 \times 5 \times 10^{-2} \times 0.8}{20 \times 10 \times 10^{-3} \times 10} = 0.12$$

ILLUSTRATION 1.35

Figure (a) shows a rod PQ of length 20 cm and mass 200 g suspended through a fixed point O by two threads of lengths 20 cm each. A magnetic field of strength 0.500 T exists in the vicinity of the wire PQ as shown in the figure.



The wire connecting PQ with the battery are loose and exert no force on PQ.

(a) Find the tension in the threads when the switch S is open.

(b) A current of 2 A is established when the switch S is closed. Find the tension in the threads now.

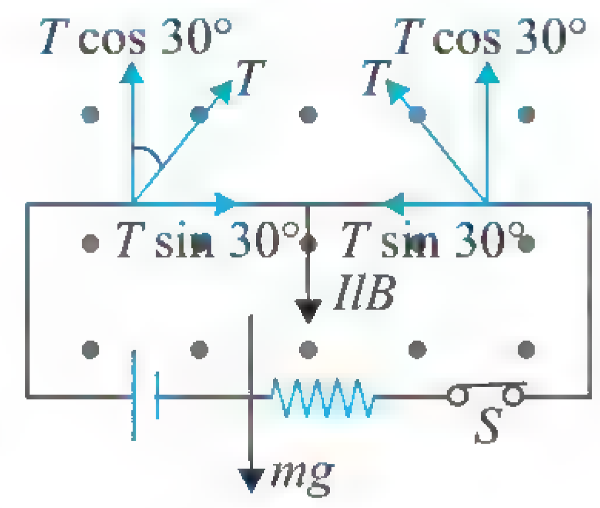
Sol.

(a) When switch S is open

$$2T\cos 30^\circ = mg$$

$$\Rightarrow\ T = mg/2\cos 30^\circ$$

$$= \frac{2}{\sqrt{3}} = 1.13\text{ N}$$



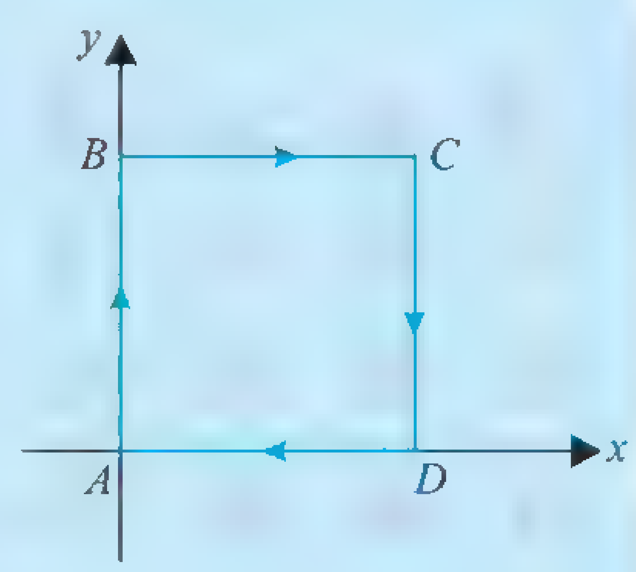
(b) When the switch is closed and a current passed through the circuit $= 2$ A

Then, $2T'\cos 30^\circ = mg + ilB$

$$2T'\left(\frac{\sqrt{3}}{2}\right) = 2.2 \ \Rightarrow\ T' = \frac{2.2}{\sqrt{3}}\text{ N}$$

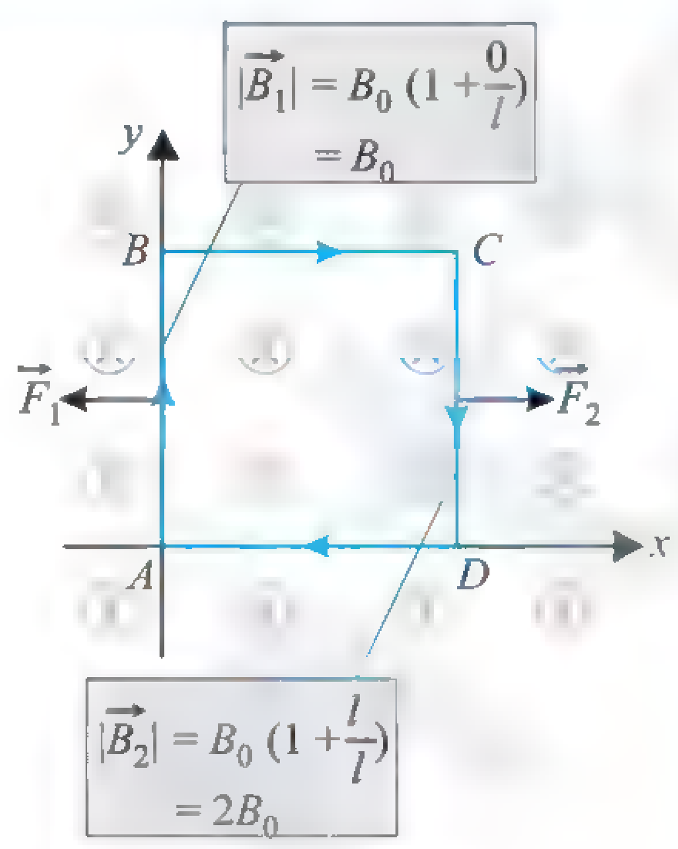
ILLUSTRATION 1.36

A square loop of edge l and carrying a current i, is placed with its edges parallel to the XY axes. There is present a non-uniform magnetic field in the region $\vec{B} = B_o\left(1+\frac{x}{l}\right)(-\hat{k})$. Find the magnitude of the net magnetic force experienced by the loop.



Sol. The magnetic field in the region

$$B = B_0\left(1+\frac{x}{l}\right)\vec{k}$$



Force on AB,

$$F_1 = iB_0\left(1+\frac{0}{l}\right)l = iB_0 l$$

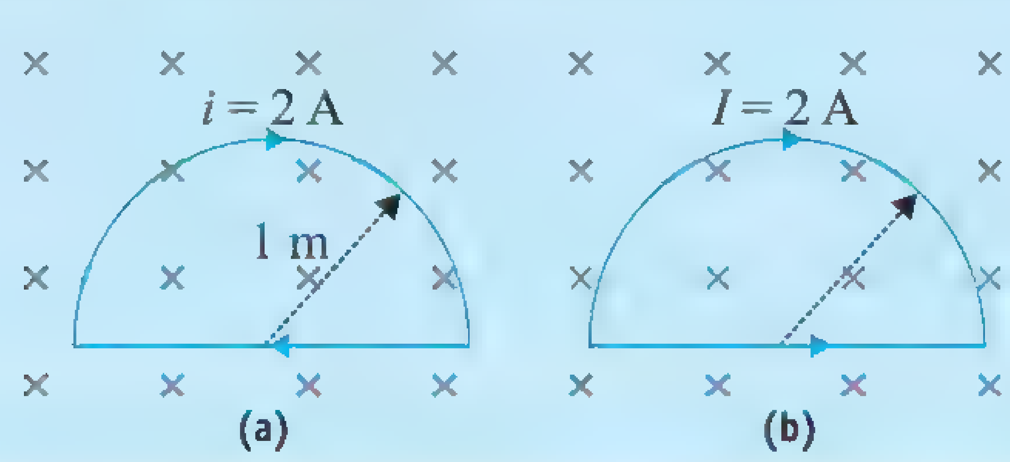
Force on CD,

$$F_2 = iB_0\left(1+\frac{l}{l}\right)l = iB_0\cdot 2l$$

The forces due to wires BC and AD will be equal and opposite. Hence, the force on BC and AD will cancel. Net force, $F_2 - F_1 = iB_0 l$

ILLUSTRATION 1.37

In figure, a semicircular wire loop is placed in uniform magnetic field $B = 1.0$ T. The plane of the loop is perpendicular to the magnetic field. Current $i = 2$ A flows in the loop in the direction shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is 1 m.



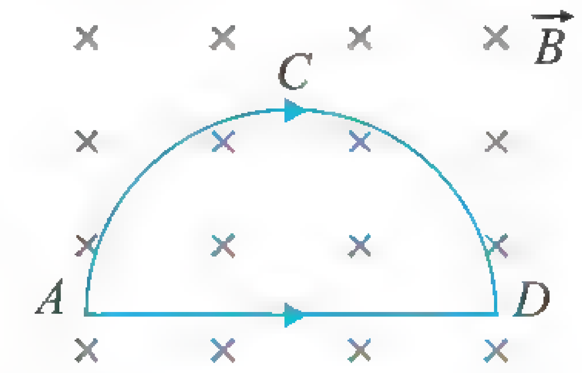
Sol. Refer Fig. (a): It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

Refer Fig. (b): In this case although it forms a closed loop, but current does not complete the loop. Hence, net force is not zero.

$$\vec{F}_{ACD} = \vec{F}_{AD}$$

$$\therefore\ \vec{F}_{\text{loop}} = \vec{F}_{ACD} + \vec{F}_{AD} = 2\vec{F}_{AD}$$

$$\therefore\ |\vec{F}_{\text{loop}}| = 2|\vec{F}_{AD}|$$



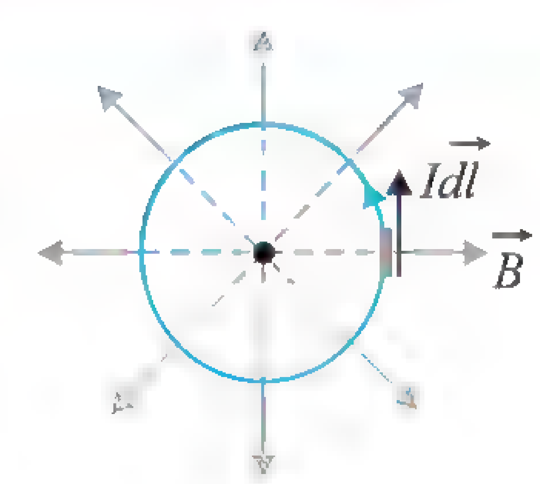
$$= 2ilB\sin\theta\ (l = 2r = 2.0\text{ m})$$

$$= (2)\,(2)\,(2)\,(1)\sin 90^\circ = 8\text{ N}$$

ILLUSTRATION 1.38

A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The center of the loop coincides with the center of the field. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire.

Sol. Let us select a current element on circular loop $I\vec{dl}$. The magnetic field in radial direction will be perpendicular to current element vector. From left hand rule or right palm rule, we can find the direction of force $d\vec{F}$ on the element which comes to be perpendicular into the plane of the paper.



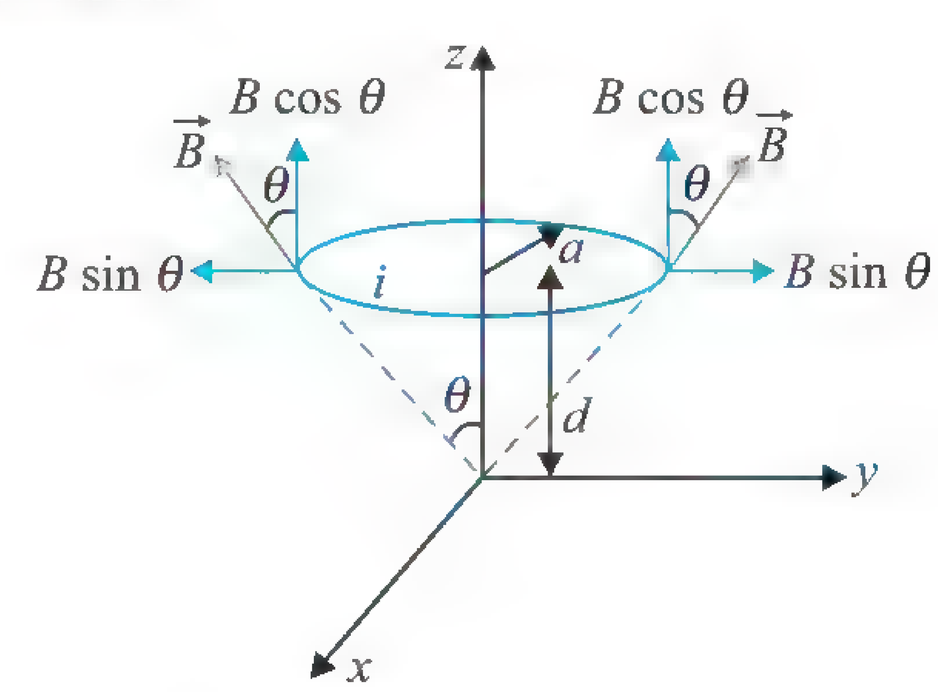
$$\text{Required force } F = \int dF = \int idlB\sin 90^\circ = \int IdlB$$

$$= 2\pi aiB \text{ perpendicular to the plane.}$$

ILLUSTRATION 1.39

A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\vec{e}_r$ where $\vec{e}_r$ denotes the unit vector along the radial direction. A circular loop of radius a, carrying a current i, is placed with its plane parallel to the X-Y plane and the center at $(0, 0, d)$. Find the magnitude of the magnetic force acting on the loop.

Sol. Magnetic force due to $B\cos\theta$ will be cancelled out. There will be no force due to magnetic field due to $B\cos\theta$ in downward direction.



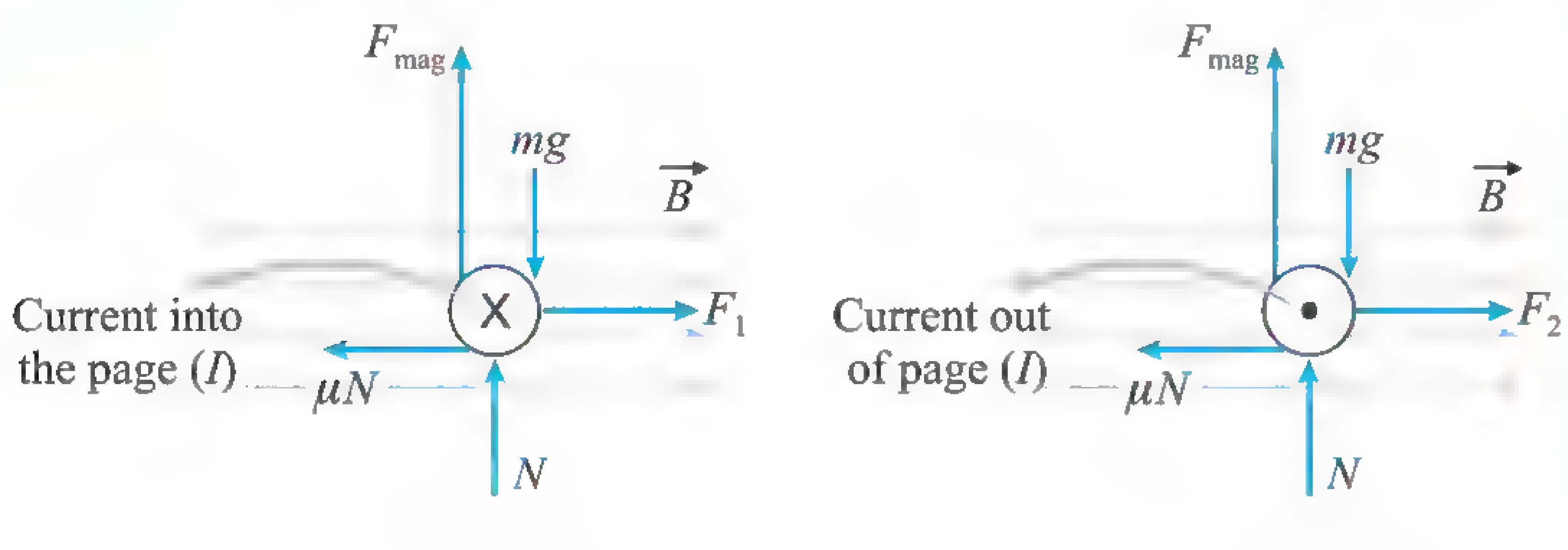
Required magnetic force

$$F = \int dF = \int_0^{2\pi a} idlB\sin\theta = \frac{2\pi i a^2 B}{\sqrt{a^2+d^2}}$$

ILLUSTRATION 1.40

A conducting wire of length l is placed on a rough horizontal surface, where a uniform horizontal magnetic field B perpendicular to the length of the wire exists. Least values of the forces required to move the rod when a current I is established in the rod are observed to be F_1 and F_2 $(< F_1)$ for the two possible directions of the current through the rod, respectively. Find the weight of the rod and the coefficient of friction between the rod and the surface.

Sol. Changing the direction of current in the wire, we can change the normal reaction on the wire by the surface.



(a) **(b)**

In one case, magnetic force on the wire will be in upward direction while in the other case it will be in the downward direction. Hence, normal reaction, $N = mg \pm Bil$.

$$f_{\text{(friction limiting)}} = \mu(mg \pm Bil) \text{ as } F_1 > F_2$$

$$F_1 = \mu(mg + Bil) \quad \text{...(i)}$$

and $$F_2 = \mu(mg - Bil) \quad \text{...(ii)}$$

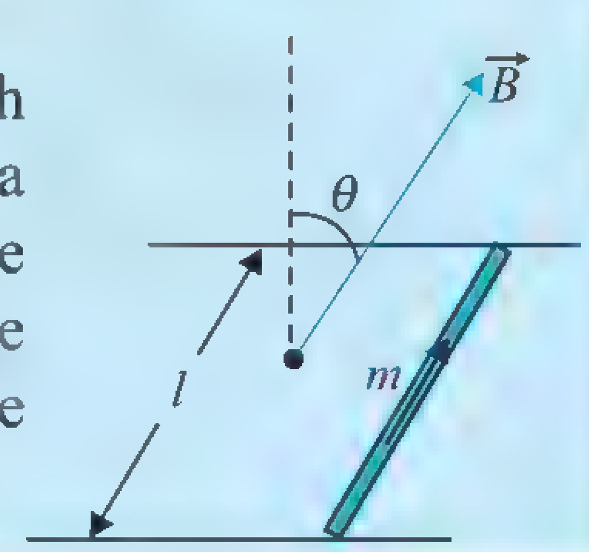
From Eqs (i) and (ii), $$\frac{F_1}{F_2} = \frac{mg + Bil}{mg - Bil}$$

$$mg = Bil\left[\frac{F_1 + F_2}{F_1 - F_2}\right] \quad \text{...(iii)}$$

From Eqs. (i) and (iii), we get $\mu = \dfrac{F_1 - F_2}{2Bil}$

ILLUSTRATION 1.41

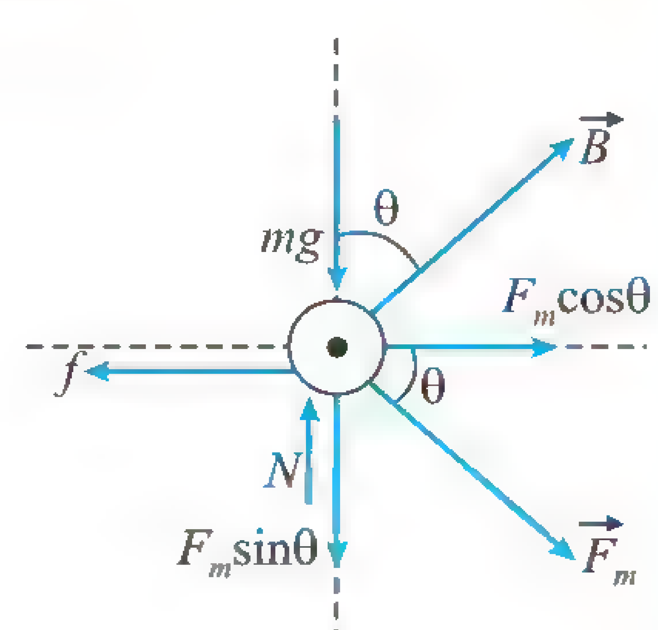
A conductor (rod) of mass m, length l carrying a current i is subjected to a magnetic field of induction B. If the coefficients of friction between the conducting rod and rail is μ, find the value of i if the rod starts sliding.



Sol. The magnetic force is $F_m = ilB \sin \varphi$, where φ_B = Angle between current and magnetic field = 90°.

or $F_m = ilB$

but it acts at an angle θ with horizontal as shown in figure according to Fleming's left hand rule or right palm rule.



Then the net force along x- and y-axes are

$$F_y = N - (mg + F_m \sin \theta) = 0$$

or $$N = mg + F_m \sin \theta \quad \text{...(i)}$$

$$F_x = f - F_m \cos \theta = 0 \quad \text{...(ii)}$$

or $$f = F_m \cos \theta$$

Law of friction, $f = \mu N$ ...(iii)

By using the above equations

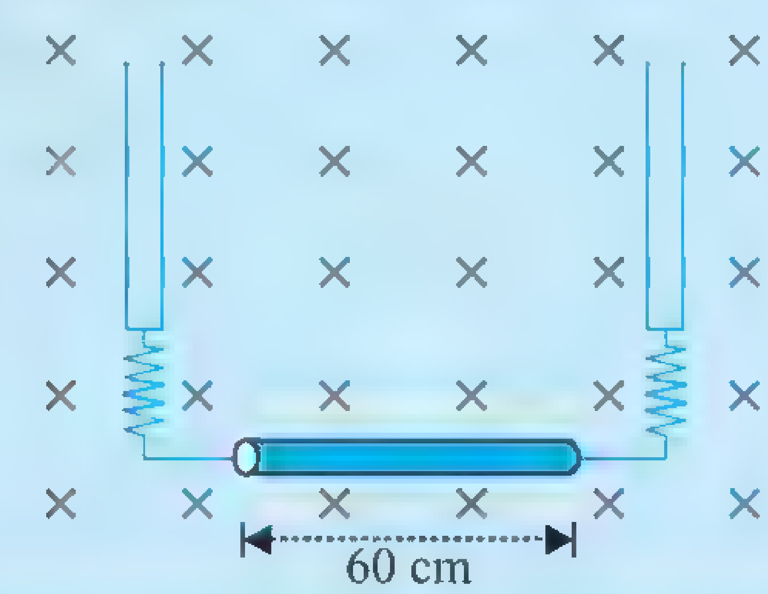
$$\mu(mg + F_m \sin \theta) = F_m \cos \theta$$

or $$F_m = \frac{\mu mg}{\cos\theta - \mu \sin\theta} \quad \text{or} \quad ilB = \frac{\mu mg}{\cos\theta - \mu\sin\theta}$$

or $$i = \frac{\mu mg}{lB(\cos\theta - \mu\sin\theta)}$$

ILLUSTRATION 1.42

A wire of 60 cm length and mas 10 gm is suspended by a pair of flexible leads in a magnetic field of induction 0.40 T. What are the magnitude and direction of the current required to remove the tension in the supporting leads?



Sol. We know that the magnetic force on a wire is given by

$$F = ilB \sin \theta$$

Here B is at right angle to i and hence $\theta = 90°$.

So $F = ilB$

Setting $F = mg$, the weight of the wire, the magnitude of the current required to remove the tension in the supporting lead is

$$i = \frac{mg}{lB} = \frac{(1.0 \times 10^{-2}\,\text{kg})(10\,\text{m s}^{-2})}{(0.6\,\text{m})(0.4\,\text{Wb m}^{-2})} = 0.41 \text{ A.}$$

Since the magnetic force should act in upward direction so the direction of current should be from left to right.

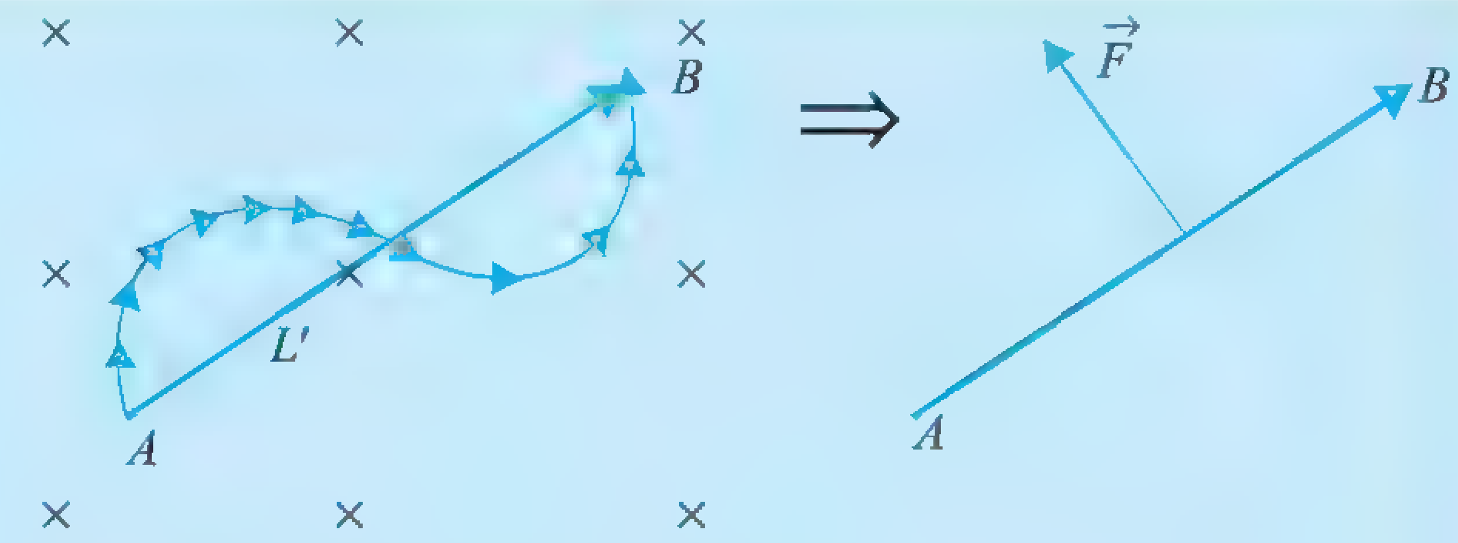
Important Points:

- In case of current carrying conductor in a magnetic field, if the field is uniform, i.e., $\vec{B}$ = constant,

$$\vec{F} = \int I\, d\vec{L} \times \vec{B} = I\left[\int d\vec{L}\right] \times \vec{B}.$$

For a conductor, $\int d\vec{L}$ represents the vector sum of all the length elements from initial to final point, which in accordance with the law of vector addition is equal to the length vector $\vec{L}$ joining initial to final point. So, a current carrying conductor of any arbitrary shape in a uniform field experiences a force

$$\vec{F} = i\left[\int d\vec{L}\right] \times \vec{B} = I\vec{L} \times \vec{B}$$



where $\vec{L}$ is the length vector joining initial and final points of the conductor as shown in figure.

- If the current carrying conductor in the form of a closed loop of any arbitrary shape is placed in a uniform field,

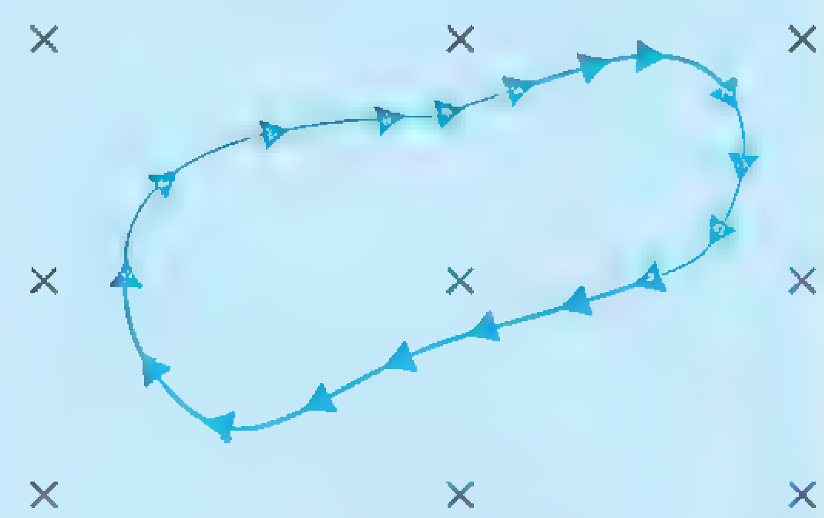
$$\vec{F} = \oint I d\vec{L} \times \vec{B} = I\left[\oint d\vec{L}\right] \times \vec{B}.$$

For a closed loop, the vector sum of $d\vec{L}$ is always zero.

So, $\vec{F} = 0$

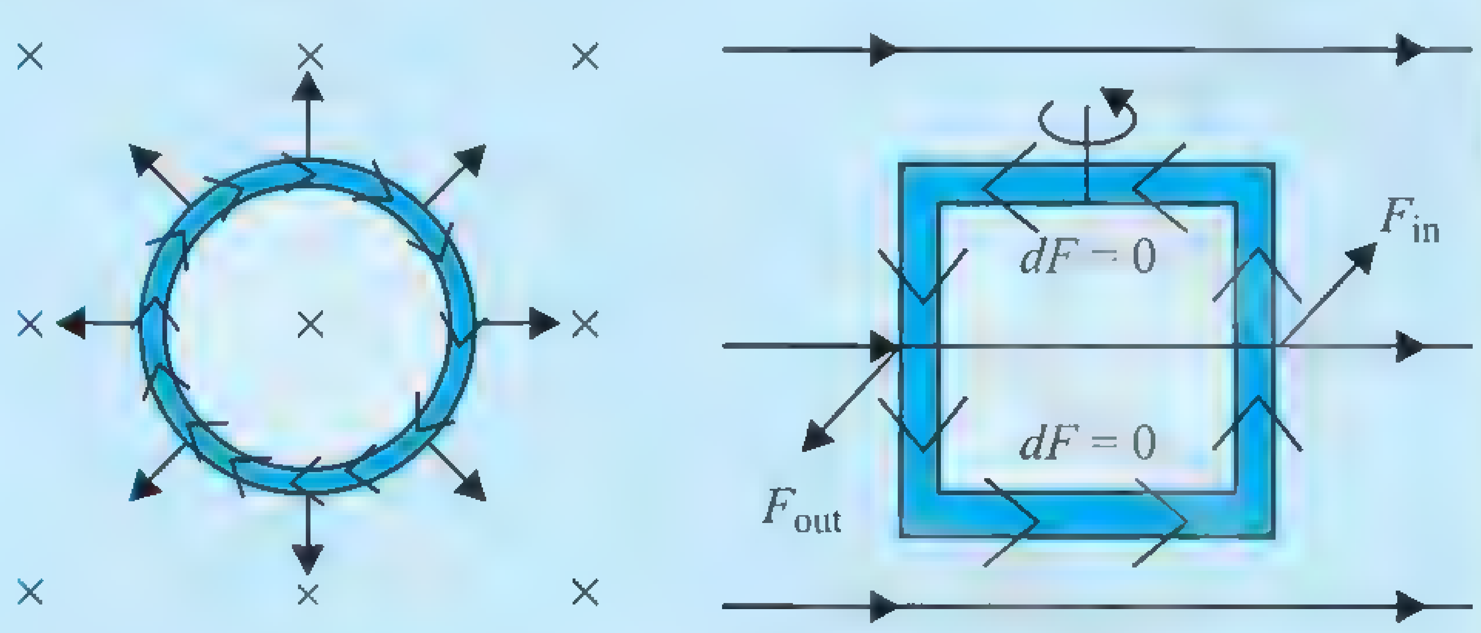
$$\left[\text{as} \oint d\vec{L} = 0\right]$$

i.e., the net magnetic force on a current loop in a uniform magnetic field is always zero as shown in figure.



$\vec{F} = I(\oint d\vec{L}) \times \vec{B} = 0$

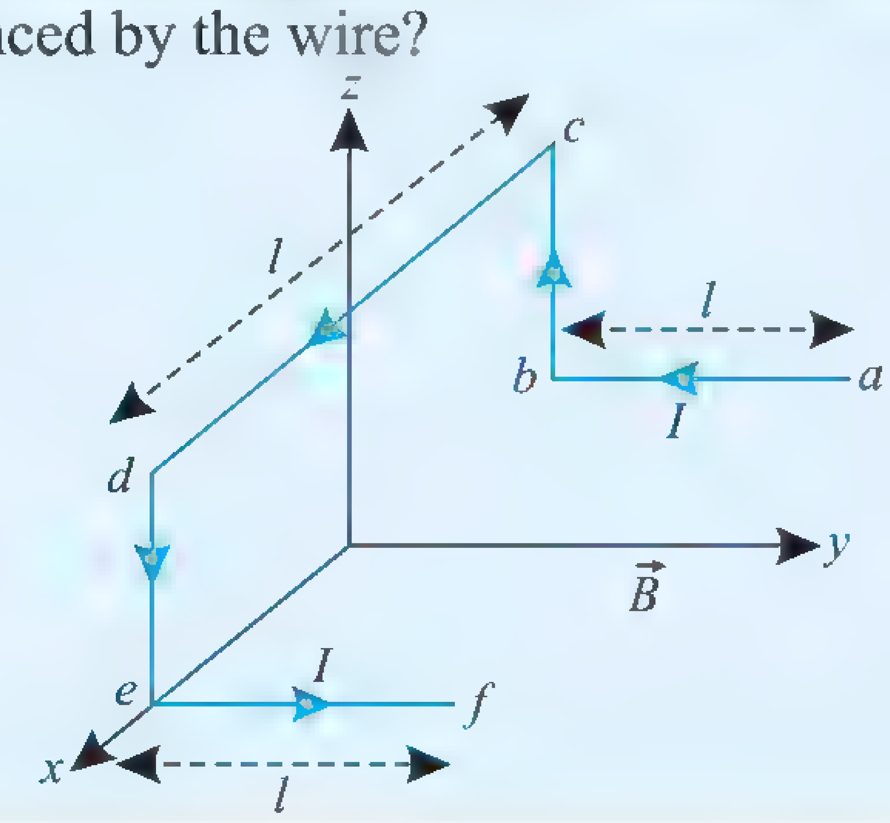
- A current carrying loop in a uniform magnetic field Here, it must be kept in mind that in this situation different parts of the loop may experience elemental force due to which the loop may be under tension or may experience a torque as shown in figure.



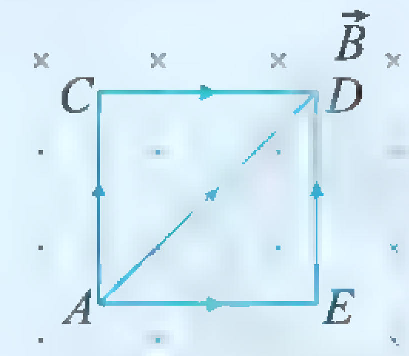
$\vec{F} = 0$, $\vec{\tau} = 0$ Current loop in a uniform field $\vec{F} = 0$, $\vec{\tau} \neq 0$

Concept Application Exercise 1.3

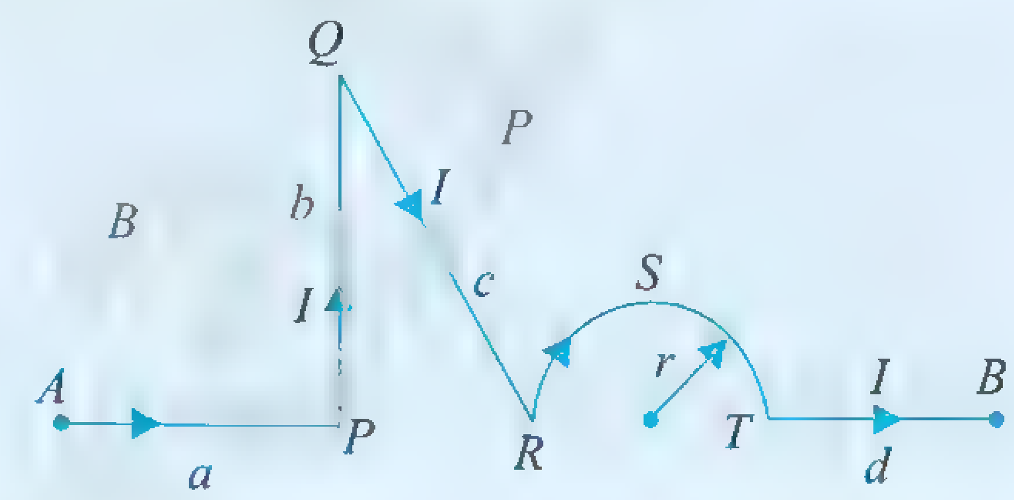
1. A wire *abcdef* with each side of length l bent as shown in figure and carrying a current I is placed in a uniform magnetic field B parallel to $+y$ direction What is the force experienced by the wire?



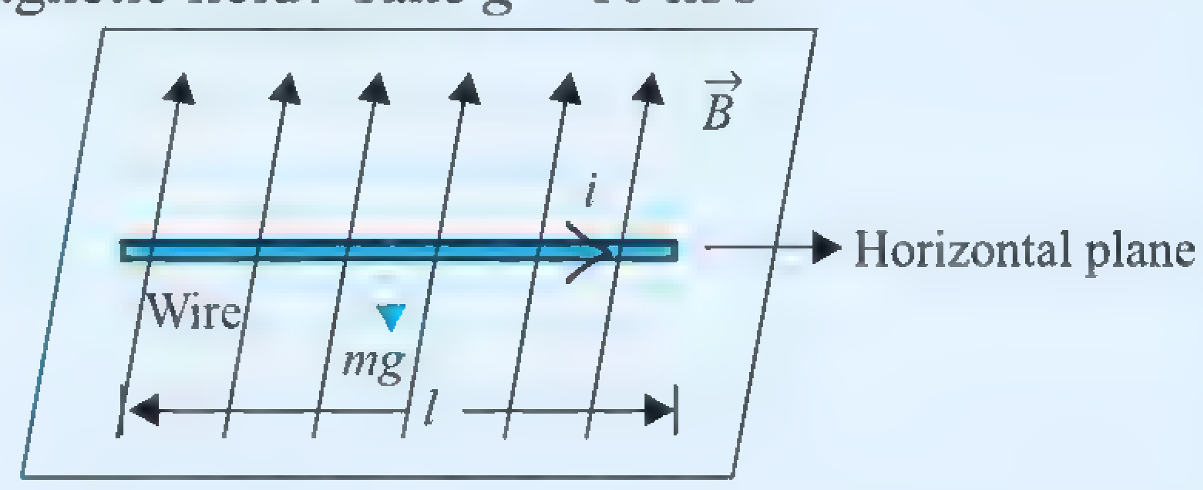
2. A square of side 2.0 m is placed in a uniform magnetic field $\vec{B} = 2.0\ T$ in a direction perpendicular to the plane of the square inwards. Equal current $i = 3.0$. A is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.



3. Calculate the force on a current carrying wire in a uniform magnetic field as shown in figure.

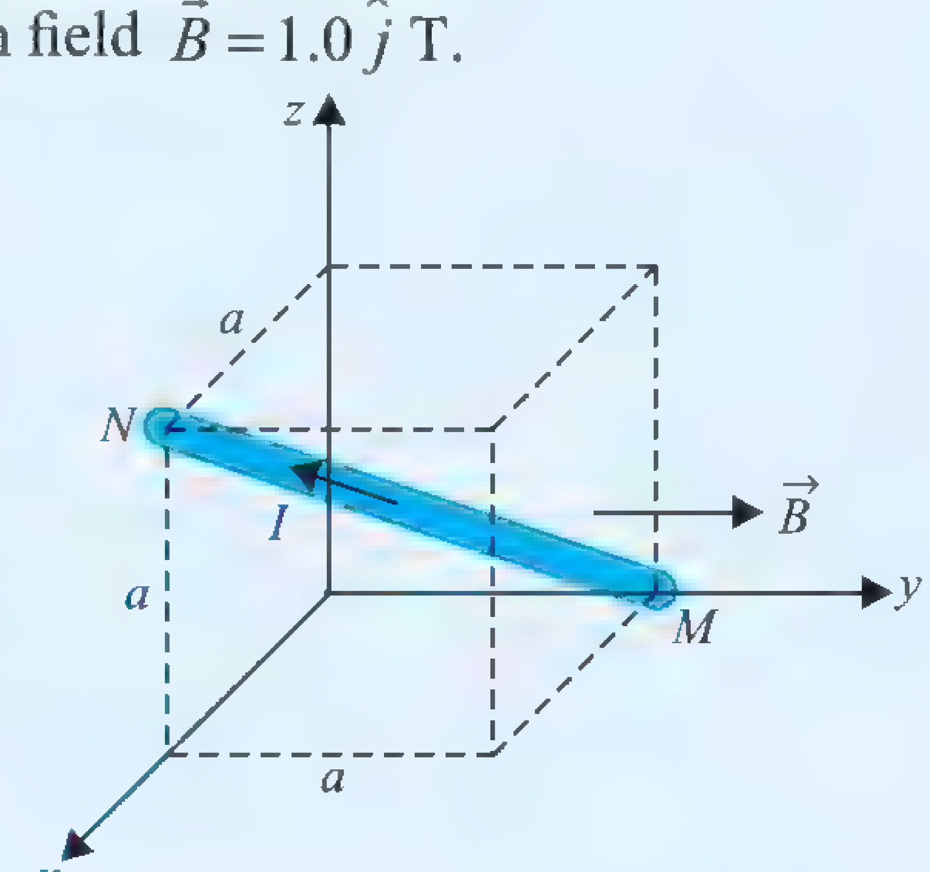


4. A straight wire of mass 200 g and length 1.0 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field? Take $g = 10\ \text{m/s}^2$



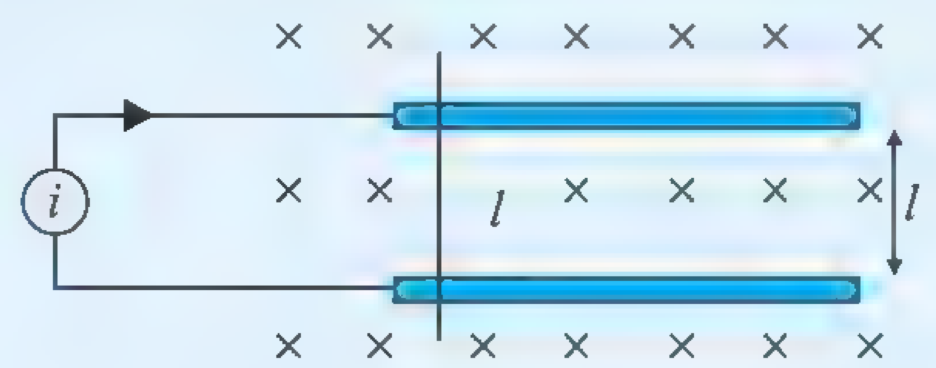
5. The horizontal component of the earth's magnetic field at a certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is
   (a) east to west;
   (b) south to north?

6. A straight wire (MN) lies along a body diagonal of an imaginary cube of side $a = \sqrt{2}$ m, and carries a current of 5 A (as shown in figure). Find the force on it due to a uniform field $\vec{B} = 1.0\,\hat{j}$ T.



7. The figure below shows two long metal rails placed horizontally and parallel to each other at a separation

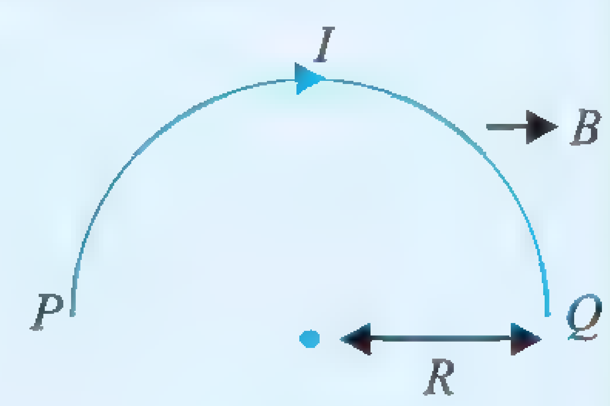
l. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.



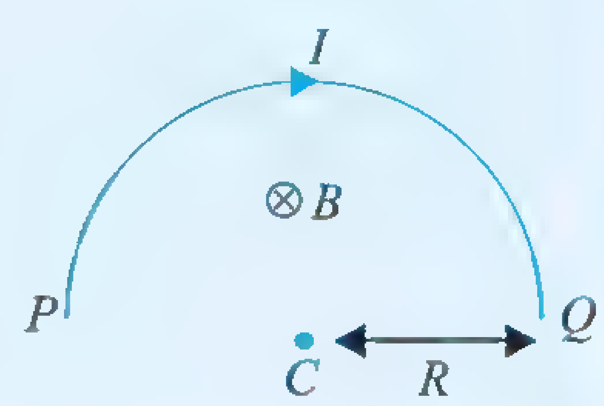
(a) What should be the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part.

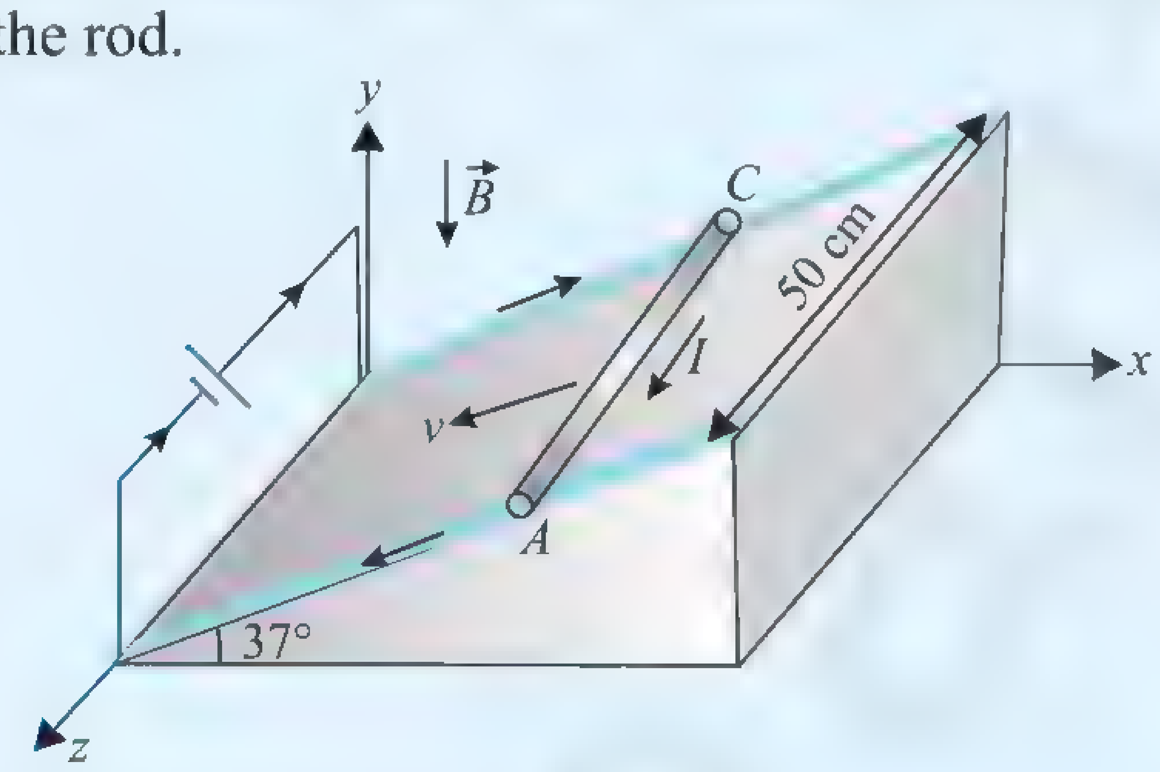
8. In the figure, a semicircular wire is placed in a uniform field $\vec{B}$ directed towards right. Find the resultant magnetic force and torque on it.



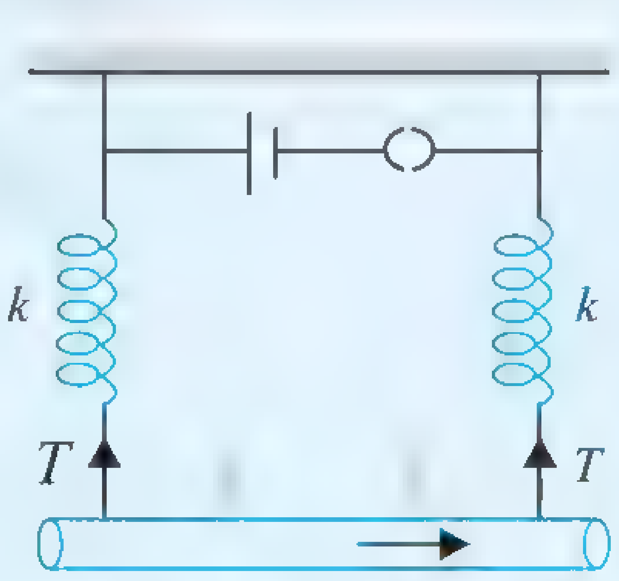
9. In the figure, find the resultant magnetic force and torque about C and P.



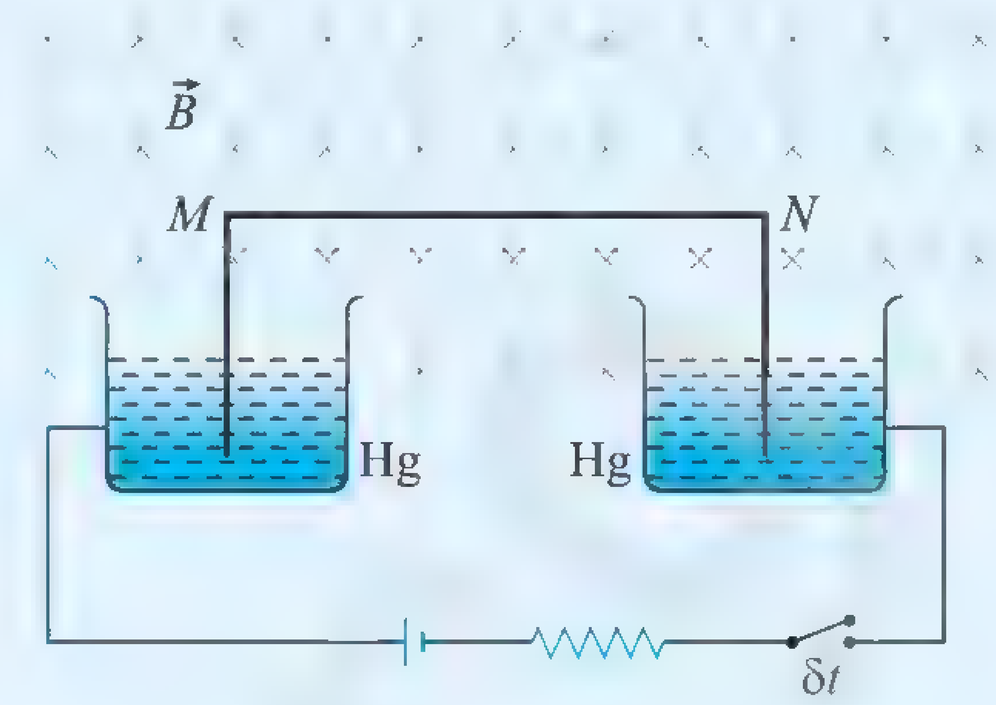
10. In figure, the bar AC has a mass of 50 g. It slides frictionlessly on the metal strips 50 cm apart at the edges of the incline. A current I flows through these strips and the bar, as shown. There is a magnetic field $|B_y| = 0.02$ T directed in the $-y$ direction. How much must I be if the rod is to remain motionless? Neglect the slight overhang of the rod.



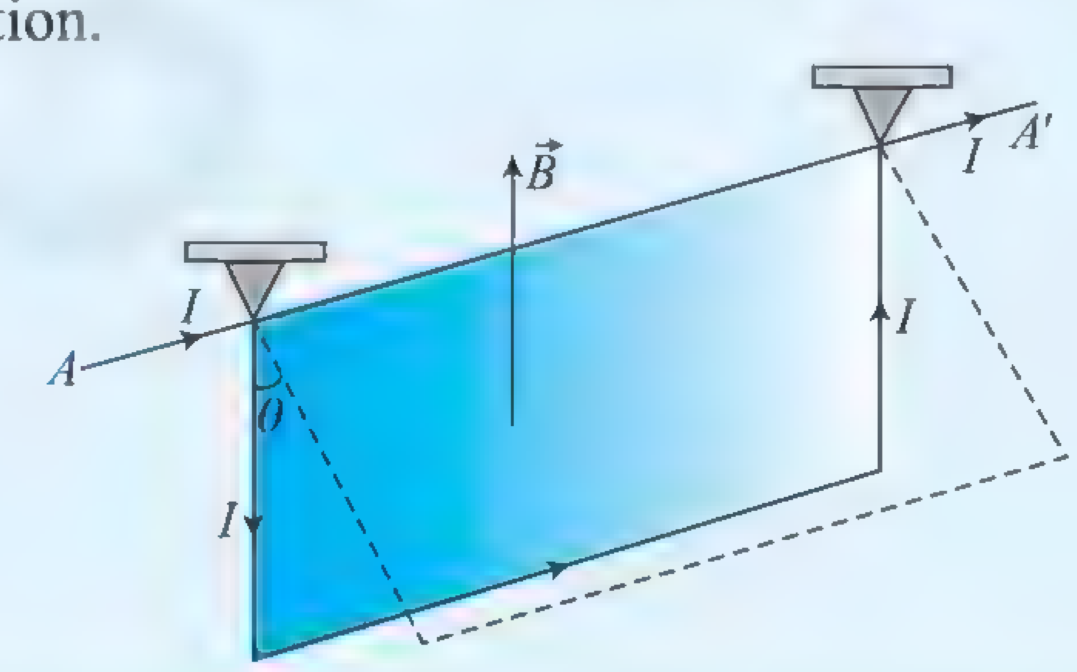
11. A metal rod of mass 10 g and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 A current passes through the rod, it rises by 1 cm. Determine the magnetic field assuming acceleration due to gravity to be 10 ms^{-1}.



12. The figure shows a horizontal wire MN of length l and mass m is placed in a magnetic field B. The ends of wire are bent and dipped in two bowls containing Hg which are connected to an external circuit as shown. If the key is pressed for a short time δt due to which a charge q suddenly flows in the circuit, find the maximum height above initial level the wire MN will jump.



13. A copper wire with density ρ with cross-sectional area S bent to make three sides of a square frame which can turn about a horizontal axis OO' as shown in figure. The wire is located in uniform vertical magnetic field. Find the magnetic induction if on passing a current I through the wire the frame deflects by an angle θ in its equilibrium position.



ANSWERS

1. $BIl(\hat{k})$ **2.** $36\sqrt{2}$ N. Direction of this force is towards EC.

3. $IB(a + \sqrt{c^2 - b^2} + 2r + d)$ **4.** 1.0 T

5. (a) 3×10^{-5} N (b) zero **6.** 10 N

7. (a) $\dfrac{ilb}{mg}$ (b) The wire will slide towards right with acceleration $\dfrac{ilB}{2m}$

8. 0; $\dfrac{i\pi R^2}{2} B(-\hat{j})$ **9.** $2IRB$, 0, $2IBR^2$

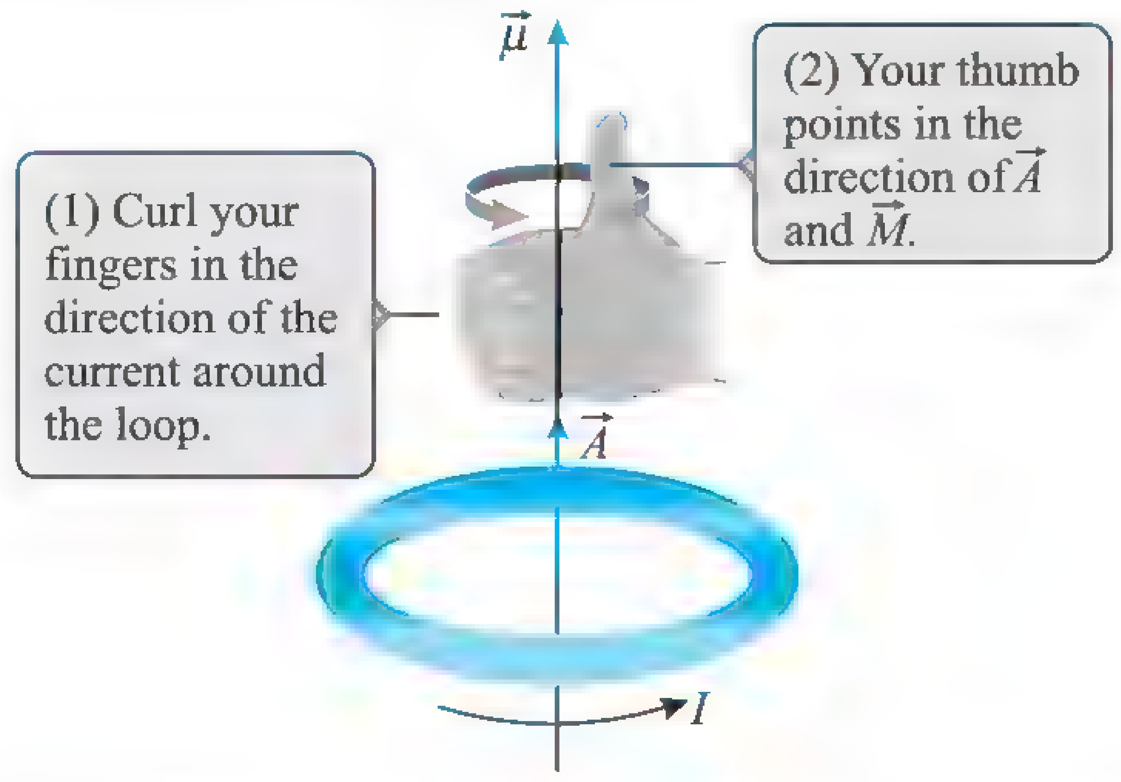
10. 37.5 A **11.** 1.5×10^{-3} T **12.** $\dfrac{B^2q^2l^2}{2m^2g}$ **13.** $\dfrac{2aS\rho g}{Ia}\tan\theta$

MAGNETIC DIPOLE AND DIPOLE MOMENT

Magnetic dipole is the magnetic equivalent of electric dipole.

The magnetic field pattern produced by a small current loop is similar to a bar magnet. Therefore, it also acts like a magnetic dipole. The magnetic moment of a flat current loop is defined as the product of the current I and the area A enclosed by it, i.e., $\vec{M} = I\vec{A}$.

The direction of the magnetic moment coincides with the direction of the area vector (which is the direction of the magnetic field).

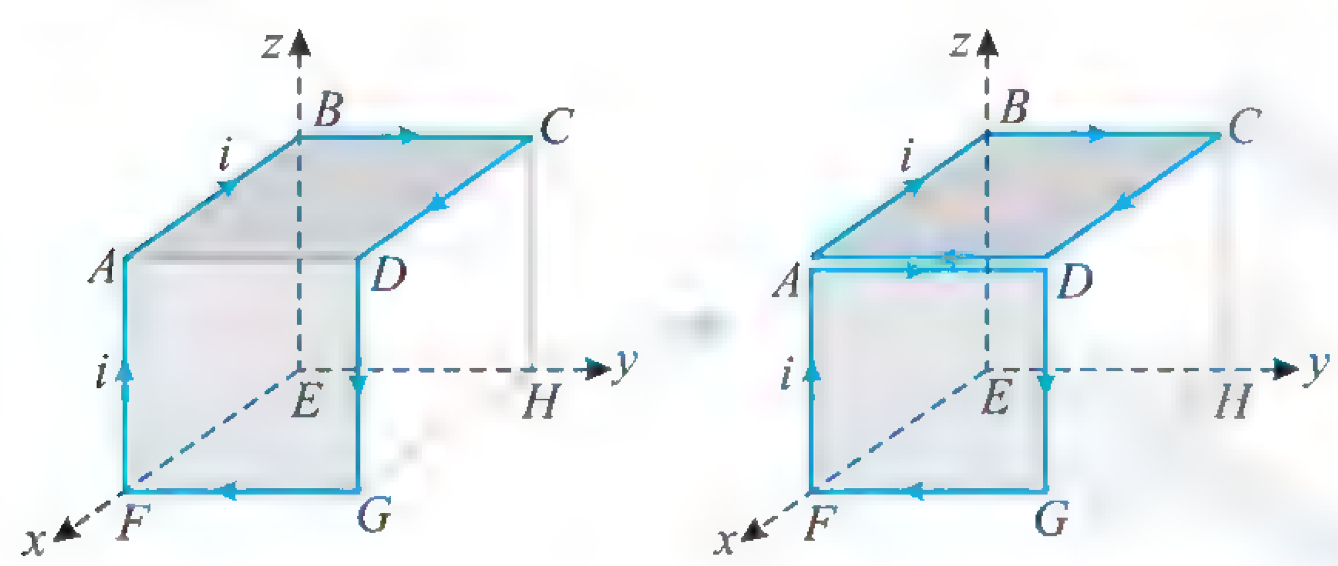


If the loop contains N number of turns, the magnetic moment is given by $M = NIA$

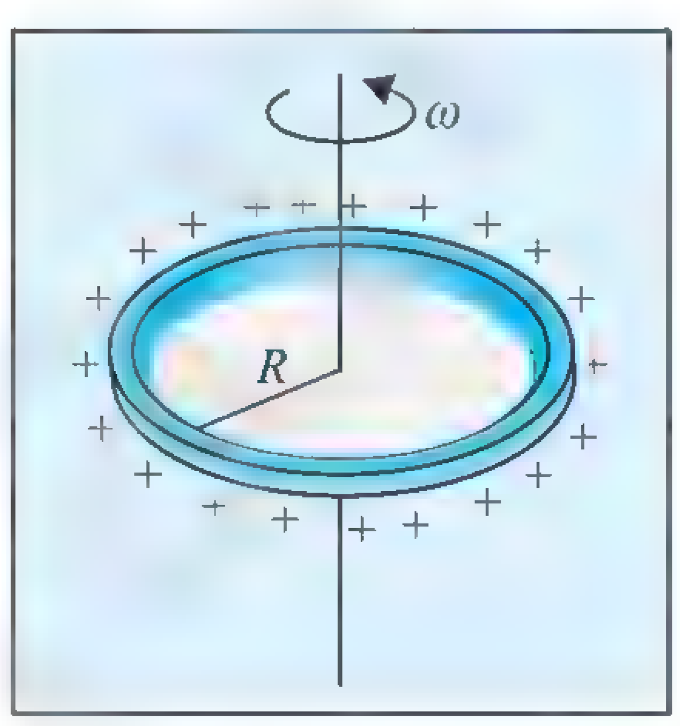
- Sometimes a current carrying loop does not lie in a single plane. But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes. Now, the net magnetic moment of the given loop is the vector sum of individual loops. For example, in figure, six sides of a cube of side l carry a current i in the directions shown. By assuming two equal and opposite currents in wire AD, two loops in two different planes (xy and yz) are completed.

$$\vec{M}_{ABCDA} = -il^2\hat{k} \quad \text{and} \quad \vec{M}_{ADGFA} = -il^2\hat{i}$$

$$\Rightarrow \quad \vec{M}_{\text{net}} = -il^2(\hat{i} + \hat{k})$$



- Sometimes a non-conducting body is rotated with some angular speed. In this case, the ratio of magnetic moment and angular momentum is constant which is equal to $q/2m$, where q is the charge and m is the mass of the body. For example, in case of a ring of mass m, radius R, and charge q distributed on its circumference,



Angular momentum,

$$L = I\omega = (mR^2)(\omega) \quad \text{...(i)}$$

Magnetic moment,

$$M = iA = (qf)(\pi R^2) \quad \text{...(ii)}$$

Here, f = frequency $= \dfrac{\omega}{2\pi}$

$$\therefore \quad M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2)$$

$$= q\frac{\omega R^2}{2}$$

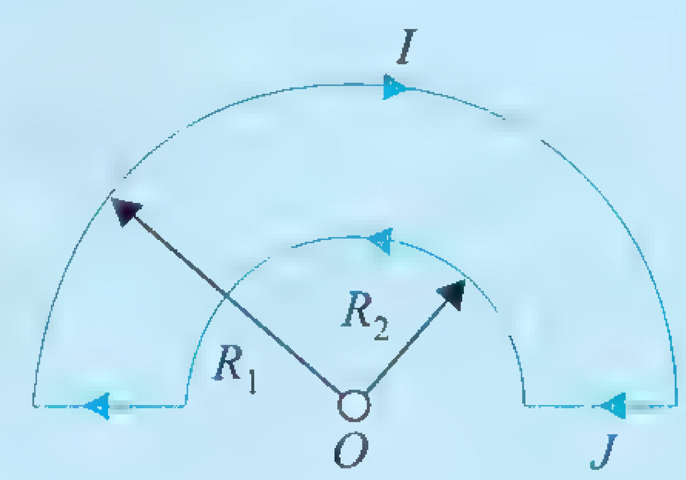
From Eqs (i) and (ii), $\dfrac{M}{L} = \dfrac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also.

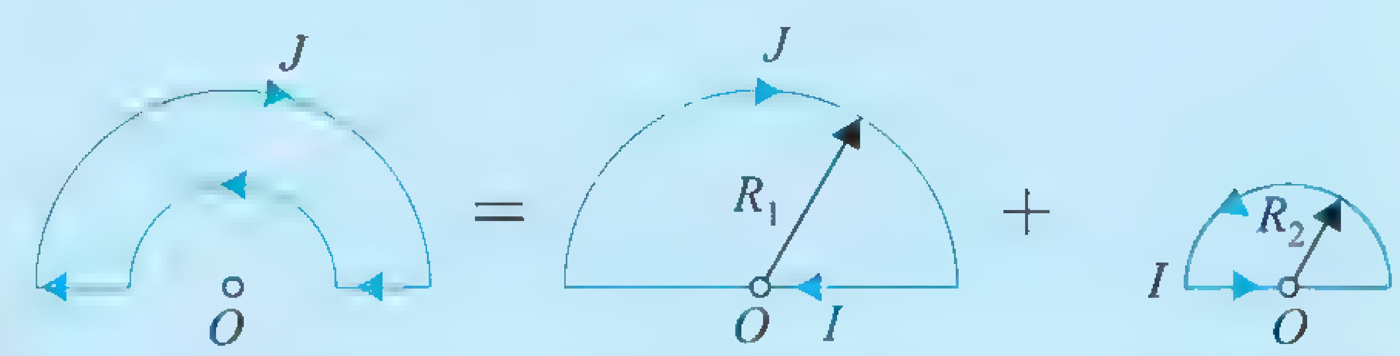
Rigid body	**Ring**	**Disc**	**Solid sphere**	**Spherical shell**
Moment of inertia (I)	mR^2	$\dfrac{mR^2}{2}$	$\dfrac{2}{5}mR^2$	$\dfrac{2}{3}mR^2$
Magnetic moment $M = \dfrac{qI\omega}{2m}$	$\dfrac{q\omega R^2}{2}$	$\dfrac{q\omega R^2}{4}$	$\dfrac{q\omega R^2}{5}$	$\dfrac{q\omega R^2}{3}$

ILLUSTRATION 1.43

Compute the magnetic dipole moment of the loop shown in figure.



(a)



(b)

Sol. The given loop may be considered as the superposition of the two loops, as shown in the figure.

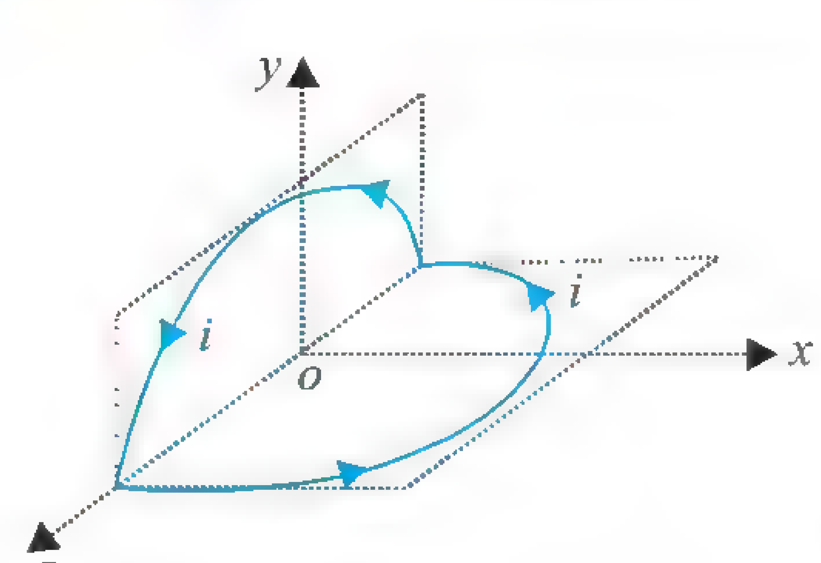
The resultant dipole moment is $M = \dfrac{\pi R_1^2 I}{2} - \dfrac{\pi R_2^2 I}{2}$

$$\text{or} \quad M = \frac{\pi I}{2}\left(R_1^2 - R_2^2\right) \text{ (inward)}$$

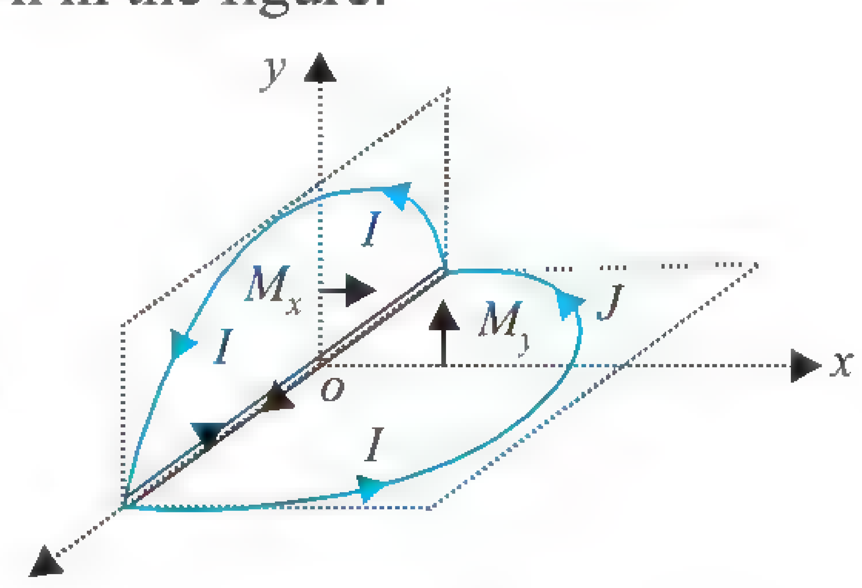
ILLUSTRATION 1.44

A circular loop of wire of radius R is bent about its diameter along two mutually perpendicular planes as shown in the figure. If the loop carries a current I, then determine its magnetic moment.

Sol. The given loop may be obtained by the superposition of two semicircular loops as shown in the figure.



(a)



(b)

The magnetic moment of the semicircle in the y-z plane is along the x-axis and that in the x-z plane is along the y-axis.

$$M_x = \frac{\pi R^2 I}{2}, \quad M_y = \frac{\pi R^2 I}{2}$$

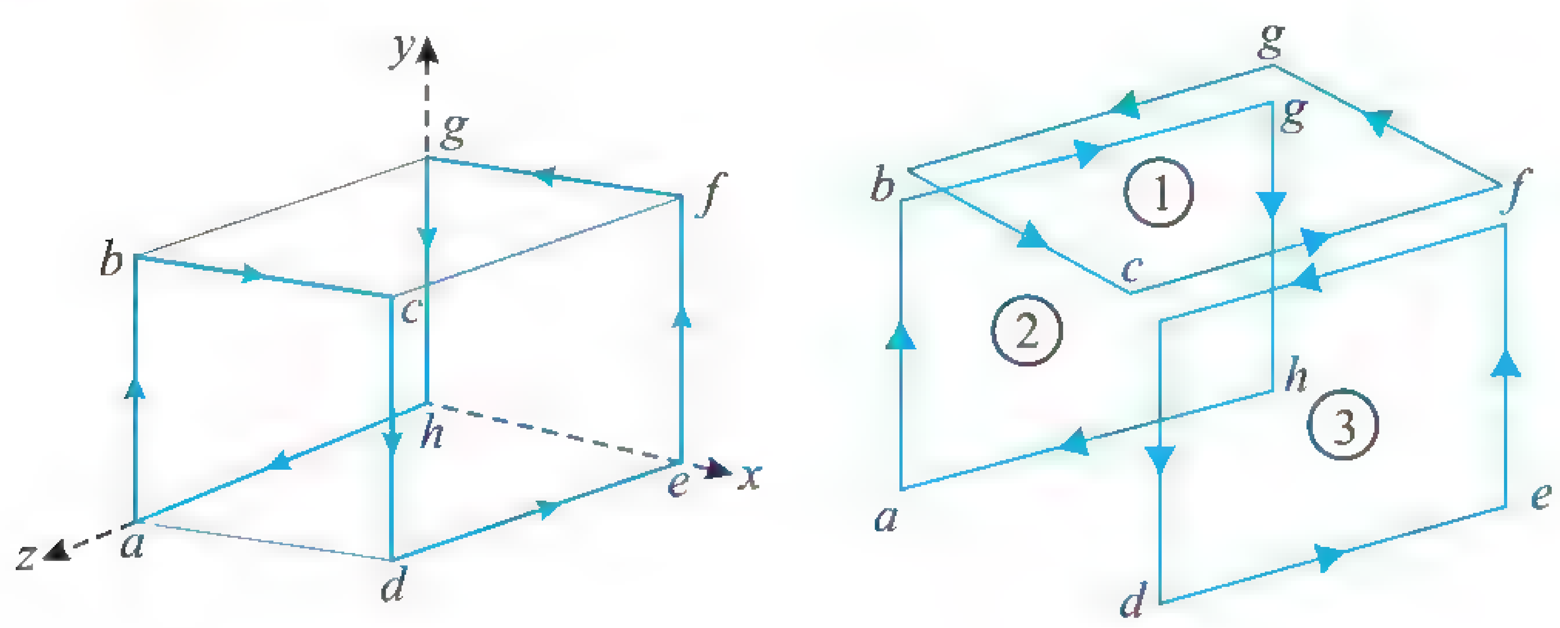
The total magnetic moment is $\vec{M} = M_x\hat{i} + M_y\hat{j}$

or $M = \dfrac{\pi R^2 I}{2}\left(\hat{i} + \hat{j}\right)$

ILLUSTRATION 1.45

A conductor carries a constant current I along the closed path *abcdefgha* involving 8 of the 12 edges of length l. Find the magnetic dipole moment of the closed path.

Sol. The closed path is a superposition of three loops: *bcfgb*, *abgha* and *cdefc*.



(a) (b)

The magnetic moments of the three loops are:

Loop 1 (*bcfgb*): $\vec{M}_1 = l^2 I\hat{j}$

2 (*abgha*): $\vec{M}_2 = -l^2 I\hat{i}$

3 (*cdefc*): $\vec{M}_3 = l^2 I\hat{i}$

The total magnetic moment is

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 \text{ or } \vec{M} = l^2 I\hat{j}$$

ILLUSTRATION 1.46

A non-conducting disk of mass M and radius R has a surface charge density σ and rotates with an angular velocity ω about its axis. Show that magnetic dipole moment and angular momentum are related as $\vec{M} = \left(\dfrac{Q}{2M}\right)\vec{L}$.

Sol. The charge is distributed on the surface of the disk.

We consider a differential ring of radius r and thickness dr.

The charge on the element is $dq = \sigma dA = \sigma(2\pi r dr)$

The magnetic moment of the ring $dM = (dI)A = (dI)\pi r^2$

The current in the differential ring

$$dI = (dq)f = (\sigma dA)\frac{\omega}{2\pi} = (\sigma 2\pi r dr)\frac{\omega}{2\pi} = \sigma\omega r dr$$

The magnetic moment of the differential ring,

$$dM = (\sigma\omega r dr)\,\pi r^2 = \pi\sigma\omega r^3 dr$$

$$M = \int dM = \int_0^R \pi\sigma\omega r^3 dr = \frac{1}{4}\pi\sigma\omega R^4$$

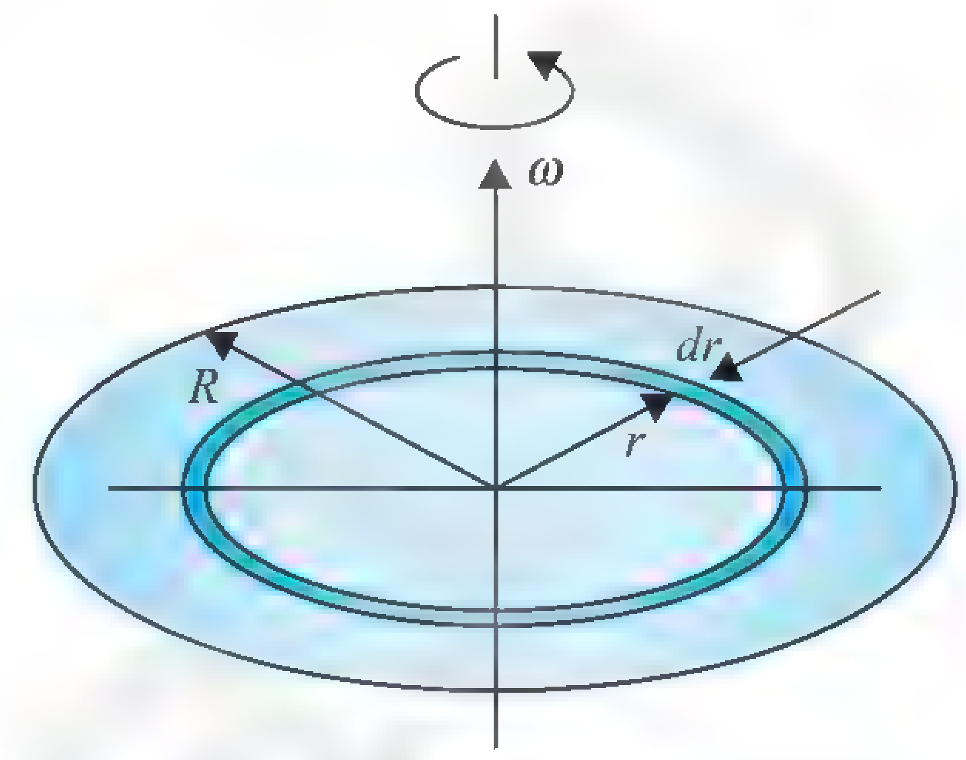
The magnetic moment vector $\vec{M}$ is parallel to $\vec{\omega}$ if charge is positive.

$$\vec{M} = \frac{1}{4}\pi\sigma R^4\vec{\omega}$$

In terms of total charge $Q = \sigma\pi R^2$, the magnetic moment is

$$\vec{M} = \frac{1}{4}QR^2\vec{\omega}$$

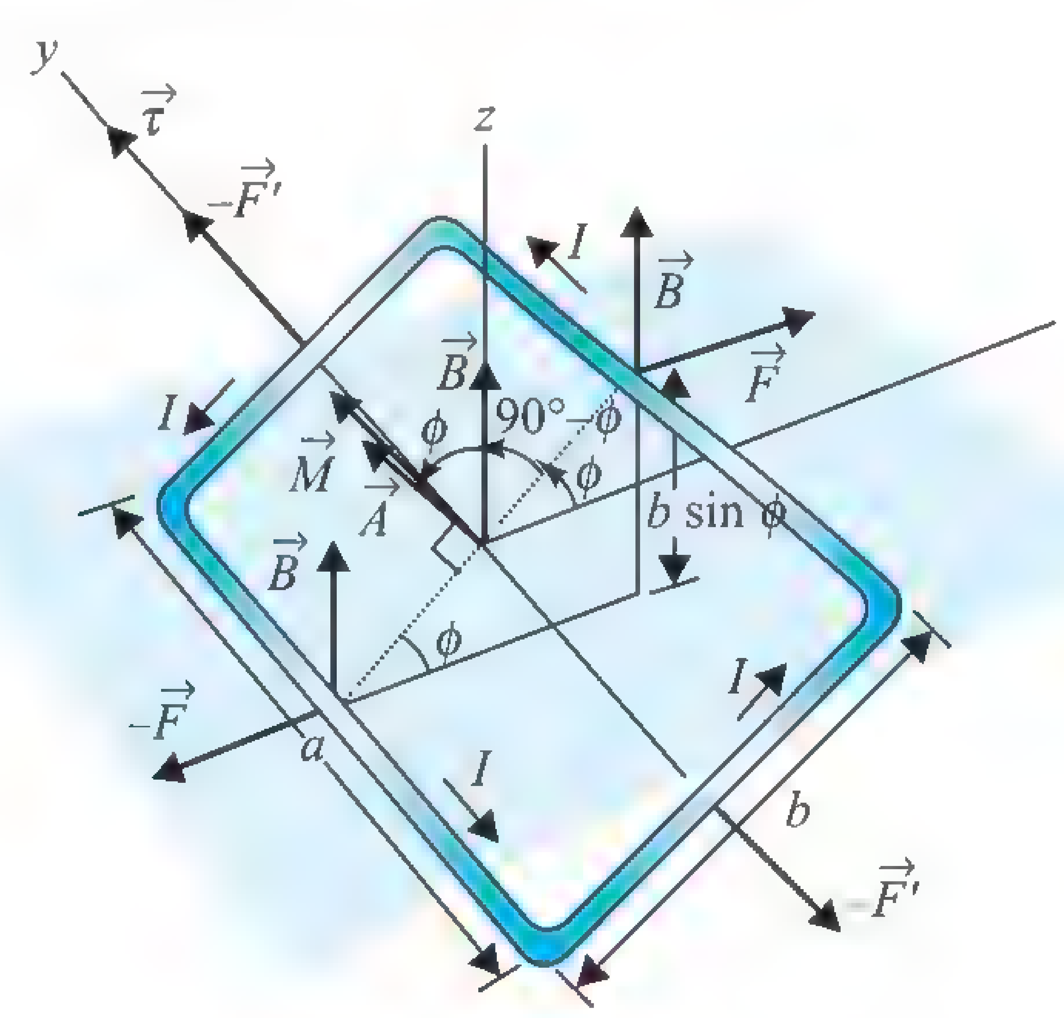
The angular momentum of disk is $\vec{L} = \left(\dfrac{1}{2}MR^2\right)\vec{\omega}$ and $\vec{M} = \left(\dfrac{Q}{2M}\right)\vec{L}$.

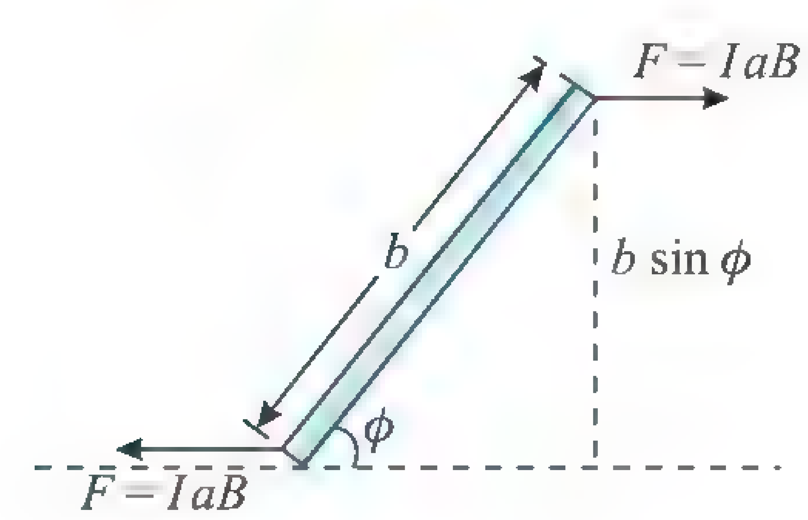


This is a general result for any rigid body of any arbitrary shape having mass M and charge Q.

TORQUE ON A CURRENT-CARRYING PLANAR LOOP IN A UNIFORM MAGNETIC FIELD

Figure shows a rectangular loop of wire with side lengths a and b. A line perpendicular to the plane of the loop (i.e., a normal to the plane) makes an angle ϕ with the direction of the magnetic field $\vec{B}$, and the loop carries a current I.





The force $\vec{F}$ on the right side of the loop (length a) in the $+x$ direction as shown. On this side, $\vec{B}$ is perpendicular to the current direction and the force on this side has magnitude

$$F = IaB \qquad \text{...(i)}$$

A force $-\vec{F}$ with the same magnitude but opposite direction acts on the opposite side of the loop, as shown in figure.

On sides of length b equal and opposite forces F' act whose lines of action are same.

The total force on the loop is zero because the forces on opposite sides cancel out in pairs. The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero. The two forces $\vec{F}$ and $-\vec{F}$ lie along different lines, and each gives rise to a torque about the y-axis. According to the right hand rule for determining the direction of torques, the vector torques due to $\vec{F}$ and $-\vec{F}$ are both in the $+y$ direction; hence the net vector torque $\vec{\tau}$ is in the $+y$ direction as well. The magnitude of the net torque is

$$\tau = F(b)\sin\phi = (IBa)(b\sin\phi) \quad \text{...(ii)}$$

The area A of the loop is equal to ab, so we can rewrite Eq. (ii) as

$$\tau = IBA\sin\phi$$

(magnitude of torque on a current loop) ...(iii)

where ϕ is the angle between the normal to the loop (the direction of the vector area $\vec{A}$) and $\vec{B}$.

The product IA is called the magnetic dipole moment or magnetic moment of the loop

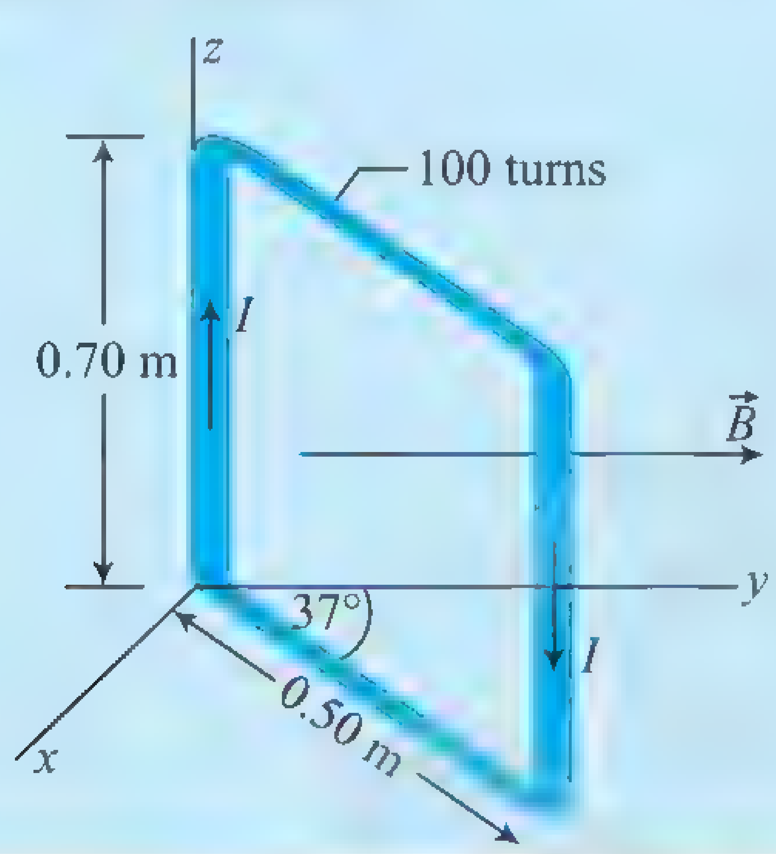
$$M = IA \quad \text{...(iv)}$$

We can express this interaction in terms of the torque vector $\vec{\tau}$, which we used for electric–dipole interactions. The magnitude of $\vec{\tau}$ is equal to the magnitude of $\vec{M}\times\vec{B}$. So, we have

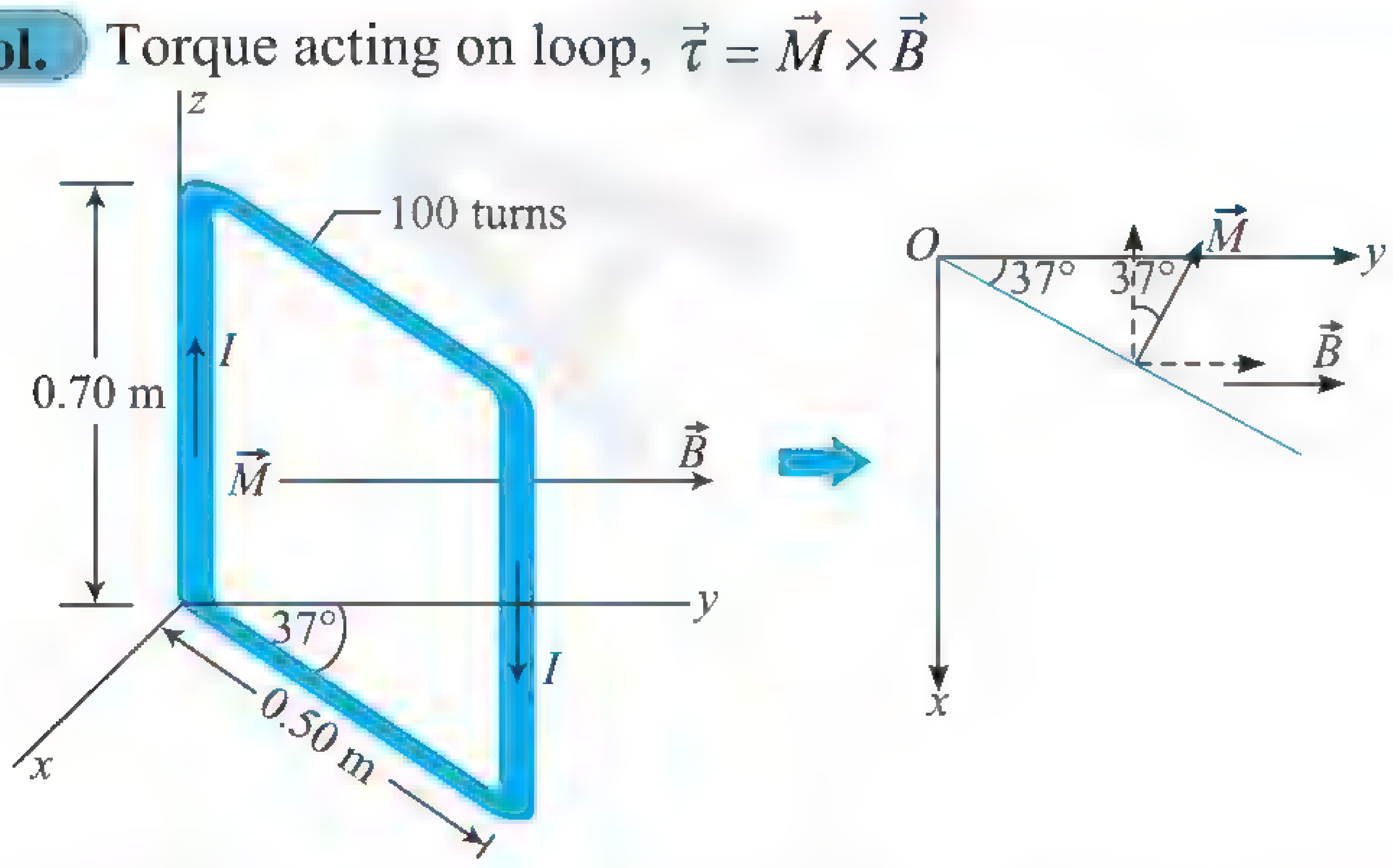
$$\vec{\tau} = \vec{M}\times\vec{B} \text{ (vector torque on a current loop)} \quad \text{...(v)}$$

ILLUSTRATION 1.47

The rectangular loop in the drawing consists of 100 turns and carries a current of $I = 5.0$ A. Magnetic field of magnitude 2.0 T magnetic field is directed along the $+y$ axis. The loop is free to rotate about the z axis. **(a)** Determine the magnitude of the net torque exerted on the loop and **(b)** state whether the 37° angle will increase or decrease.



Sol. Torque acting on loop, $\vec{\tau} = \vec{M}\times\vec{B}$



The direction of the torque vector is into the screen or along $-z$ direction. It means the loop will rotate in clockwise sense as seen from top. It means the 37° angle will increase.

Unit vector in the direction of M,

$$\hat{M} = -\cos 37°\hat{i} + \sin 37°\hat{j} = \left(-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right)$$

The magnitude of the magnetic moment of the loop,

$$M = NiA = 100 \times 5 \times (0.70 \times 0.50) = 175 \text{ A} - \text{m}^2$$

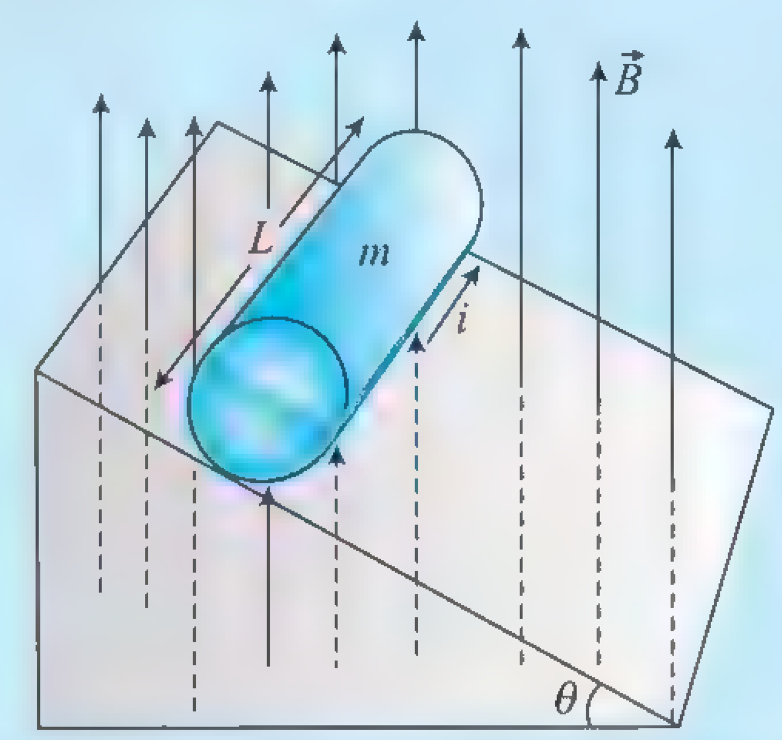
$$\vec{M} = M\hat{M} = 175\left(-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right) = 35(-4\hat{i} + 3\hat{j})\text{A} - \text{m}^2$$

Torque acting on loop, $\vec{\tau} = \vec{M}\times\vec{B} = [35(-4\hat{i}+3\hat{j})]\times[2\hat{j}]$

$$\vec{\tau} = -35\times 4\times 2(\hat{i}\times\hat{j}) = -280\hat{k} \text{ N} - \text{m}$$

ILLUSTRATION 1.48

The figure shows a wood cylinder of mass m and length L with N turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the inclined plane. If there is a vertical uniform magnetic field of magnitude B, what is the least current i through the coil that keeps the cylinder in equilibrium?



Sol. As the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. For translational equilibrium of the cylinder

$$f = mg\sin\theta \quad \text{...(i)}$$

Now let us consider rotational equlibrium, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $MB\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. For rotational equilibrium of the cylinder

$$fr = MB\sin\theta \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$mg\sin\theta . r = MB\sin\theta \Rightarrow mgr = MB \quad \text{...(iii)}$$

The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $M = NiA = Ni(2rL)$. Hence Eq. (iii) becomes

$$mgr = 2NirLB \Rightarrow i = \frac{mg}{2NLB}$$

ILLUSTRATION 1.49

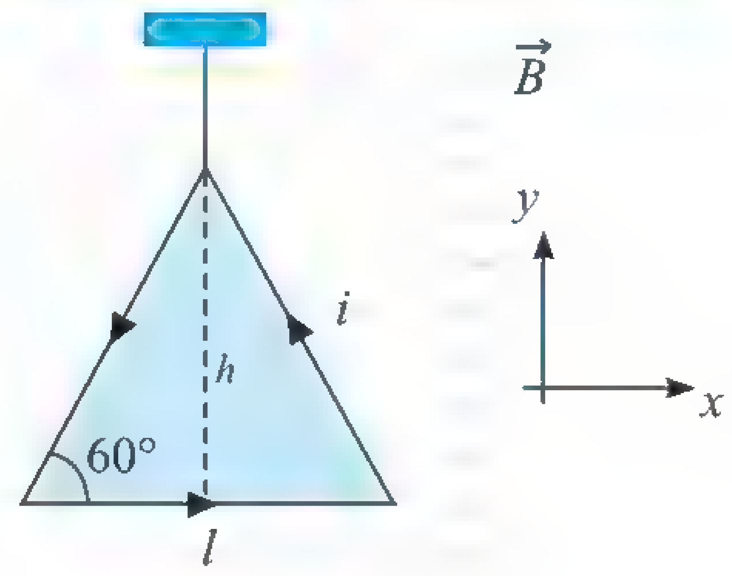
A coil in the shape of an equilateral triangle of side 0.20 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent

magnet producing a horizontal magnetic field of 5.0 T. Find the couple acting on the coil when a current of $\sqrt{3}$ A is passed through it and the magnetic field is parallel to its plane.

Sol. The magnetic moment of the loop will be along positive z direction.

The couple acting on a closed loop is given by $\tau = MB \sin\theta$.

θ = angle between magnetic field and magnetic moment of the loop.



The magnetic moment of the loop, $M = NiA$

$$M = NiA = Ni\left(\frac{1}{2}l \times h\right) = Ni\left(\frac{1}{2}l \times l \sin 60^\circ\right) = \frac{\sqrt{3}}{4}Nil^2$$

$$M = \frac{\sqrt{3}}{4} \times 1 \times \sqrt{3} \times (0.2)^2 = 3 \times 10^{-2}\ \text{A}-\text{m}^2$$

$$\tau = 3 \times 10^{-2} \times 5 \times 1 = 0.15\ \text{Nm}$$

ILLUSTRATION 1.50

A circular coil of wire 8 cm in diameter has 12 turns and carries a current of 5 A. The coil is in a field where the magnetic induction is 0.6 T.

(a) What is the maximum torque on the coil?

(b) In what position would the torque be half as great as in (i)?

Sol.

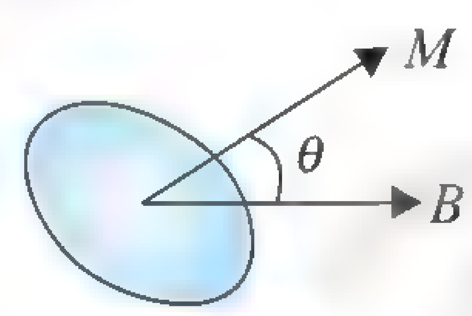
(a) The dipole moment of the coil: $M = NiA$

Here $N = 12$ turns, $i = 5$ A and $A = \pi r^2 = \pi(4 \times 10^{-2})^2$

$\therefore\quad M = 12 \times 5 \times \pi \times (4 \times 10^{-2})^2 = 0.3\ \text{Am}^2$

Maximum torque = $MB = 0.3 \times 0.60 = 0.18$ Nm

(b) If θ be the angle between the axis of the coil and the field torque = $MB \sin\theta$



According to the problem,

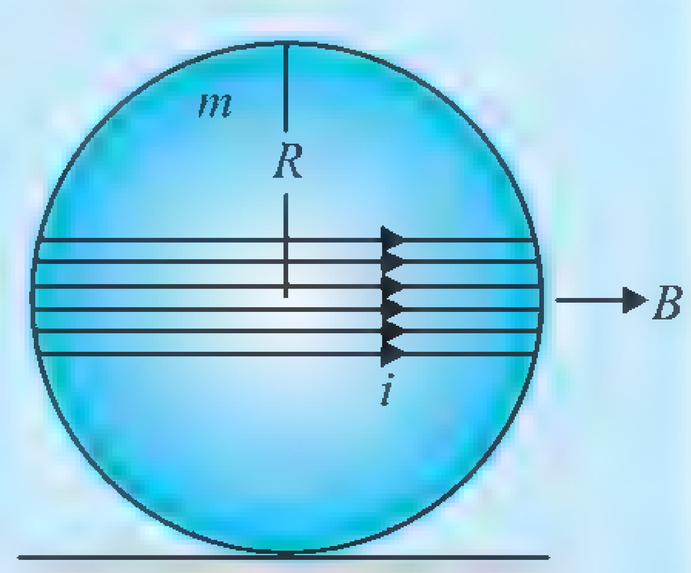
torque = $MB/2$

So $\dfrac{MB}{2} = MB\sin\theta \;\Rightarrow\; \sin\theta = \dfrac{1}{2}$ or $\theta = 30^\circ$

Thus, the normal to the coil is at 30° to the field.

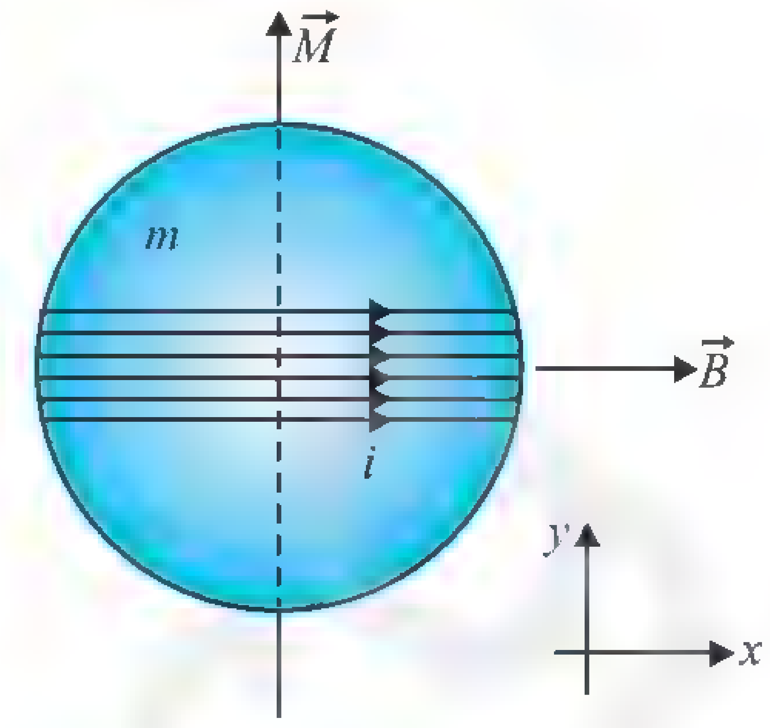
ILLUSTRATION 1.51

A wire is wrapped N times over a solid sphere of mass m which is placed on a smooth horizontal surface. A horizontal magnetic field of induction $\vec{B}$ is present. Find the **(a)** torque **(b)** angular acceleration experienced by the sphere. Assume that the mass of the wire is negligible compared to the mass of the sphere.



Sol.

(a) The net torque acting on the sphere is



$$\vec{\tau} = \vec{M} \times \vec{B} = (NiA\hat{j}) \times (B\hat{i}) = -NiAB\hat{k} \quad \text{where } A = \pi R^2$$

or $\quad \vec{\tau} = -N\pi R^2 iB\hat{k}$

(b) $\vec{\alpha} = \dfrac{\vec{\tau}}{I_c}$ ($\because$ the sphere is free to rotate, it must rotate about the centroidal axis)

$$= \frac{-N\pi R^2 iB}{\frac{2}{5}mR^2}\hat{k} = \frac{5N\pi iB}{2m}\hat{k} \qquad \left(\because\ I_c = \frac{2}{5}mR^2\right)$$

ENERGY OF DIPOLE

When a magnetic dipole changes orientation in a magnetic field, the field does work on it. In an infinitesimal angular displacement $d\phi$, the work dW is given by $\tau\, d\phi$, and there is a corresponding change in potential energy.

Energy needed to rotate the loop through an angle $d\theta$ is

$$dU = \tau d\theta$$

$$\Rightarrow\quad \Delta U = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

$$\Rightarrow\quad \Delta U = MB(\cos\theta_1 - \cos\theta_2)$$

If we choose θ_1 such that at $\theta_1 = 90^\circ$, $U_1 = 0$

$$U = -\vec{M}\cdot\vec{B}$$

This is the energy stored in the loop.

Important Points:

Forces on the sides of a current carrying loop in a uniform magnetic field.

- The resultant force is zero; the net torque has magnitude $\tau = IAB \sin\phi$.
- The torque is maximum when the normal to the loop is perpendicular to $\vec{B}$.
- When the normal to the loop is parallel to $\vec{B}$, the torque is zero and the equilibrium is stable. If the normal is antiparallel to $\vec{B}$, the torque is also zero but the equilibrium is unstable.

WORK DONE IN ROTATING A CURRENT-CARRYING COIL IN MAGNETIC FIELD

Let a current-carrying coil is rotated in a uniform magnetic field in such a way that its orientation changes with angle between magnetic field and magnetic moment of coil from θ_1 to θ_2. Then the potential energy of the coil in magnetic field in initial and

final state is given as

$$U_i = -MB\cos\theta_1 \quad ...(i)$$

and $$U_f = -MB\cos\theta_2 \quad ...(ii)$$

To change the orientation of coil externally, we need to apply a torque against the magnetic torque of equal magnitude thus work done in in changing the orientation of the coil during its rotation is given as

$$W = U_f - U_i$$

$$\Rightarrow \quad W = (-MB\cos\theta_2) - (-MB\cos\theta_1)$$

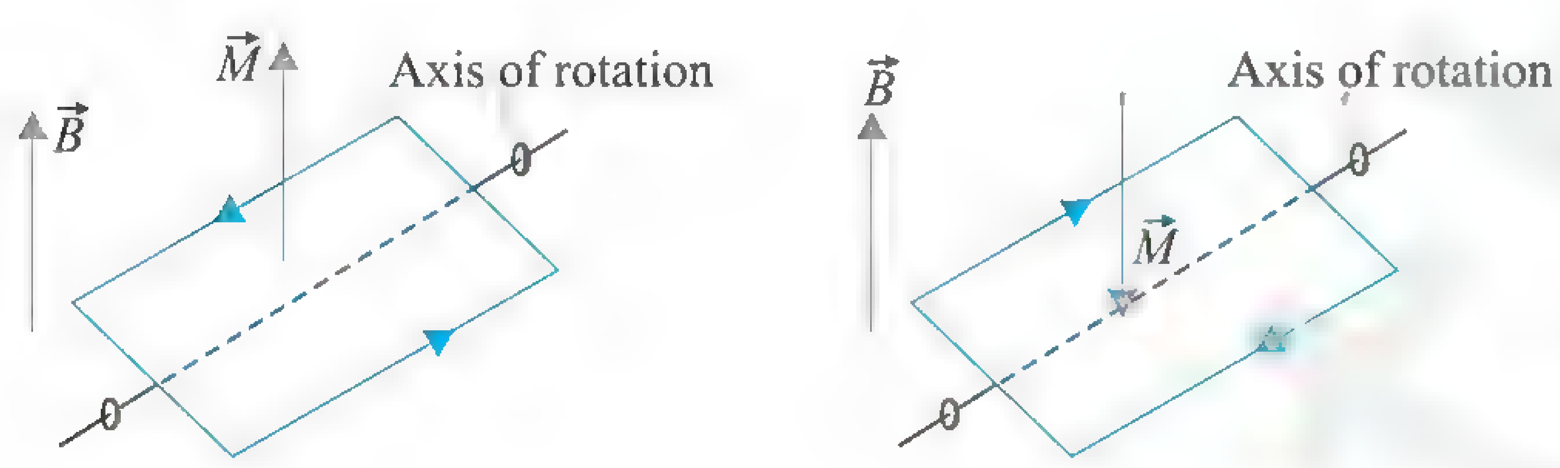
$$\Rightarrow \quad W = MB(\cos\theta_1 - \cos\theta_2) \quad ...(iii)$$

Stable and Unstable Equilibrium of a Current-Carrying Loop in Magnetic Field

Let us consider a current-carrying circular loop placed in a plane perpendicular to the direction of magnetic field. The loop carries a current in clockwise direction due to which the direction vector of its magnetic moment is along the direction of magnetic induction due to which the angle between these two vectors is $\theta = 0$ and thus torque on loop is also $\vec{M} \times \vec{B} = 0$ and also net force acting on it will be zero as it is placed in uniform magnetic field. It means the loop is in equilibrium.

The interaction potential energy of the coil is given as

$$U = -MB\cos\theta = -MB \quad ...(i)$$



(a) (b)

If we slightly tilt the loop about the axis of rotation it will experience net torque and this torque will have a tendency to bring back the loop in its initial position thus therea restoring torque will be developed, hence the equilibrium of coil is stable equilibrium.

We can also see from Eq. (i) that the potential energy of loop in this state is at its minimum value which corresponds to the case of stable equilibrium.

In Fig. (b), a similar situation is shown with the current direction in the loop is in anticlockwise direction. In this case the angle between magnetic moment and magnetic induction is $\theta = 180°$ and thus torque on loop is again $\vec{M} \times \vec{B} = 0$ and loop is in equilibrium.

The interaction potential energy of the coil in this situation is given as

$$U = -MB\cos\theta$$
$$= -MB(-1) = +MB \quad ...(ii)$$

If in Fig. (b) we slightly tilt the loop about the axis of rotation the torque developed will have a tendency to further rotate the coil away from its initial position thus in this case the equilibrium of coil is unstable equilibrium.

We can also see from Eq. (ii) that the potential energy of loop in this state is at its maximum value which corresponds to the case of unstable equilibrium.

ILLUSTRATION 1.52

A circular coil of 100 turns and having a radius of 0.05 m carries a current of 0.1 A. Calculate the work required to turn the coil in an external field of 1.5 T through 180° about an axis perpendicular to the magnetic field? The plane of coil is initially at right angles to magnetic field.

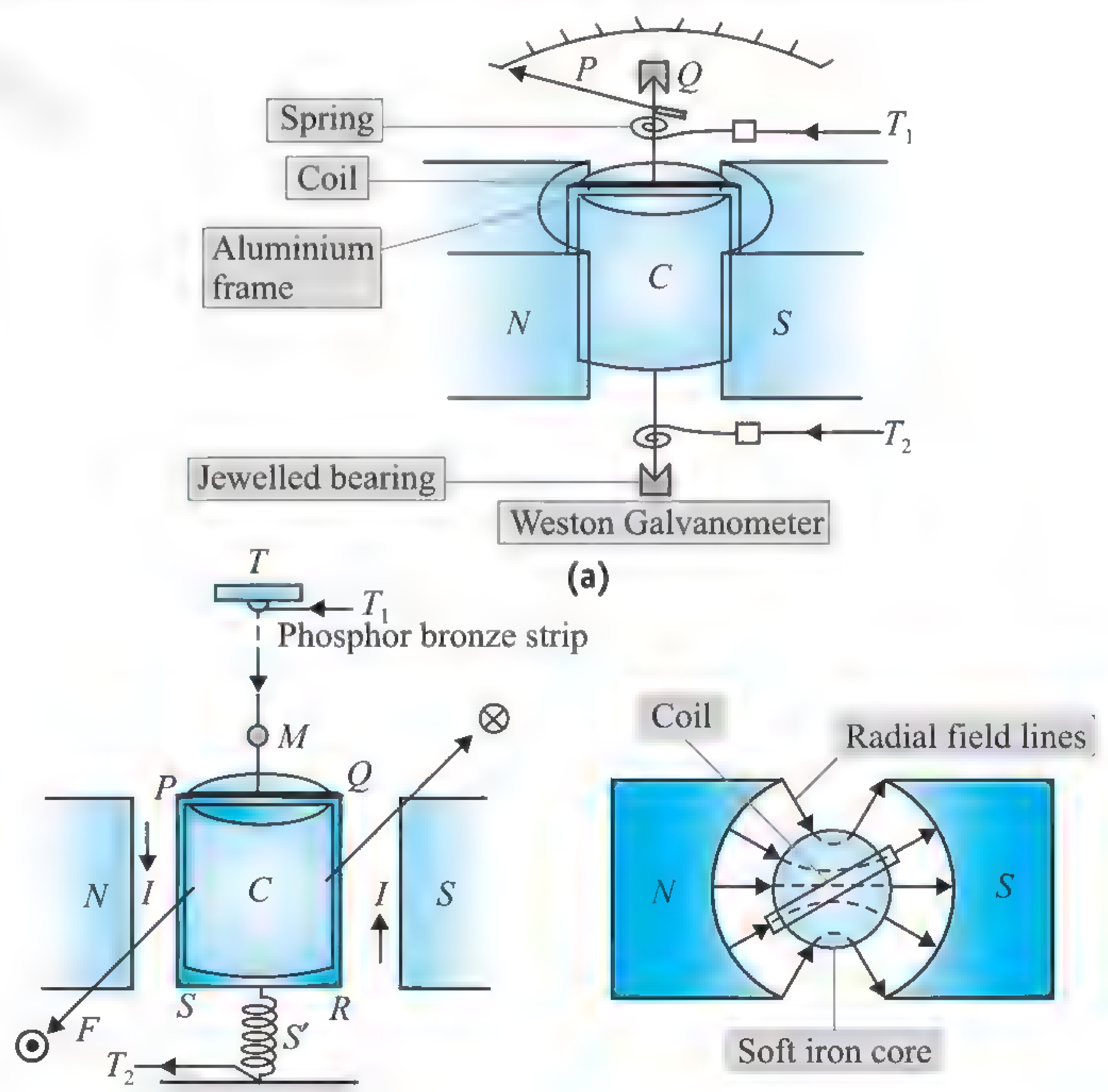
Sol. Work done

$$W = MB(\cos\theta_1 - \cos\theta_2)$$
$$= NIAB(\cos\theta_1 - \cos\theta_2)$$

$$\Rightarrow \quad W = NI\pi r^2 B(\cos\theta_1 - \cos\theta_2)$$
$$= 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5(\cos 0° - \cos\pi)$$
$$= 0.2355 \text{ J}$$

MOVING COIL GALVANOMETER

This is a device which is used for detection and measurement of small electric current. The principle of a moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



(a) (b) (c)

CONSTRUCTION

A moving coil ballistic galvanometer is shown in Fig. (a). It essentially consists of a rectangular coil *PQRS* or a cylindrical coil of large number of turns of fine insulated wire wound over a non-conducting frame of ivory or bamboo [Fig. (b)].This coil is suspended by means of phosphor bronze wire between the pole pieces of a powerful horse shoe magnet NS. The poles of the magnet are curved to make the field radial as shown in Fig. (c). The lower end of the coil is attached to a spring of

phosphorbronze wire. The spring and the free ends of phosphor bronze wire are joined to two terminals T_2 and T_1, respectively, on the top of the case of the instrument. L is a soft iron core. A small mirror M is attached on the suspension wire. Using lamp and scale arrangement, the deflection of the coil can be recorded. The whole arrangement is enclosed in a non-metallic case.

Theory

Let the coil be suspended freely in the magnetic field.

Suppose, N = number of turns in the coil

A = area of the coil

B = magnetic field induction of radial magnetic field in which the coil is suspended.

Here, the magnetic field is radial, i.e., the plane of the coil always remains parallel to the direction of magnetic field, and hence the torque acting on the coil

$$\tau = NiAB \quad \text{...(i)}$$

Due to this torque, the coil rotates. As a result, the suspension wire gets twisted. Now a restoring torque is developed in the suspension wire. The coil will rotate till the deflecting torque acting on the coil due to flow of current through it is balanced by the restoring torque developed in the suspension wire due to twisting. Let C be the restoring couple per unit twist in the suspension wire and θ be the angle through which the coil has turned. The couple for this twist θ is $C\theta$.

In equilibrium, deflecting couple = restoring couple

$$NiAB = C\theta$$

or $i = C\theta/(NAB)$

or $i = K\theta$ (where $C/NAB = K$) ...(ii)

K is a constant for galvanometer and is known as galvanometer constant.

Hence, $i \propto \theta$

Therefore, the deflection produced in the galvanometer is directly proportional to the current flowing through it.

Current Sensitivity of the Galvanometer

The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit current is passed through it.

We know that, $NiAB = C\theta$

$$\therefore \text{ Current sensitivity, } i_s = \frac{\theta}{i} = \frac{NAB}{C}$$

The unit of current sensitivity is radian per ampere or deflection per ampere.

Voltage Sensitivity of the Galvanometer

The voltage sensitivity of the galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across the terminals of the galvanometer.

$$\therefore \text{ Voltage sensitivity, } V_s = \frac{\theta}{V}$$

If R be the resistance of the galvanometer and a current i is passed through it, then

$$V = iR$$

$$\text{Voltage sensitivity, } V_s = \frac{\theta}{iR} = \frac{NAB}{CR}$$

The unit of voltage sensitivity is radian per volt or deflection per volt.

Conditions for Sensitive Galvanometer

A galvanometer is said to be more sensitive if it shows a large deflection evern for a small value of current.

We know that $\theta = \dfrac{NAB}{C} i$

For a given value of i, θ will be large if

(1) N is large, (2) A is large, (3) B is large, and (4) C is small.

The value of C for quartz and phosphor-bronze is very small so the suspension wire of quartz or phosphor-bronze is used. N and A cannot be increased beyond a certain limit, because otherwise size of the galvanometer increases. B can be increased using strong magnets.

Illustration 1.53

A galvanometer having 30 divisions has current sensitivity of 20 μA/division. Find the maximum current it can measure.

Sol. maximum current that can be measured:

I_g = current sensitivity × number of divisions

= (20 μA/division) × (30) = 600 μA

Illustration 1.54

A rectangular coil of area 5.0×10^{-4} m^2 and 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 90 G (radial here means the field liners are in the plane of the coil for any rotation). What is the torsional constant of the hair spring connected to the coil if a current of 2.0 mA produces an angular deflection of 18°?

Sol. Torsional constant is given by: $C = \dfrac{INBA}{\theta}$

$$\Rightarrow \quad C = \frac{2 \times 10^{-3} \times 60 \times 90 \times 10^{-4} \times 5 \times 10^{-4}}{18^\circ}$$

$$= 3 \times 10^{-8} \text{ Nm/degree}$$

Illustration 1.55

The coil of a galvanometer is 0.02×0.8 m^2. It consists of 200 turns of the wire and is in a magnetic field of 0.20 T. The restoring torque constant of suspension fibre is 10^{-5} Nm/degree. Assuming magnetic field to be radial

(a) What is the maximum current that can be measured by this galvanometer if scale can accommodate 45° deflection?

(b) What is the smallest current that can be detected if minimum observed deflection is 0.1°?

Sol. We know that $I = \dfrac{C}{NBA}\theta$

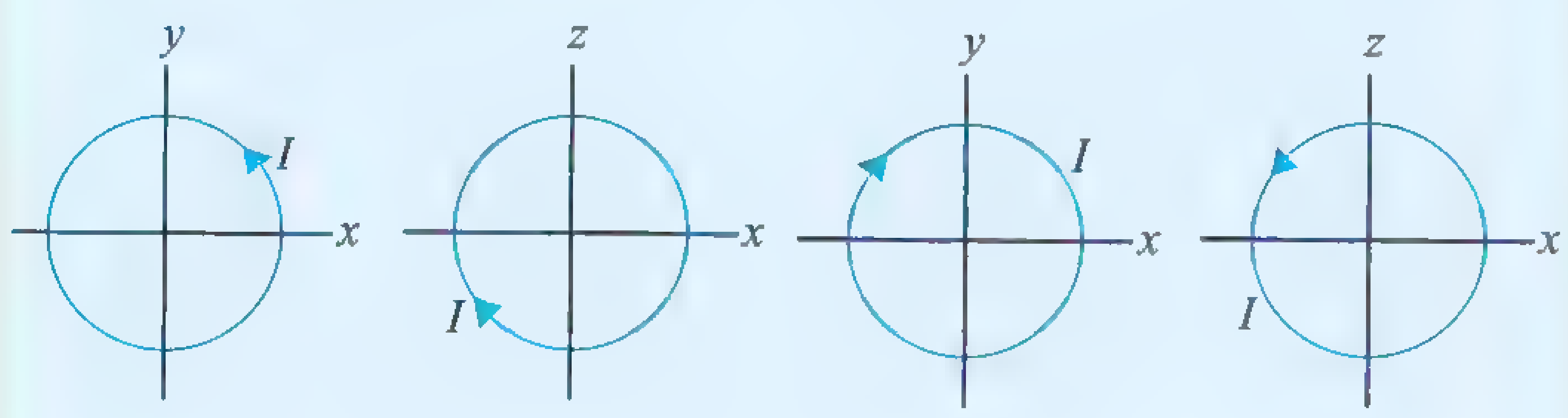
(a) $$I = \frac{10^{-5} \times 45^\circ}{200 \times 0.20 \times 0.02 \times 0.8} = 7.03 \times 10^{-4} \text{ A}$$

(b) $$I_{\min} = \frac{C}{NBA}\theta_{\min}$$

$$= \frac{10^{-5} \times 0.1^\circ}{200 \times 0.20 \times 0.02 \times 0.8} = 1.56 \times 10^{-6} \text{ A}$$

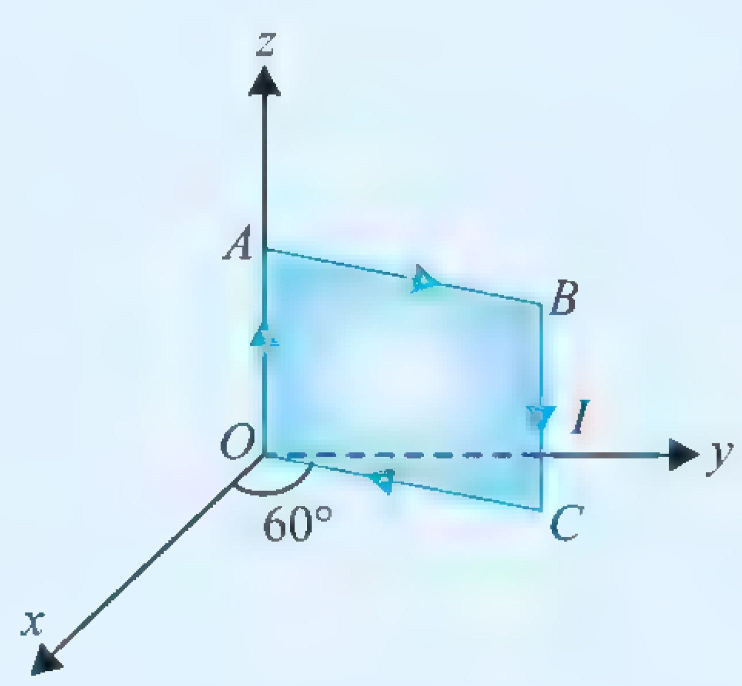
Concept Application Exercise 1.4

1. A circular coil with area A and N turns is free to rotate about a diameter that coincides with the x-axis. Current I is circulating in the coil. There is a uniform magnetic field $\vec{B}$ in the positive y direction. Calculate the magnitude and direction of the torque $\vec{\tau}$ and the value of the potential energy U, when the coil is oriented as shown in parts (a) through (d) of figure.

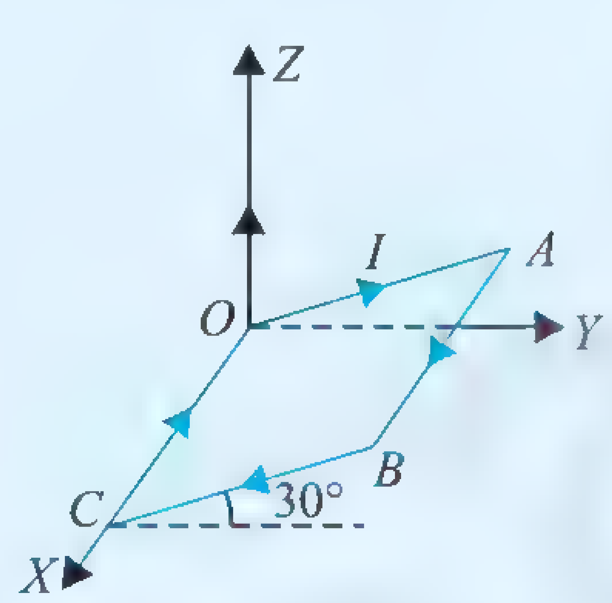


(a) **(b)** **(c)** **(d)**

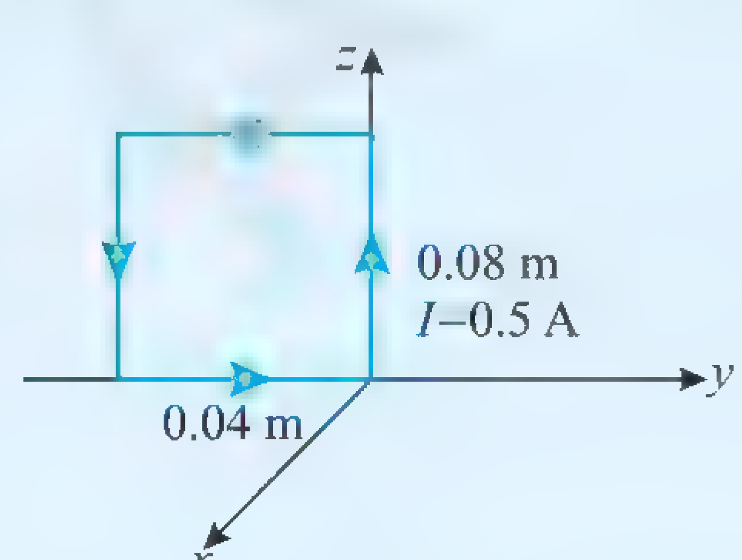
2. A square loop $OABCO$ of side l carries a current I. It is placed as shown in figure. Find the magnetic moment of the loop.



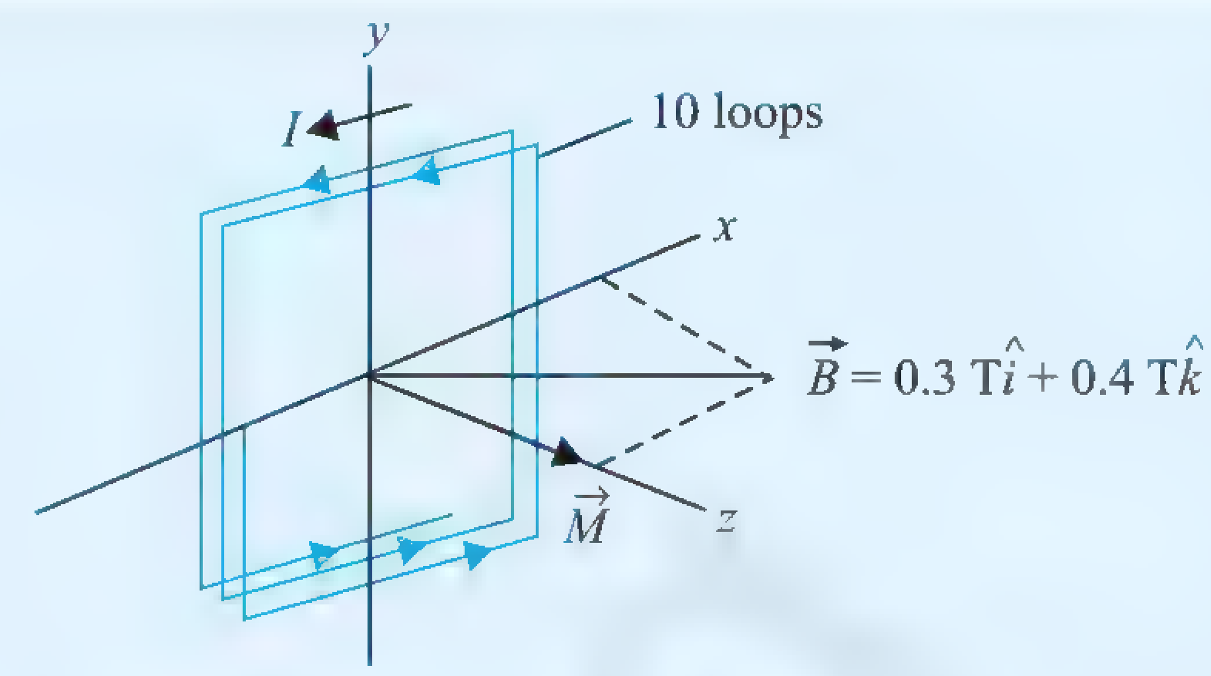
3. Find the magnetic moment of the current carrying loop $OABCO$ as shown in figure. Given that i = 4 A, OA = 20 cm, and AB = 10 cm.



4. The rectangular coil having 100 turns is placed in a uniform magnetic field of $(0.05/\sqrt{2})\hat{j}$ tesla as shown in figure. Find the torque acting on the loop.

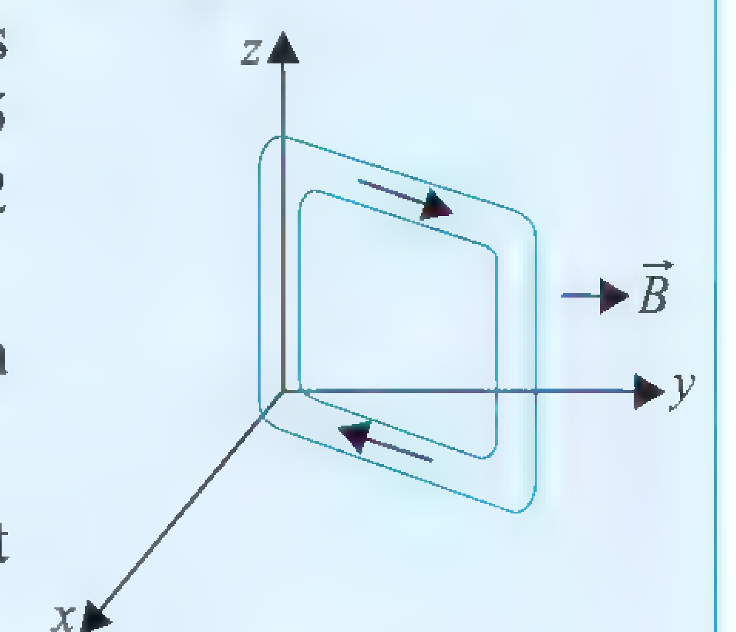


5. A square 10-turn coil with sides of length 40 cm carries a current of 5 A. It lies in the x–y plane as shown in figure in a uniform magnetic field.



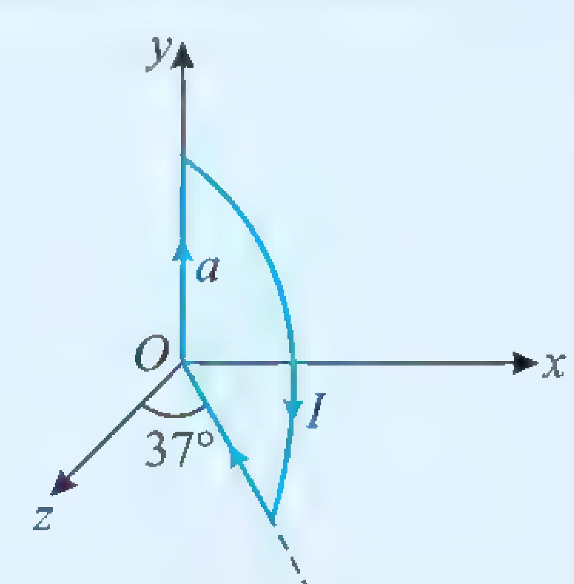
(a) Find the magnetic moment of the coil.
(b) Find the torque exerted on the coil.
(c) Find the potential energy of the coil.

6. The square loop in figure has sides of length 20 cm. It has 5 turns and carries a current of 2 A. The normal to the loop is at 37° to a uniform field $\vec{B} = 0.5\hat{j}$ T.



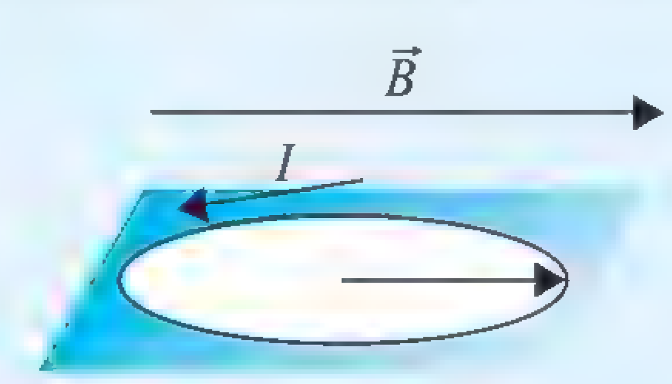
(a) Find the magnetic moment of the loop.
(b) Find the torque on the loop.
(c) Find the work needed to rotate the loop from its position of minimum energy to the given orientation.

7. The figure shows one quarter of a simple circular loop of wire that carries a current of 10 A. Its radius is a = 5 cm. A uniform magnetic field, B = 400 G, is directed in the $+x$ direction. Find the torque on the entire loop and the direction in which it will rotate.



8. A galvanometer coil is replaced by another coil of diameter one-fourth of the original diameter and the total number of turns as ten times the original number. What will be the new deflection if the same current is passed through it? Old deflection is θ.

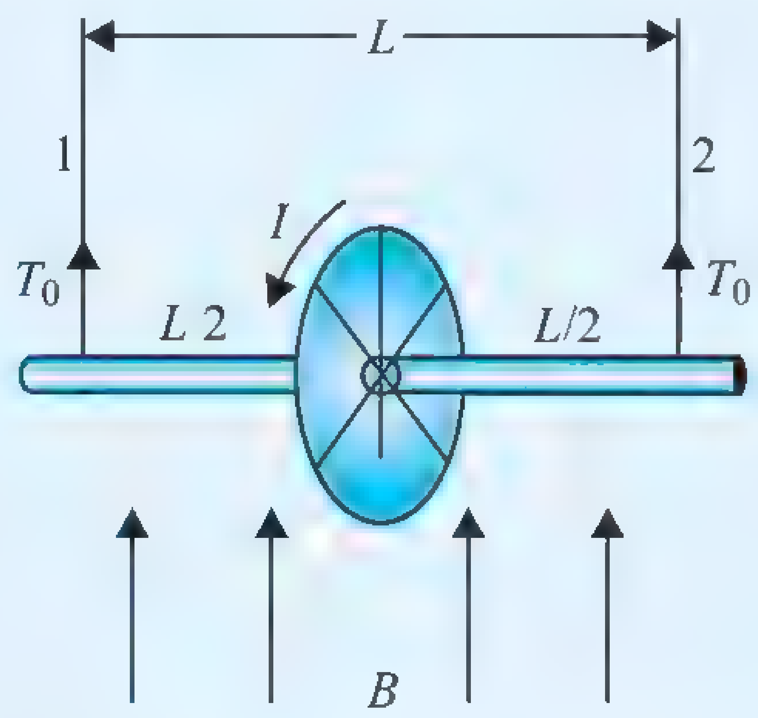
9. A circular wire loop of radius R, mass m carrying current I lies on a rough surface (as shown in figure). There is a horizontal magnetic field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface?



10. A wire is formed into a circle having a diameter of 10.0 cm and placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find

(a) the maximum torque on the wire, and
(b) the range of potential energies of the wire–field system for different orientations of the circle.

11. The circular current loop of radius b shown in the figure is mounted rigidly on the axle, midway between the two supporting cords. In the absence of an external magnetic field, the tensions in the cords are equal and are T_0.

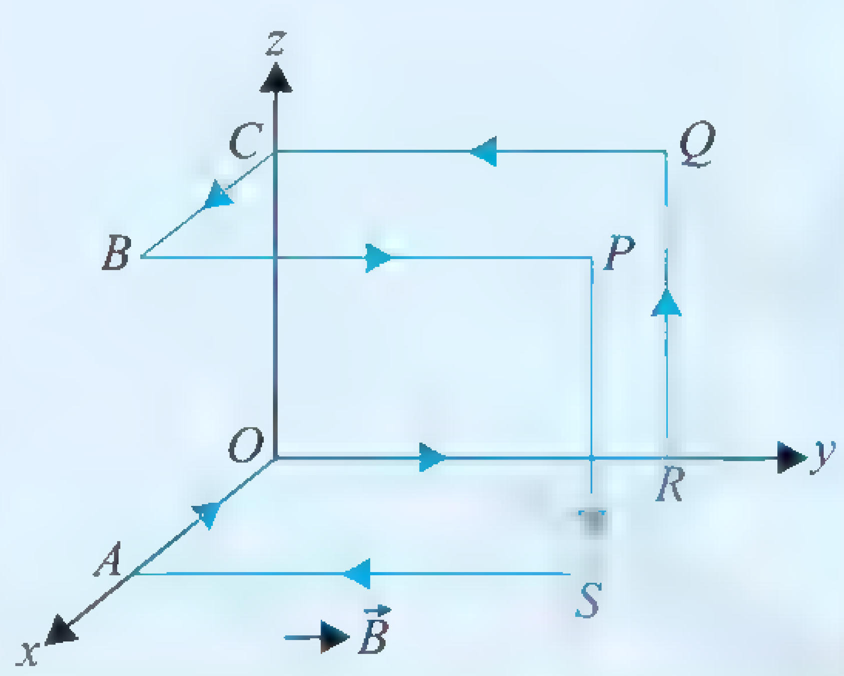


(a) What will be the tensions in the two cords when the vertical magnetic field B is present?

(b) Repeat if the field is parallel to the axis.

12. A circular coil of 100 turns and radius 5.0 cm, carrying a current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 2.0 T. The field lines run horizontally in the plane of the coil. Calculate the force and torque on coil due to the magnetic field. In which direction should a balancing torque be applied to prevent the coil from turning?

13. The figure shows a coil bent with all edges of length 2 m and carrying a current of 2A. There exists in space a uniform magnetic field of 2T in positive y-direction. Find the torque on the loop.



14. A particle of charge q moves in a circle of radius r with speed v. Treating the circular path as a current loop with an average current, find the maximum torque exerted on the loop by a uniform field of magnitude B.

Answers

1. (a) $NIAB(-\hat{i})$, 0 (b) 0, $-NIAB$ (c) $NIAB(\hat{i})$, 0 (d) 0, $NIAB$

2. $\dfrac{Il^2}{2}(-\sqrt{3}\hat{i} + \hat{j})$ **3.** $4 \times 10^{-2}(\hat{j} - \sqrt{3}\hat{k})$ Am^2

4. $4\sqrt{2} \times 10^{-3}$(Nm)$\hat{k}$

5. (a) $8.0\hat{k}$ Am^2 (b) $2.4\hat{j}$ N-m (c) -3.2 J

6. (a) $-0.24\hat{i} + 0.32\hat{j}$ Am^{-2} (b) $-0.12\hat{k}$ Nm (c) 0.04 J

7. $6\pi \times 10^{-4}$ Nm; loop will rotate about the y-axis, so as to decrease the angle labelled 37°.

8. $\dfrac{5}{8}\theta$ **9.** $\dfrac{mg}{\pi RB}$

10. (a) $375\pi \times 10^{-7}$ Nm (b) $U_{max} = +MB$, $U_{min} = -MB$

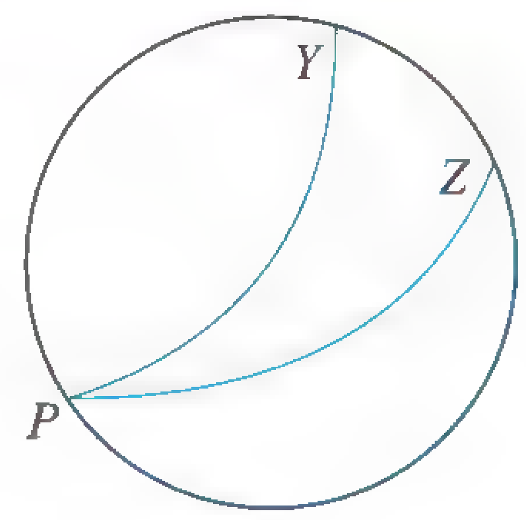
11. (a) $\dfrac{mg}{2} + \dfrac{\pi b^2 BI}{L}, \dfrac{mg}{2} - \dfrac{\pi b^2 BI}{L}$ (b) $mg/2$

12. 5π Nm **13.** 0 **14.** $\dfrac{1}{2}qvrB$

Exercises

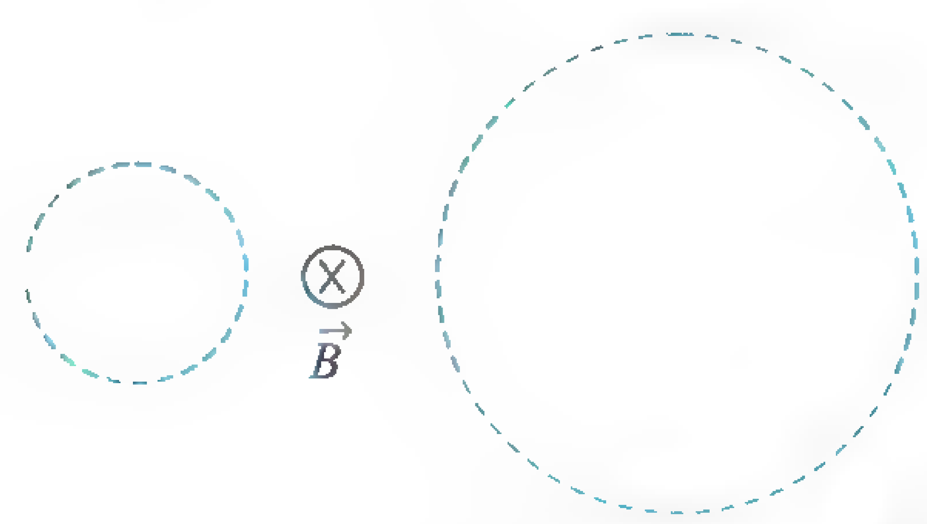
Single Correct Answer Type

1. Two particles Y and Z emitted by a radioactive source at P made tracks in a chamber as illustrated in the figure. A magnetic field acts downward into the paper. Careful measurements showed that both tracks were circular, the radius of Y track being half that of the Z track. Which one of the following statements is certainly true?



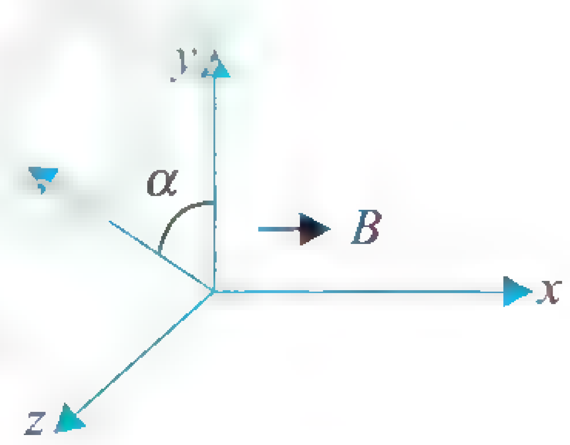
   (1) Both particles Y and Z carried a positive charge
   (2) The mass of particle Z was one-half that of particle Y
   (3) The mass of particle Z was twice that of particle Y
   (4) The charge of particle Z was twice that of particle Y

2. An electron and a proton each travel with equal speeds around circular orbits in the same uniform magnetic field as indicated (not to scale) in figure. The field is into the page on the diagram. The electron travels ______ around the ______ circle and the proton travels ______ around the ______ circle.



   (1) clockwise, smaller, counterclockwise, larger
   (2) counterclockwise, larger, counterclockwise, smaller
   (3) clockwise, larger, counterclockwise, smaller
   (4) counterclockwise, larger, clockwise, smaller

3. In a region of space, a uniform magnetic field B exists in the x-direction. An electron is fired from the origin with its initial velocity u making an angle α with the y direction in the y-z plane. In the subsequent motion of the electron,



   (1) y-coordinate of the electron will never be negative
   (2) z-coordinate of the electron will never be negative
   (3) x-coordinate of the electron will never be negative
   (4) trajectory of the electron would be helical

4. An electron is moving along positive x-axis. To get it moving on an anticlockwise circular path in x–y plane, a magnetic field is applied
   (1) along positive y-axis (2) along positive z-axis
   (3) along negative y-axis (4) along negative z-axis

5. An electron accelerated through a potential difference V passes through a uniform transverse magnetic field and experiences a force F. If the accelerating potential is increased to 2V, the electron in the same magnetic field will experience a force
   (1) F (2) $F/2$
   (3) $\sqrt{2}\,F$ (4) $2F$

6. A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible?
   (1) $E = 0, B = 0$ (2) $E = 0, B \neq 0$
   (3) $E \neq 0, B = 0$ (4) $E \neq 0, B \neq 0$

7. A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one end. If a magnetic field is switched on in the vertical direction, the tension in the string
   (1) will increase (2) will decrease
   (3) remains same (4) may increase or decrease

8. An electron is launched with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The angle θ between $\vec{v}$ and $\vec{B}$ lies between 0 and $\dfrac{\pi}{2}$. Its velocity vector $\vec{v}$ returns to its initial value in a time interval of
   (1) $\dfrac{2\pi m}{eB}$ (2) $\dfrac{2\times 2\pi m}{eB}$
   (3) $\dfrac{\pi m}{eB}$
   (4) depends upon angle between $\vec{v}$ and $\vec{B}$

9. A charged particle moves with velocity $\vec{v} = a\hat{i} + d\hat{j}$ in a magnetic field $\vec{B} = A\hat{i} + D\hat{j}$. The force acting on the particle has magnitude F. Then,
   (1) $F = 0$, if $aD = dA$. (2) $F = 0$, if $aD = -dA$.
   (3) $F = 0$, if $aA = -dD$. (4) $F \propto (a^2 + b^2)^{1/2} \times (A^2 + D^2)^{1/2}$

10. A particle with a specific charge s is fired with a speed v toward a wall at a distance d, perpendicular to the wall. What minimum magnetic field must exist in this region for the particle not to hit the wall?
   (1) v/sd (2) $2v/sd$
   (3) $v/2sd$ (4) $v/4sd$

11. A charged particle begins to move from the origin in a region which has a uniform magnetic field in the x-direction and a uniform electric field in the y-direction. Its speed is v when it reaches the point (x, y, z). Then, v will depend
   (1) only on x
   (2) only on y
   (3) on both x and y, but not z
   (4) on x, y and z

12. A proton and an α-particle enter a uniform magnetic field moving with the same speed. If the proton takes 25 μs to make 5 revolutions, then the periodic time for the α-particle would be
   (1) 50 μs (2) 25 μs
   (3) 10 μs (4) 5 μs

13. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

(1) $(R_1/R_2)^{1/2}$ (2) (R_2/R_1)
(3) $(R_1/R_2)^2$ (4) (R_1/R_2)

14. An electron of mass 0.90×10^{-30} kg under the action of a magnetic field moves in a circle of 2.0 cm radius at a speed of 3.0×10^6 ms^{-1}. If a proton of mass 1.8×10^{-27} kg was to move in a circle of the same radius in the same magnetic field, then its speed will be

(1) 3.0×10^6 m s^{-1}
(2) 1.5×10^3 m s^{-1}
(3) 6.0×10^4 m s^{-1}
(4) cannot be estimated from the given data

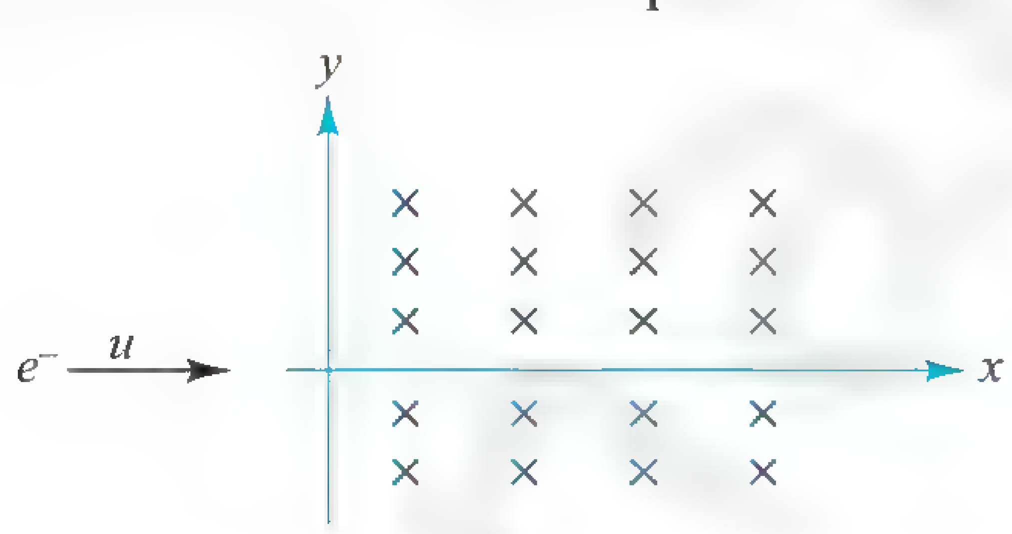
15. A charged particle of mass 10^{-3} kg and charge 10^{-5} C enters a magnetic field of induction 1 T. If $g = 10$ m s^{-2}, for what value of velocity will it pass straight through the field without deflection?

(1) 10^{-3} m s^{-1} (2) 10^3 m s^{-1}
(3) 10^6 m s^{-1} (4) 1 m s^{-1}

16. A particle of mass 2×10^{-5} kg moves horizontally between two horizontal plates of a charged parallel plate capacitor between which there is an electric field of 200 NC^{-1} acting upward. A magnetic induction of 2.0 T is applied at right angles to the electric field in a direction normal to both $\vec{B}$ and $\vec{v}$. If g is 9.8 m s^{-2} and the charge on the particle is 10^{-6} C, then find the velocity of charge particle so that it continues to move horizontally

(1) 2 m s^{-1} (2) 20 m s^{-1}
(3) 0.2 m s^{-1} (4) 100 m s^{-1}

17. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field which exists to the right of y-axis. The electron exits from the region after some time with the speed v at coordinate y, then



(1) $v > u, y < 0$ (2) $v = u, y > 0$
(3) $v > u, y > 0$ (4) $v = u, y < 0$

18. In a moving coil galvanometer, we use a radial magnetic feild so that the galvanometer scale is

(1) logarithmic (2) exponential
(3) linear (4) none of the above

19. The current that must flow through the coil of a galvanometer so as to produce a deflection of one division on its scale is called

(1) charge sensitivity of the galvanometer
(2) current sensitivity of the galvanometer
(3) micro-volt sensitivity
(4) none of the above

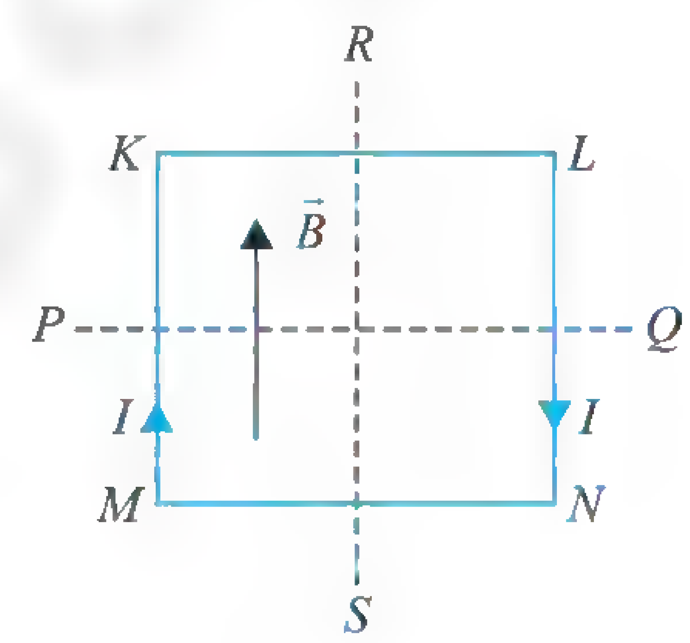
20. In a moving coil galvanometer, the deflection of the coil θ is related to the electric current i by the relation

(1) $i \propto \tan\theta$ (2) $i \propto \theta$
(3) $i \propto \theta^2$ (4) $i \propto \sqrt{\theta}$

21. A current carrying loop lies on a smooth horizontal plane. Then,

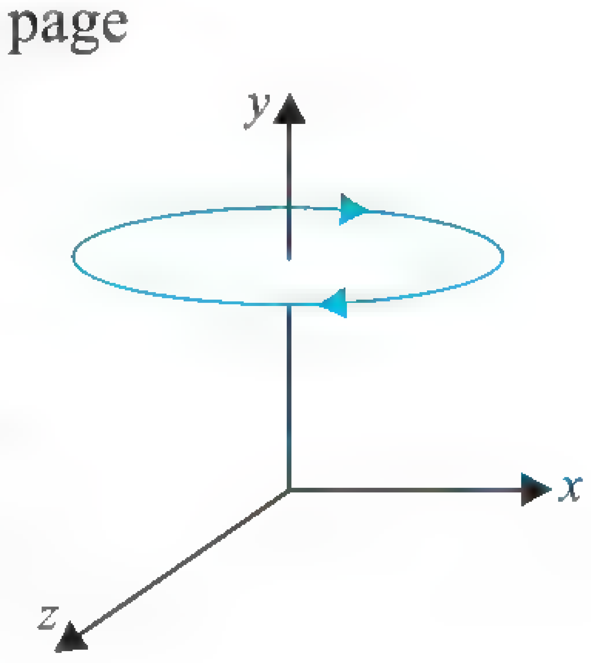
(1) it is possible to establish a uniform magnetic field in the region so that the loop starts rotating about its own axis.
(2) it is possible to establish a uniform magnetic field in the region so that the loop will tip over about any of the point.
(3) it is not possible that loop will tip over about any of the point whatever be the direction of established magnetic field (uniform).
(4) both (1) and (2) are correct.

22. A square loop of wire carrying current I is lying in the plane of paper as shown in figure. The magnetic field is present in the region as shown. The loop will tend to rotate



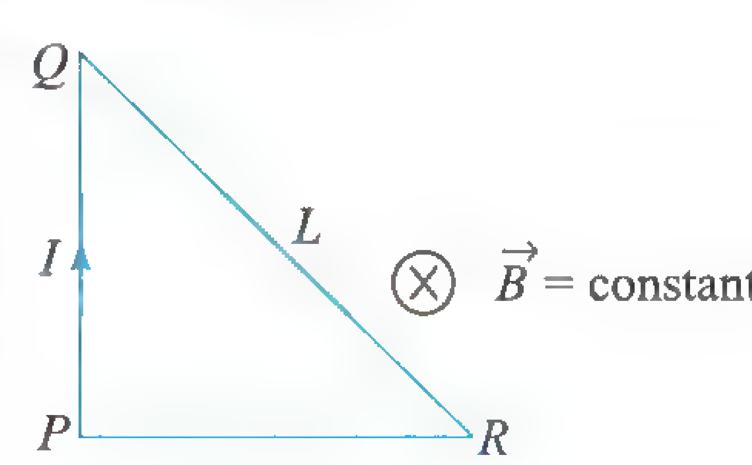
(1) about PQ with KL coming out of the page
(2) about PQ with KL going into the page
(3) about RS with MK coming out of the page
(4) about RS with MK going into the page

23. A circular coil having mass m is kept above ground (x–z plane) at some height. The coil carries a current i in the direction shown in figure. In which direction a uniform magnetic field $\vec{B}$ be applied so that the magnetic force balances the weight of the coil?



(1) Positive x-direction (2) Negative x-direction
(3) Positive z-direction (4) None of these

24. The rigid conducting thin wire frame carries an electric current I and this frame is inside a uniform magnetic field $\vec{B}$ as shown in figure. Then,



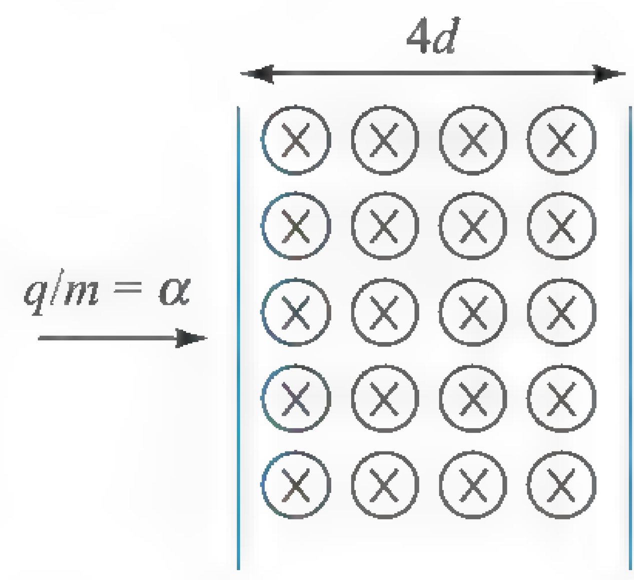
(1) the net magnetic force on the frame is zero but the torque is not zero.
(2) the net magnetic force on the frame and the torque due to magnetic field are both zero.
(3) the net magnetic force on the frame is not zero and the torque is also not zero.
(4) none of these

25. Three particles, an electron (e), a proton (p) and a helium atom (He) are moving in circular paths with constant speeds

in the x–y plane in a region where a uniform magnetic field B exists along z-axis. The times taken by e, p and He inside the field to complete one revolution are t_e, t_p and t_{He} respectively. Then,

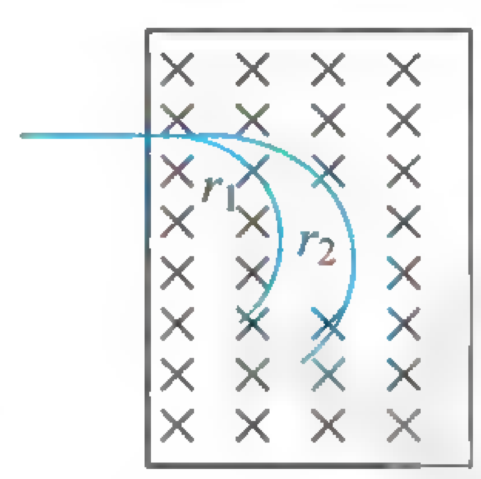
(1) $t_{He} > t_p = t_e$ (2) $t_{He} > t_p > t_e$
(3) $t_{He} = t_p = t_e$ (4) none of these

26. If a charged particle of charge to mass ratio $(q/m) = \alpha$ enters in a magnetic field of strength B at a speed $v = (2\alpha d)(B)$, then



(1) angle subtended by the path of charged particle in magnetic field at the center of circular path is 2π
(2) the charge will move on a circular path and then will come out from magnetic field at some distance from the point of insertion
(3) the time for which particle will be in the magnetic field is $\dfrac{2\pi}{\alpha B}$
(4) angle subtended by the path of charged particle in magnetic field at the center of circular path is $\pi/2$

27. A beam of mixture of α particles and protons are accelerated through same potential difference before entering into the magnetic field of strength B. If $r_1 = 5$ cm, then r_2 is



(1) 5 cm (2) $5\sqrt{2}$ cm
(3) $10\sqrt{2}$ cm (4) 20 cm

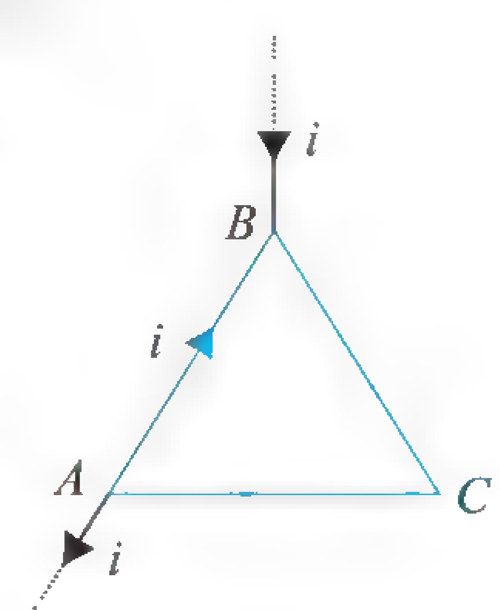
28. An electron is projected at an angle θ with a uniform magnetic field. If the pitch of the helical path is equal to its radius, then the angle of projection is

(1) $\tan^{-1} \pi$ (2) $\tan^{-1} 2\pi$
(3) $\cot^{-1} \pi$ (4) $\cot^{-1} 2\pi$

29. A charged particle moves in a uniform magnetic field perpendicular to it, with a radius of curvature 4 cm. On passing through a metallic sheet it loses half of its kinetic energy. Then, the radius of curvature of the particle is

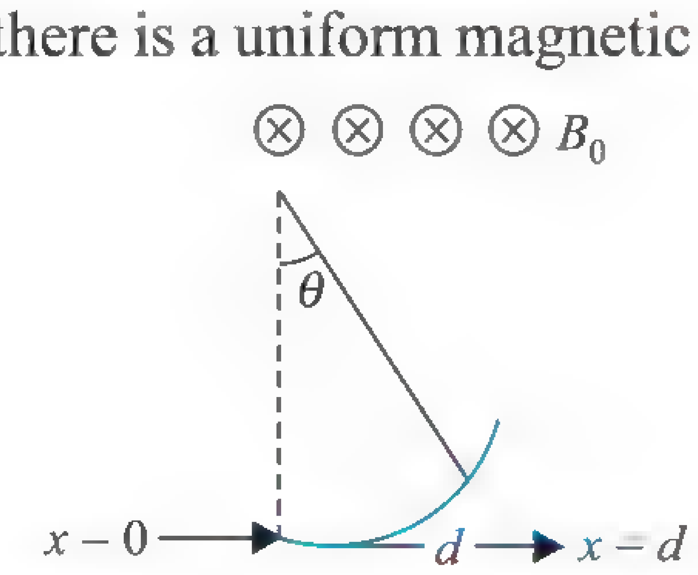
(1) 2 cm (2) 4 cm
(3) 8 cm (4) $2\sqrt{2}$ cm

30. Figure shows an equilateral triangle ABC of side l carrying currents as shown, and placed in a uniform magnetic field B perpendicular to the plane of triangle. The magnitude of magnetic force on the triangle is



(1) ilB (2) $2ilB$
(3) $3\ ilB$ (4) zero

31. A charged particle moving along +ve x-direction with a velocity v enters a region where there is a uniform magnetic field $B_0(-\hat{k})$, from $x = 0$ to $x = d$. The particle gets deflected at an angle θ from its initial path. The specific charge of the particle is



(1) $\dfrac{v\cos\theta}{Bd}$ (2) $\dfrac{v\tan\theta}{Bd}$
(3) $\dfrac{v}{Bd}$ (4) $\dfrac{v\sin\theta}{Bd}$

32. A particle of specific charge $\dfrac{q}{m} = \pi$ C kg^{-1} is projected from the origin toward positive x-axis with a velocity of 10 m s^{-1} in a uniform magnetic field $\vec{B} = -2\hat{k}$ T. The velocity $\vec{v}$ of particle after time $t = 1/12$ s will be (in ms^{-1})

(1) $5[\hat{i} + \sqrt{3}\,\hat{j}]$ (2) $5[\sqrt{3}\,\hat{i} + \hat{j}]$
(3) $5[\sqrt{3}\,\hat{i} - \hat{j}]$ (4) $5[\hat{i} + \hat{j}]$

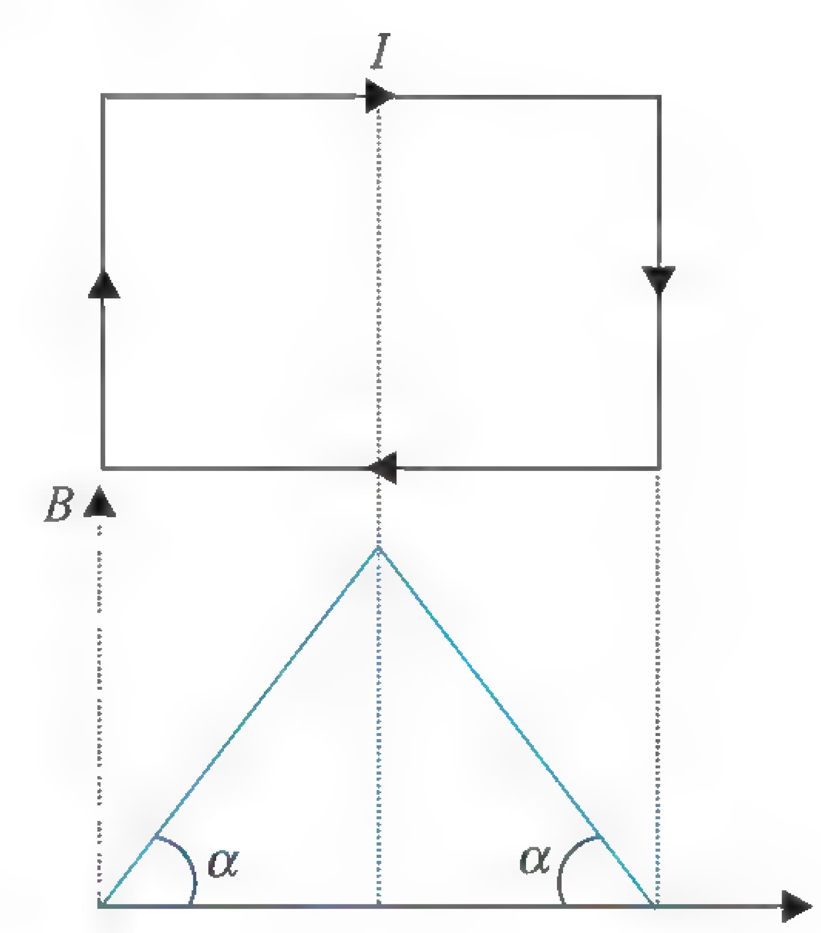
33. There exist uniform magnetic and electric fields of magnitudes 1 T and 1 V m^{-1}, respectively, along positive y-axis. A charged particle of mass 1 kg and charge 1 C is having velocity 1 m s^{-1} along x-axis and is at origin at $t = 0$. Then, the coordinates of the particle at time π s will be

(1) (0, 1, 2) m (2) (0, $-\pi$ 2, –2) m
(3) (2, $\pi^2/2$, 2) m (4) (0, $\pi^2/2$, 2) m

34. A uniform magnetic field exists in a region which forms an equilateral triangle of side a. The magnetic field is perpendicular to the plane of the triangle. A charge q enters into this magnetic field perpendicular to a side with speed v. The charge enters from midpoint and leaves the field from mid-point of other side. Magnetic induction in the triangle is

(1) $\dfrac{mv}{qa}$ (2) $\dfrac{2mv}{qa}$
(3) $\dfrac{mv}{2qa}$ (4) $\dfrac{mv}{4qa}$

35. A current carrying loop is placed in the non-uniform magnetic field whose variation in space is shown in figure. Direction of magnetic field is into the plane of paper. The magnetic force experienced by the loop is



(1) non-zero
(2) zero
(3) cannot say anything
(4) none of the above

36. A proton of mass 1.67×10^{-27} kg and charge 1.67×10^{-19} C is projected with a speed of 2×10^6 m s^{-1} at an angle of 60° to the X-axis. If a uniform magnetic field of 0.10 T is applied along Y-axis, the path of proton is

(1) a circle of radius 0.2 m and time period $\pi \times 10^{-7}$ s
(2) a circle of radius 0.1 m and time period $2\pi \times 10^{\ 7}$ s
(3) a helix of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(4) a helix of radius 0.2 m and time period $4\pi \times 10^{\ 7}$ s

37. An electron is accelerated from rest through a potential difference V. This electron experiences a force F in a uniform magnetic field. On increasing the potential difference to V', the force experienced by the electron in the same magnetic field becomes $2F$. Then, the ratio (V'/V) is equal to

(1) 4/1 (2) 2/1
(3) 1/2 (4) 1/4

38. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time

(1) $t = \dfrac{2mv_0}{qE}$ (2) $t = \dfrac{2Bq}{mv_0}$

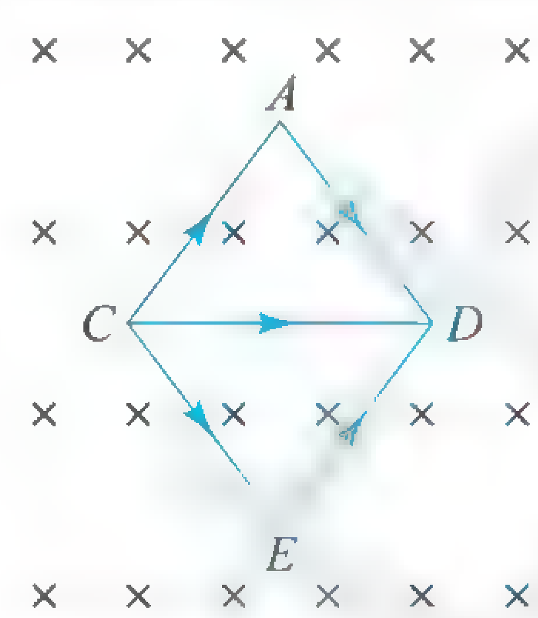
(3) $t = \dfrac{\sqrt{3}Bq}{mv_0}$ (4) $t = \dfrac{\sqrt{3}mv_0}{qE}$

39. A conducting rod of mass m and length l is placed over a smooth horizontal surface. A uniform magnetic field B is acting perpendicular to the rod. Charge q is suddenly passed through the rod and it acquires an initial velocity v on the surface, then q is equal to

(1) $\dfrac{2mv}{Bl}$ (2) $\dfrac{Bl}{2mv}$

(3) $\dfrac{mv}{Bl}$ (4) $\dfrac{Blv}{2m}$

40. Let current $i = 2$ A be flowing in each part of a wire frame as shown in figure. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field B = 4 T acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



(1) 24 N (2) zero
(3) 16 N (4) 8 N

41. A charged particle enters a uniform magnetic field with velocity vector at an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p. The radius of the helix will be

(1) $\dfrac{p}{\sqrt{2}\pi}$ (2) $\sqrt{2}p$

(3) $\dfrac{p}{2\pi}$ (4) $\dfrac{\sqrt{2}p}{\pi}$

42. The plane of a rectangular loop of wire with sides 0.05 m and 0.08 m is parallel to a uniform magnetic field of induction 1.5×10^{-2} T. A current of 10.0 ampere flows through the loop. If the side of length 0.08 m is normal and the side of length 0.05 m is parallel to the lines of induction, then the torque acting on the loop is

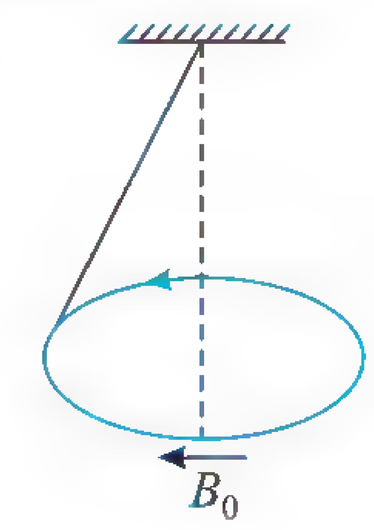
(1) 6000 Nm (2) zero
(3) 1.2×10^{-2} Nm (4) 6×10^{-4} Nm

43. A loop of flexible conducting wire of length l lies in magnetic field B which is normal to the plane of loop. A current I is passed through the loop. The tension developed in the wire to open up is

(1) $\dfrac{\pi}{2}BIl$ (2) $\dfrac{BIl}{2}$

(3) $\dfrac{BIl}{2\pi}$ (4) BIl

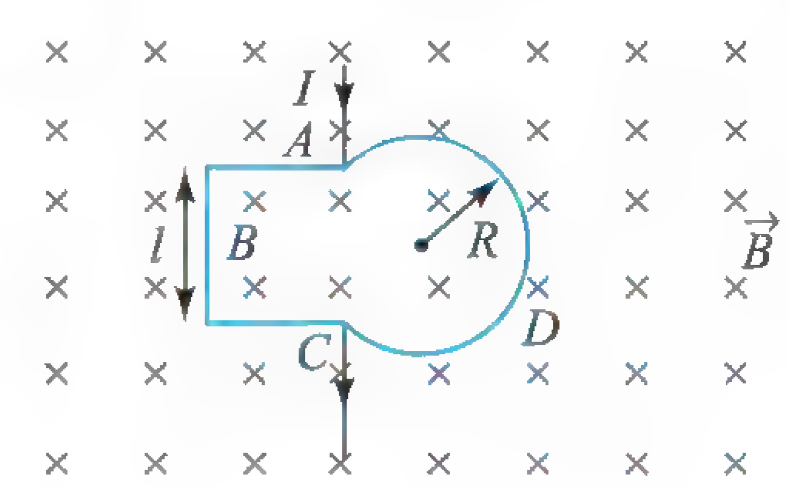
44. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown in figure. A uniform magnetic field B_0 exists in the region to keep the ring in horizontal position, then the current in the ring is (l = length of string)



(1) $\dfrac{mg}{\pi RB_0}$ (2) $\dfrac{mg}{RB_0}$

(3) $\dfrac{mg}{3\pi RB_0}$ (4) $\dfrac{mg\,l}{\pi R^2 B_0}$

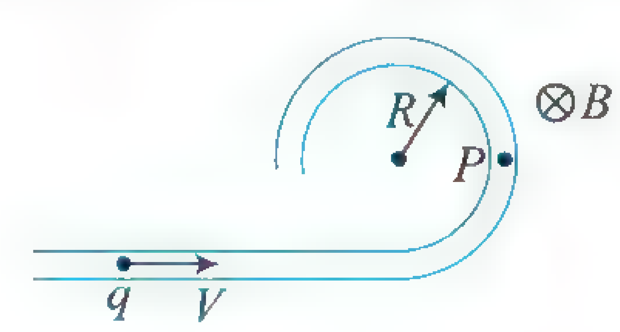
45. Figure shows a conducting loop $ABCDA$ placed in a uniform magnetic field (strength B) perpendicular to its plane. The part ABC is the (three-fourth) portion of the square of side length l. The part ADC is a circular arc of radius R. The points A and C are connected to a battery which supplies a current I to the circuit. The magnetic force on the loop due to the field B is



(1) zero (2) BIl

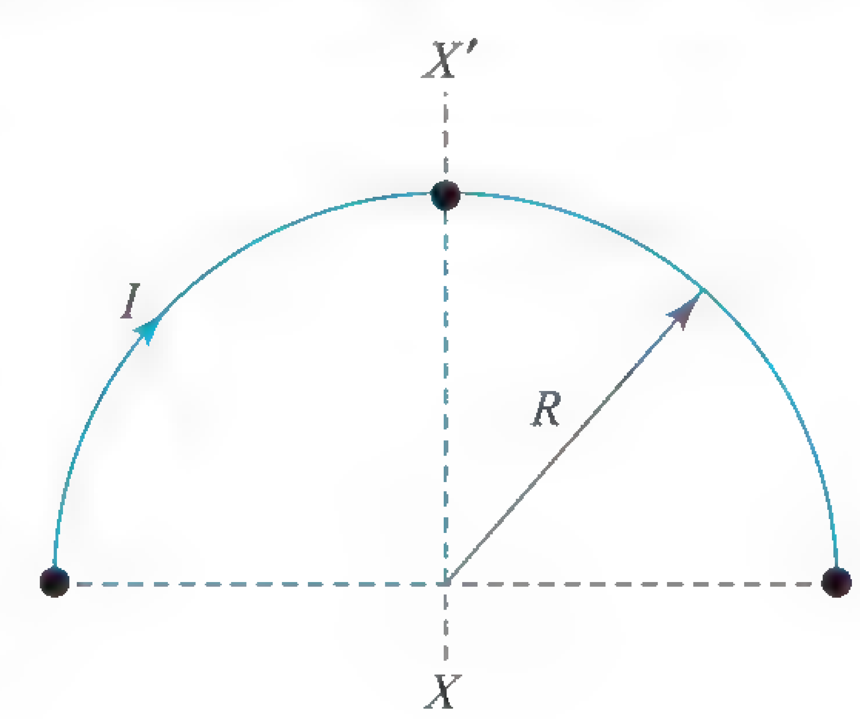
(3) $2BIR$ (4) $\dfrac{BIlR}{I+R}$

46. A charged particle moves insides a pipe which is bent as shown in figure. If $R < \dfrac{mv}{qB}$, then force exerted by the pipe on charged particle at P is (Neglect gravity)



(1) toward center (2) away from center
(3) zero (4) none of these

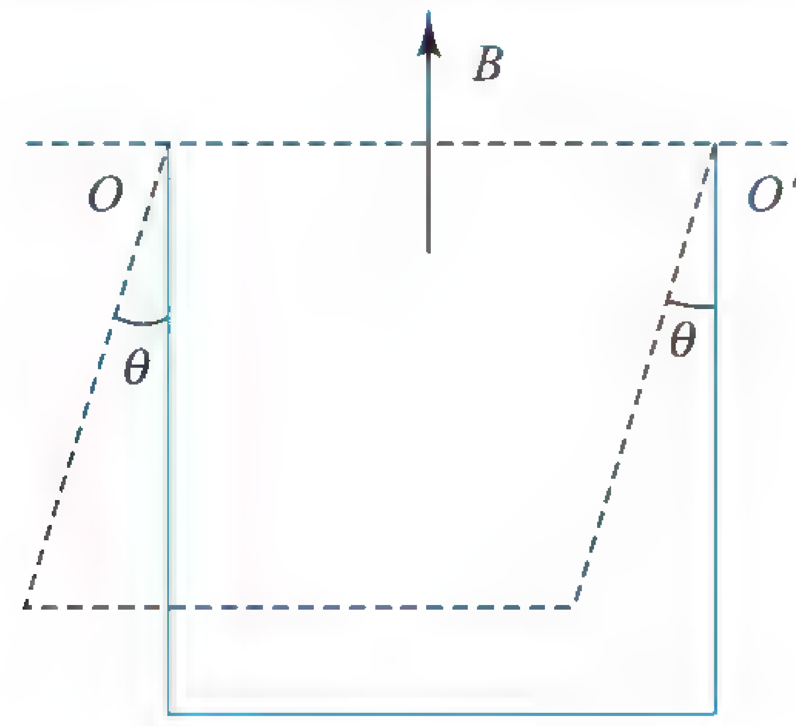
47. A semicircular wire of radius R, carrying current I, is placed in a magnetic field as shown in figure. On left side of $X'X$, magnetic field strength is B_0, and on right side of $X'X$, magnetic field strength is $2B_0$. Both fields are directed inside the page. The magnetic force experienced by the wire would be



(1) $3IB_0R$ (2) $2IB_0R$

(3) $\sqrt{10}\ IB_0R$ (4) $\sqrt{5}\ IB_0R$

48. A wire of cross-sectional area A forms three sides of a square and is free to rotate about axis OO'. If the structure is deflected by an angle θ from the vertical when current i is passed through it in a magnetic field B acting vertically upward and density of the wire is ρ, then the value of θ is given by



(1) $\dfrac{2A\rho g}{iB} = \cot\theta$ (2) $\dfrac{2A\rho g}{iB} = \tan\theta$

(3) $\dfrac{A\rho g}{iB} = \sin\theta$ (4) $\dfrac{A\rho g}{2iB} = \cos\theta$

49. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction being parallel to the axis along east to west. A current carrying wire in north–south direction passes through this region. The wire intersects the axis and experiences a force of 1.2 N downward. If the wire is turned from north south to northeast–southwest direction, then magnitude and direction of force is

(1) 1.2 N, upward (2) $1.2\sqrt{2}$, downward

(3) 1.2 N, downward (4) $\dfrac{1.2}{\sqrt{2}}$ N, downward

50. A particle of positive charge q and mass m enters with velocity $V\hat{j}$ at the origin in a magnetic field $B(-\hat{k})$ which is present in the whole space. The charge makes a perfectly inelastic collision with an identical particle (having same charge) at rest but free to move at its maximum positive y-coordinate. After collision, the combined charge will move on trajectory

$\left(\text{where } r = \dfrac{mV}{qB}\right)$

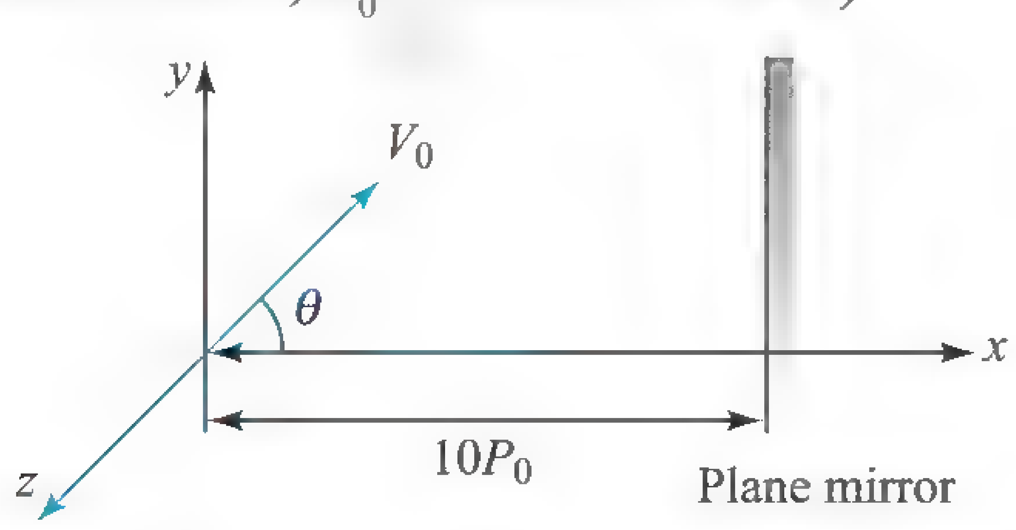
(1) $y = \dfrac{mv}{qB}x$

(2) $(x + r)^2 + (y - r/2)^2 = r^2/4$

(3) $(x + r)^2 + (y - r/2)^2 = r^2/8$

(4) $(x - r)^2 + (y + r/2)^2 = r^2/4$

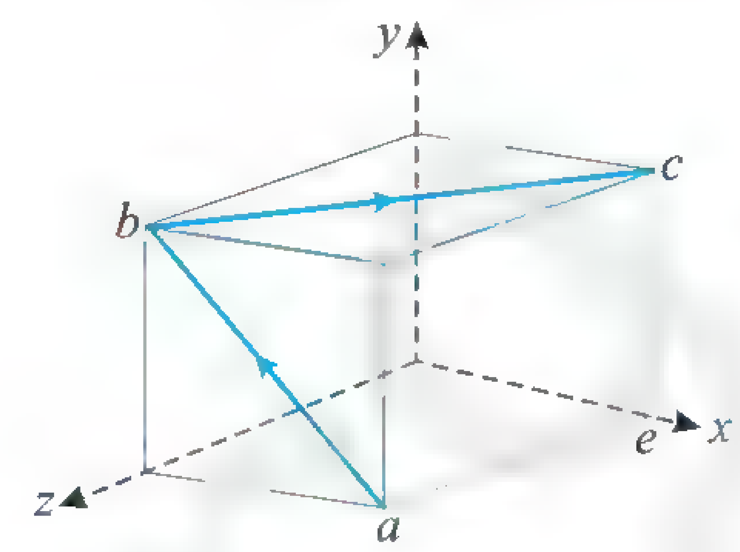
51. In the plane mirror, the coordinates of image of a charged particle (initially at origin as shown in figure) after two and a half time periods are (initial velocity of charge particle is V_0 in the x-y plane and the plane mirror is perpendicular to the x-axis. A uniform magnetic field $B\hat{i}$ exists in the space. P_0 is pitch of helix, R_0 is radius of helix.)



(1) $17P_0, 0, -2R_0$ (2) $3P_0, 0, -2R_0$

(3) $17.5P_0, 0, -2R_0$ (4) $3P_0, 0, 2R_0$

52. Two straight segments of wire ab and bc each carrying current I, are placed as shown in figure. The cube edge is 50 cm and magnetic field is uniform along Y-axis having magnitude 0.4 T. If $I = 3$ A, the force experienced by wire abc in the presence of magnetic field is



(1) $0.6\hat{i}$ (2) $1.2(\hat{i} + \hat{k})$

(3) $0.6\left(\sqrt{2}\hat{i} + \hat{j} - \sqrt{2}\hat{k}\right)$ (4) $0.6\left(\sqrt{2}\hat{i} - \hat{k}\right)$

53. A particle of specific charge α is projected from origin with velocity $\vec{v} = v_0\hat{i} - v_0\hat{k}$ in a uniform magnetic field $\vec{B} = -B_0\hat{k}$. Find time dependence of velocity of the particle.

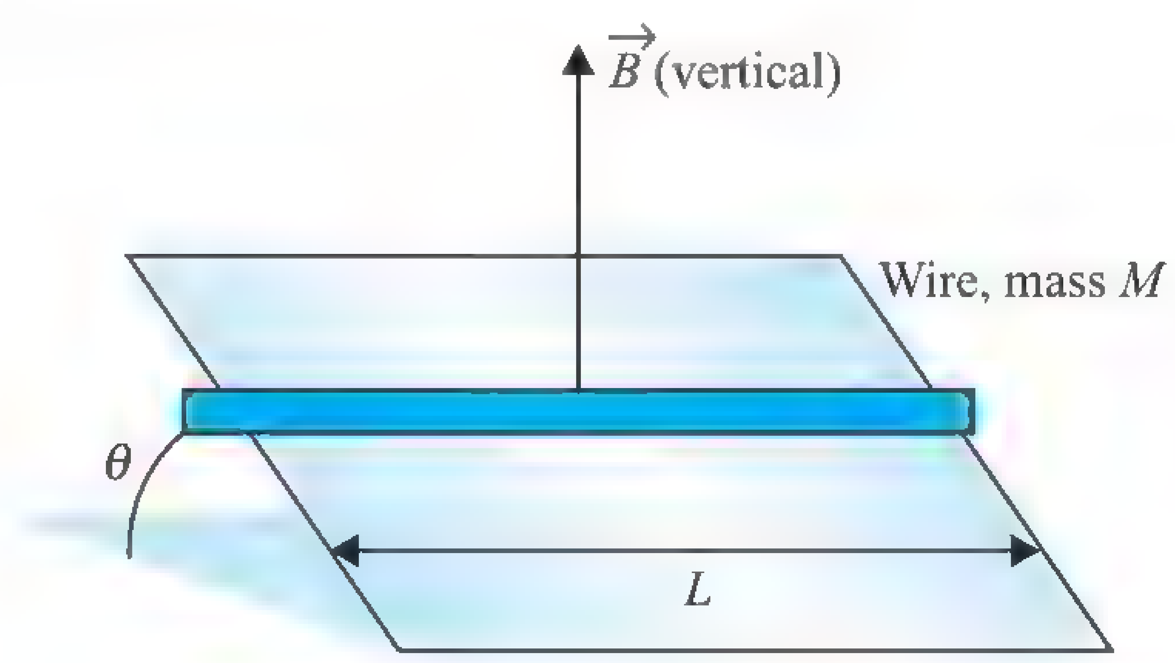
(1) $\vec{v}_{(t)} = v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} - v_0\hat{k}$

(2) $\vec{v}_{(t)} = -v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} + v_0\hat{k}$

(3) $\vec{v}_{(t)} = -v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} - v_0\hat{k}$

(4) $\vec{v}_{(t)} = v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} + v_0\hat{k}$

54. A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (as shown in figure). There is a uniform, vertical magnetic field at all points (produced by an arrangement of magnets not shown in figure). To keep the wire from sliding down the incline, a voltage source is

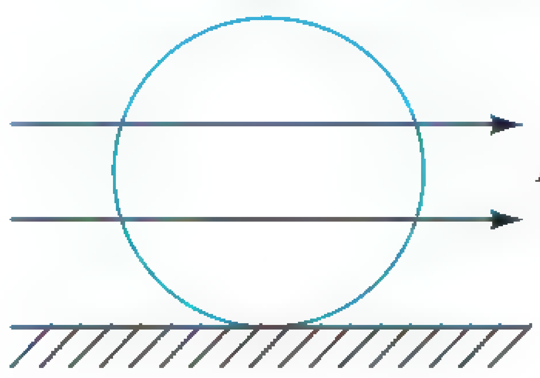


attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest.

(1) $\dfrac{Mg\tan\theta}{2LB}$ to the left (2) $\dfrac{Mg\tan\theta}{LB}$ to the right

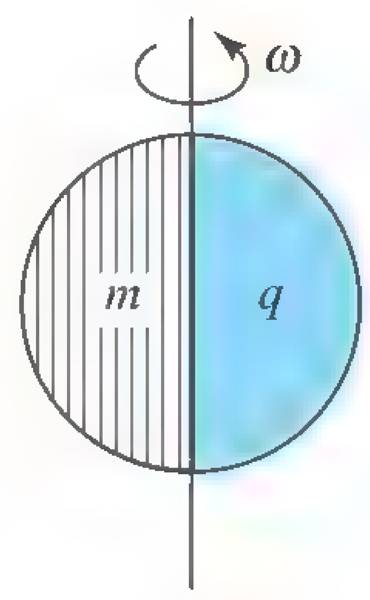
(3) $\dfrac{Mg\tan\theta}{LB}$ to the left (4) $\dfrac{3Mg\tan\theta}{2LB}$ to the left

55. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current of $i = 4$ A. A horizontal magnetic field $B = 10$ T is switched on at time $t = 0$ as shown in figure. The initial angular acceleration of the ring will be



(1) 40π rad s^{-2} (2) 20π rad s^{-2}
(3) 5π rad s^{-2} (4) 15π rad s^{-2}

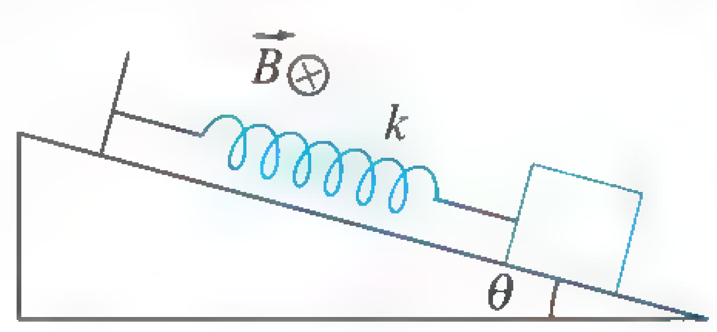
56. Consider a hypothetical spherical body. The body is cut into two parts about the diameter. One of hemispherical portion has mass distribution m while the other portion has identical charge distribution q. The body is rotated about the axis with constant speed ω. Then, the ratio of magnetic moment to angular momentum is



(1) $\frac{q}{2m}$ (2) $>\frac{q}{2m}$

(3) $<\frac{q}{2m}$ (4) cannot be calculated

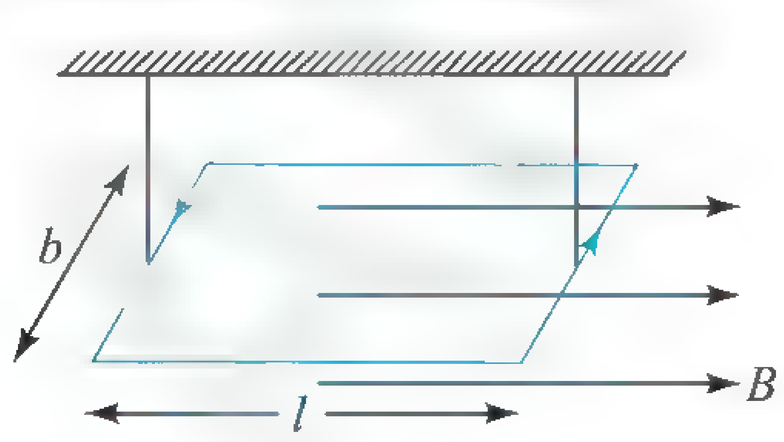
57. A small block of mass m, having charge q, is placed on a frictionless inclined plane making an angle θ with the horizontal. There exists a uniform magnetic field B parallel to the inclined plane but perpendicular to the length of spring. If m is slightly pulled on the incline in downward direction, the time period of oscillation will be (assume that the block does not leave contact with the plane)



(1) $2\pi\sqrt{\frac{m}{K}}$ (2) $2\pi\sqrt{\frac{2m}{K}}$

(3) $2\pi\sqrt{\frac{qB}{K}}$ (4) $2\pi\sqrt{\frac{qB}{2K}}$

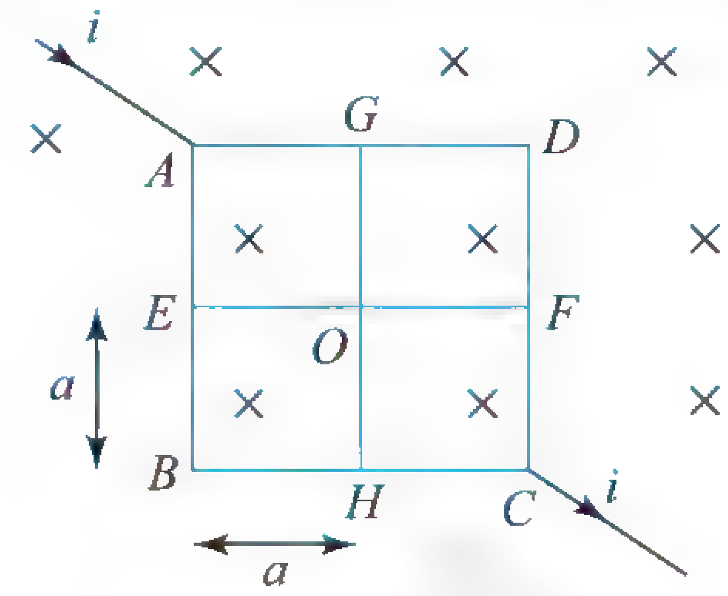
58. A uniform conducting rectangular loop of sides l, b and mass m carrying current i is hanging horizontally with the help of two vertical strings. There exists a uniform horizontal magnetic field B which is parallel to the longer side of loop. The value of tension which is least is



(1) $\frac{mg - Bb}{2}$ (2) $\frac{mg + Bb}{2}$

(3) $\frac{mg - 2iBb}{2}$ (4) $\frac{mg + 2Bb}{2}$

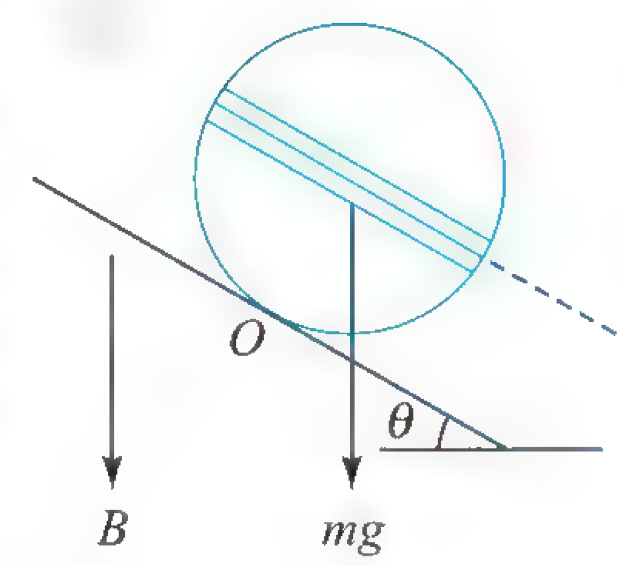
59. In figure, there is a uniform conducting structure in which each small square has side a. The structure is kept in a uniform magnetic field B. Then the magnetic force on the structure will be



(1) $2\sqrt{2}\, iBa$ (2) $\sqrt{2}\, iBa$

(3) $2\, iBa$ (4) iBa

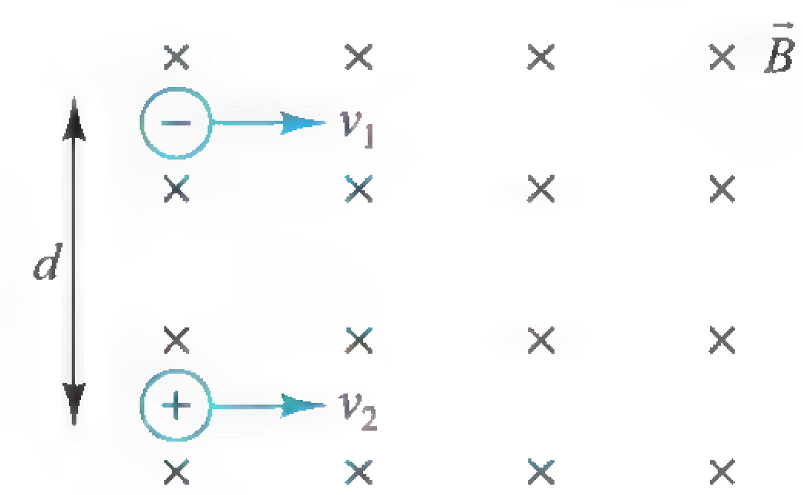
60. In figure, a coil of single turn is wound on a sphere of radius r and mass m. The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere. If the sphere is in rotational equilibrium, the value of B is [Current in the coil is i]



(1) $\frac{mg}{\pi i r}$ (2) $\frac{mg\sin\theta}{\pi i}$

(3) $\frac{mg\sin\theta}{\pi i}$ (4) none of these

61. Two identical particles having the same mass m and charges $+q$ and $-q$ separated by a distance d enter a uniform magnetic field B directed perpendicular to paper inwards with speeds v_1 and v_2 as shown in figure. The particles will not collide if



(1) $d > \frac{m}{Bq}(v_1 + v_2)$ (2) $d < \frac{m}{Bq}(v_1 + v_2)$

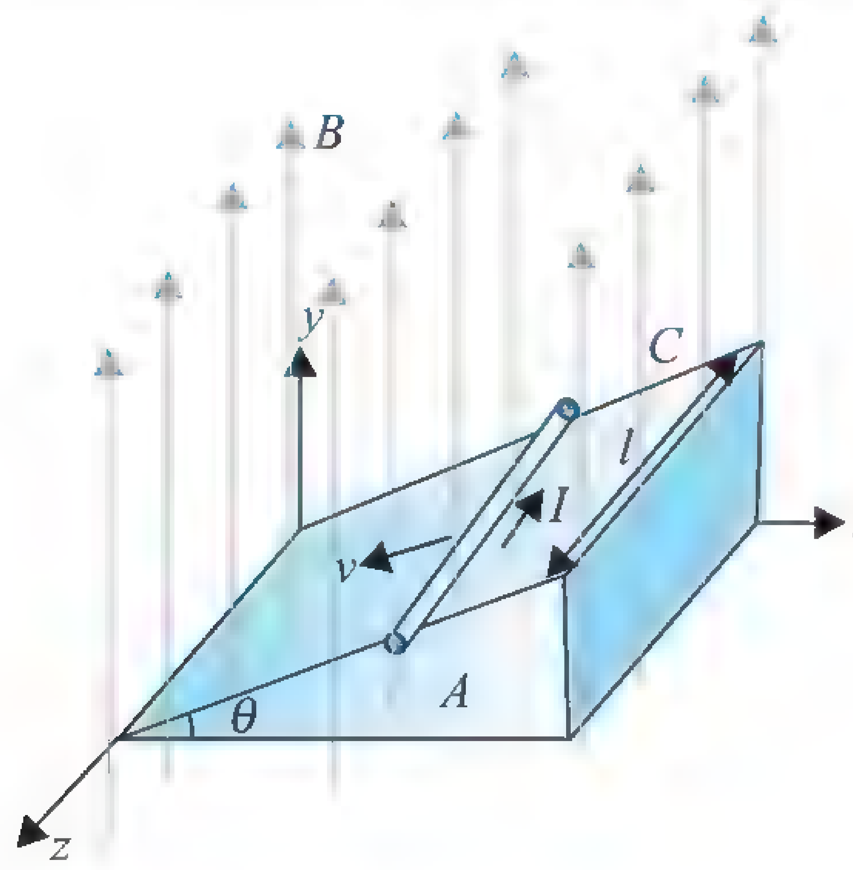
(3) $d > \frac{2m}{Bq}(v_1 + v_2)$ (4) $v_1 = v_2$

62. A charged particle of specific charge (charge/mass) α is released from origin at time $t = 0$ with velocity $\vec{v} = v_0(\hat{i} + \hat{j})$ in uniform magnetic field $\vec{B} = B_0\hat{i}$. Coordinates of the particle at time $t = \pi/(B_0\alpha)$ are

(1) $\left(\frac{v_0}{2B_0\alpha}, \frac{\sqrt{2}v_0}{\alpha B_0}, \frac{-v_0}{B_0\alpha}\right)$ (2) $\left(\frac{-v_0}{2B_0\alpha}, 0, 0\right)$

(3) $\left(0, \frac{2v_0}{B_0\alpha}, \frac{v_0\pi}{2B_0\alpha}\right)$ (4) $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

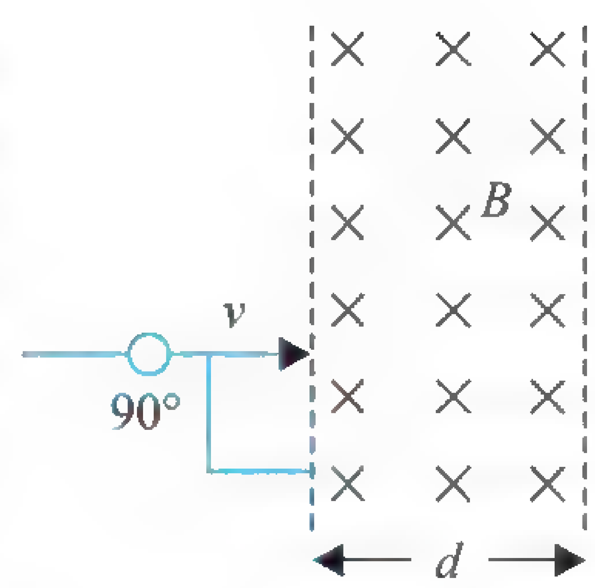
63. A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current i is flowing in the conductor in a direction perpendicular to paper inwards. A vertically upward magnetic field $\vec{B}$ exists in space. Then, magnitude of magnetic field $\vec{B}$ is



(1) $\dfrac{mg}{il}\sin\theta$ (2) $\dfrac{mg}{il}\tan\theta$

(3) $\dfrac{mg\cos\theta}{il}$ (4) $\dfrac{mg}{il\sin\theta}$

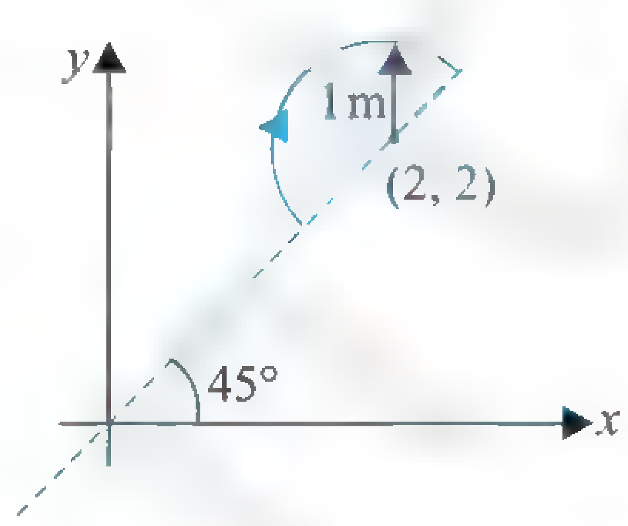
64. A positive charge particle of mass m and charge q is projected with velocity v as shown in figure. If radius of curvature of charge particle in magnetic field region is greater than d, then find the time spent by the charge particle in magnetic field.



(1) $\dfrac{2m}{qB}\sin^{-1}\left(\dfrac{dqB}{mv}\right)$ (2) $\dfrac{2m}{qB}\sin^{-1}\left(\dfrac{mv}{dqB}\right)$

(3) $\dfrac{m}{qB}\sin^{-1}\left(\dfrac{dqB}{mv}\right)$ (4) $\dfrac{m}{qB}\sin^{-1}\left(\dfrac{mv}{dqB}\right)$

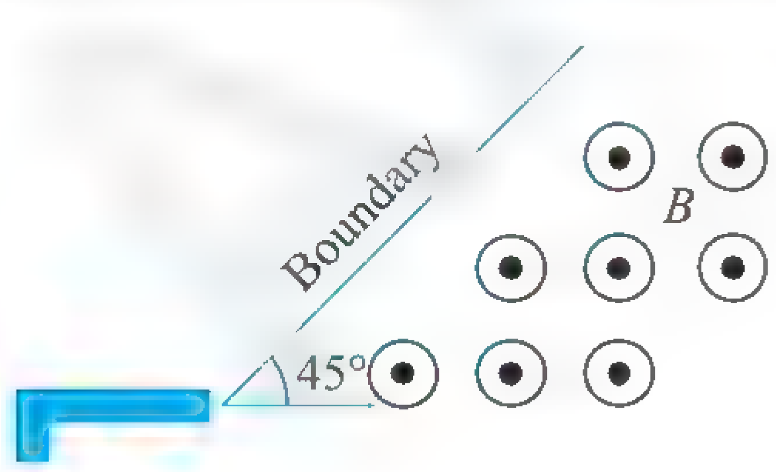
65. A uniform magnetic field $\vec{B} = 3\hat{i} + 4\hat{j} + \hat{k}$ exists in region of space. A semicircular wire of radius 1 m carrying current 1 A having its centre at (2, 2, 0) is placed in x-y plane as shown in figure. The force on semicircular wire will be



(1) $\sqrt{2}\,(\hat{i}+\hat{j}+\hat{k})$ (2) $\sqrt{2}\,(\hat{i}-\hat{j}+\hat{k})$

(3) $\sqrt{2}\,(\hat{i}+\hat{j}-\hat{k})$ (4) $\sqrt{2}\,(-\hat{i}+\hat{j}+\hat{k})$

66. An electron gun ejects electrons at an angle of 45° with magnetic field boundary as shown in figure. Find the angular deviation of electrons as it comes out of field.



(1) 45° (2) 90°

(3) 60° (4) None of these

67. In a certain region of space, there exists a uniform and constant electric field of strength E along x-axis and uniform constant magnetic field of induction B along z-axis. A charged particle having charge q and mass m is projected with speed v parallel to x-axis from a point $(a, b, 0)$. When the particle reaches a point $2a$, $b/2$, 0 its speed becomes $2v$. Find the value of electric field strength in terms of m, v and co-ordinates.

(1) $\dfrac{3}{2}\dfrac{mv^2}{qa}$ (2) $\dfrac{mv^2}{qB}$

(3) $\dfrac{2mv^2}{qBa}$ (4) $\dfrac{3}{2}vB$

68. A particle of specific charge $q/m = \pi$ C/kg is projected from the origin towards positive x-axis with a velocity of 10 m/s in a uniform magnetic field $\vec{B} = -2\hat{k}$ Tesla. The velocity $\vec{V}$ of the particle after time $t = 1/6$ s will be

(1) $(5\hat{i}+5\sqrt{3}\,\hat{j})$ m/s (2) $10\hat{j}$ m/s

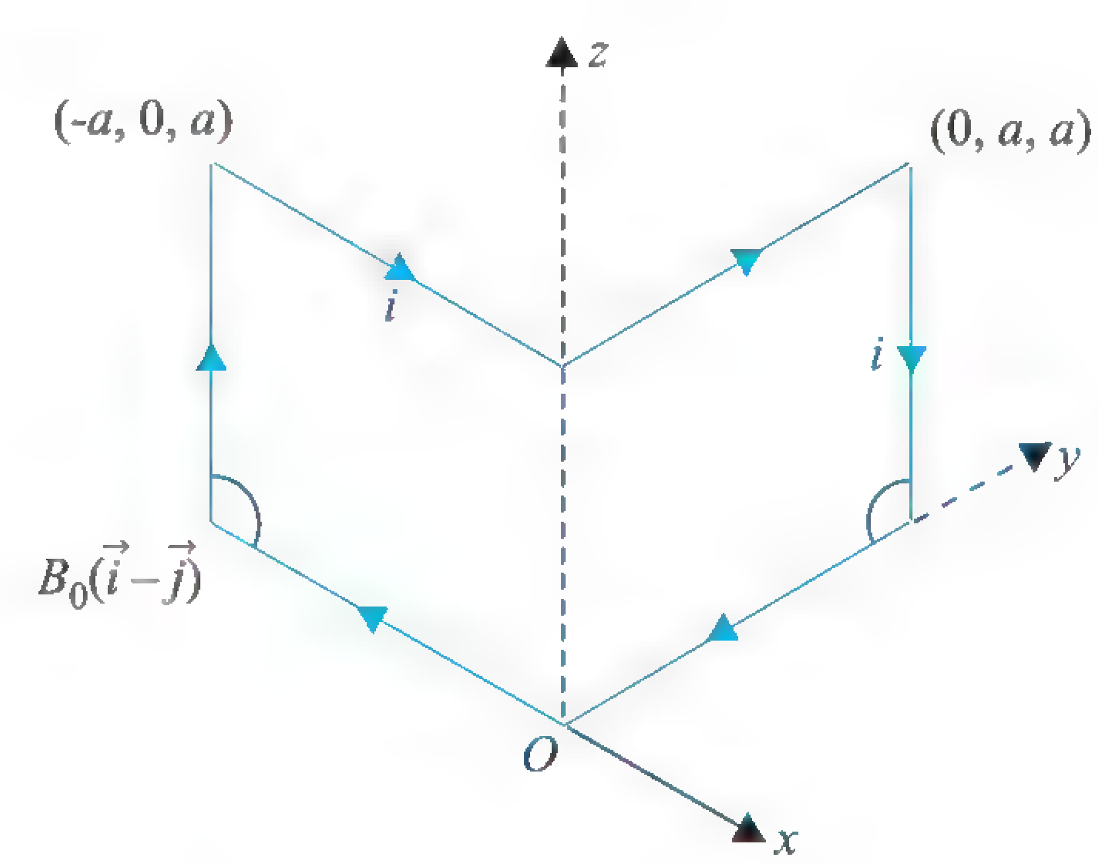
(3) $(5\sqrt{3}\,\hat{i}-5\hat{j})$ m/s (4) $-10\hat{j}$ m/s

69. An α-particle and a proton are both simultaneously projected in opposite directions into a region of constant magnetic field perpendicular to the direction of the field. After some time it is found that the velocity of the α-particle has changed in a direction by 45°. Then at this time, the angle between velocity vectors of α-particle and proton is

(1) 90° (2) 45°

(3) 135° (4) none

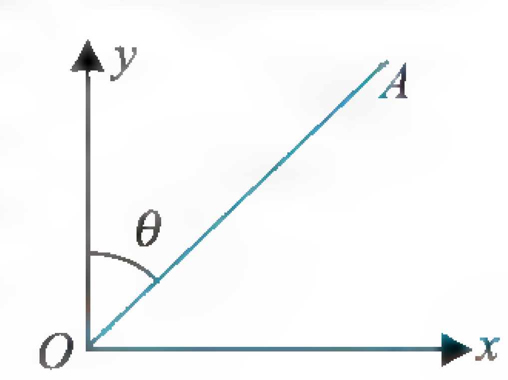
70. The torque experienced by a given current carrying loop in a uniform magnetic field $\vec{B}$ given by $B_0(\vec{i}-\vec{j})$ would have magnitude



(1) $\sqrt{2}B_0a^2I$ (2) zero

(3) $2B_0a^2I$ (4) B_0a^2I

71. A uniform magnetic field B and electric field E exist along y and negative z axis respectively. Under the influence of these fields a charge particle moves along OA undeflected. If electric field is swithed off, find the pitch of helical trajectory in which the particle will move.

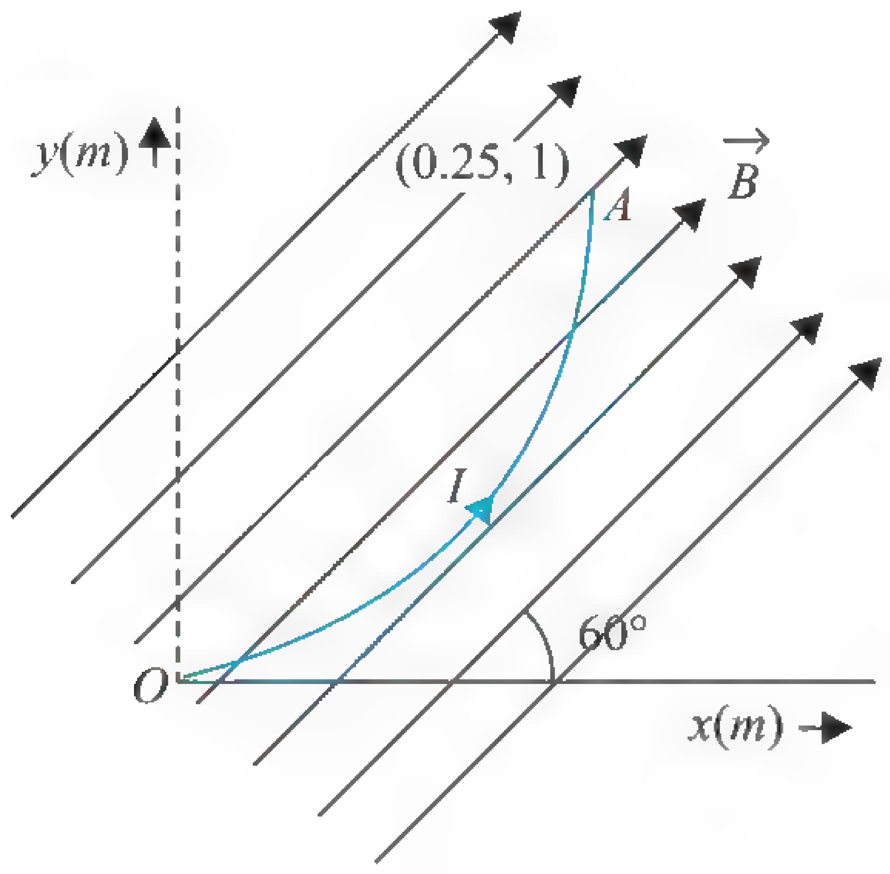


(1) $\dfrac{2\pi mE}{qB^2\cot\theta}$ (2) $\dfrac{4\pi mE}{qB^2\tan\theta}$

(3) $\dfrac{4\pi mE}{qB^2\cot\theta}$ (4) $\dfrac{2\pi mE}{qB^2\tan\theta}$

72. A parabolic section of wire OA is located in the x-y plane and carries current I = 12 A. A uniform magnetic field $B = 4.0$ T making an angle 60° with x axis exists in x-y plane.

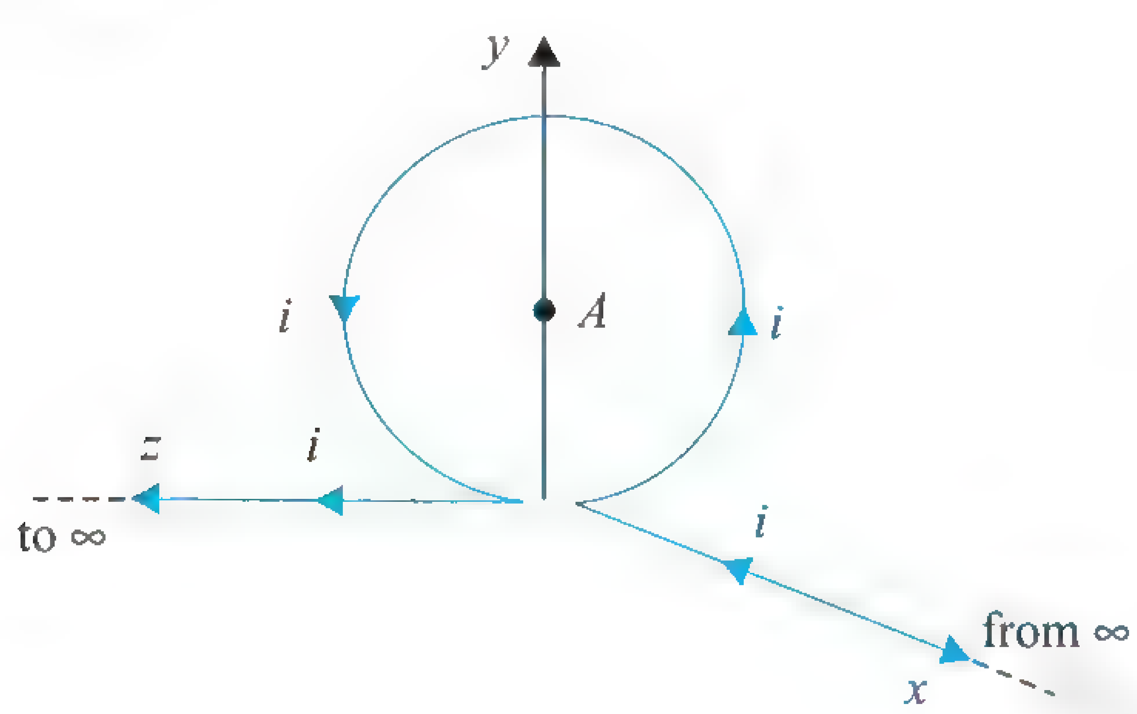
Calculate the magnetic force on the wire OA. Co-ordinates of A are (0.25 m, 1 m).



(1) $6(\sqrt{3}-4)\hat{k}$ N (2) $6(4-\sqrt{3})\hat{k}$ N

(3) $3(\sqrt{3}-4)\hat{k}$ N (4) $3(4-\sqrt{3})\hat{k}$ N

73. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors along which a current i is flowing at a point A (O, R, O), that is the centre of a circular conductor of radius R. The ring is in yz plane.



(1) $B=\dfrac{\mu_0 i}{4\pi R}\sqrt{\left(2\pi^2-2\pi+1\right)}$

(2) $B=\dfrac{\mu_0 i}{4\pi R}\sqrt{2\left(2\pi^2-2\pi+1\right)}$

(3) $B=\dfrac{\mu_0 i}{2\pi R}\sqrt{\left(2\pi^2-2\pi+1\right)}$

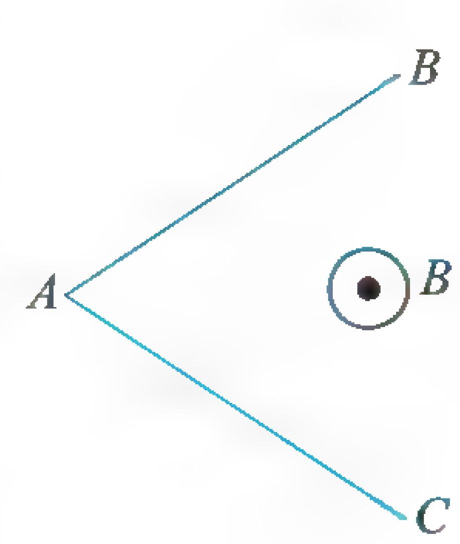
(4) None of these

74. A charged particle (charge q, mass m) has velocity v_0 at origin in $+x$ direction. In space there is a uniform magnetic field B in $-z$ direction. Find the y coordinate of particle when is crosses y axis.

(1) $\dfrac{mv_0}{qB}$ (2) $\dfrac{2mv_0}{qB}$

(3) $\dfrac{mv_0}{2qB}$ (4) None of these

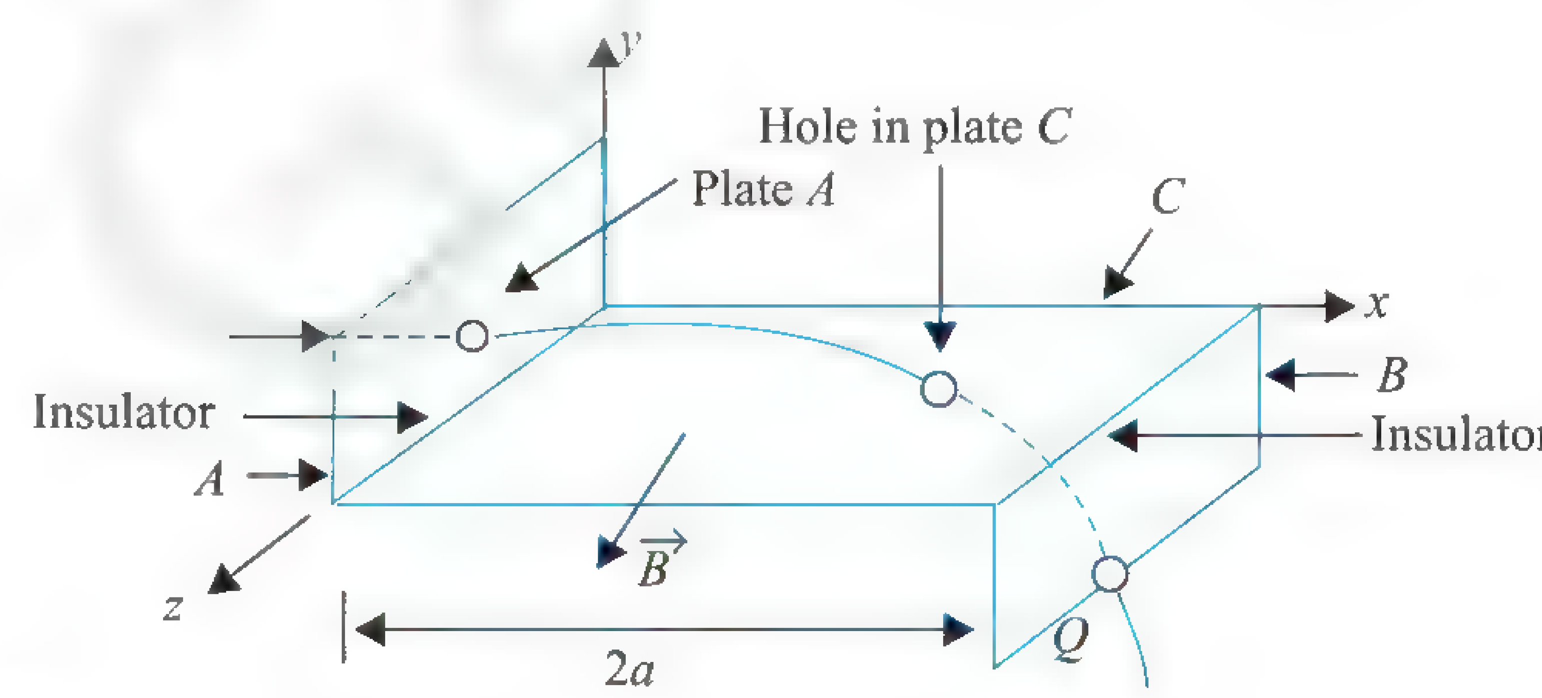
75. AB and AC are the boundary lines within which a magnetic field B exists. If the magnetic field is absent, a charged particle of mass m and charge q must have passed through a point P on angle bisector of $-BAC$, at a distance $r\sqrt{2}$ from A, if it has fallen on AB normally at a point Q such that $QP = r$. If the magnetic field is present, how much time the charged particle will take to come out of the magnetic field



(1) $\dfrac{m\pi}{Bq}$ (2) $\dfrac{2m\pi}{Bq}$

(3) $\dfrac{4m\pi}{Bq}$ (4) $\dfrac{m\pi}{2Bq}$

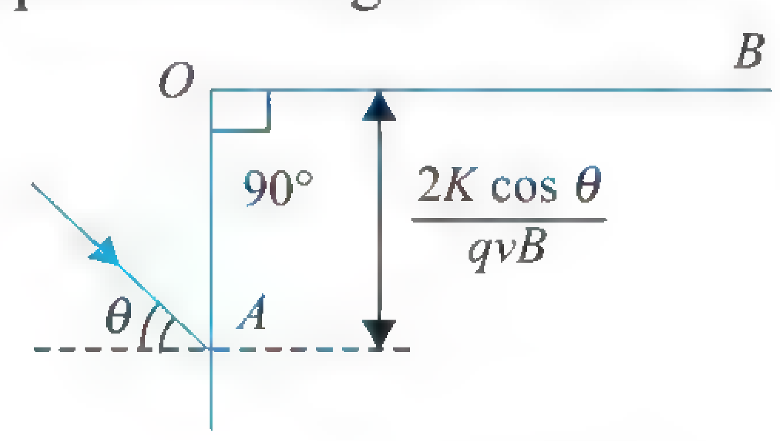
76. Two rectangular plates A and B placed at a distance $2a$ apart, are connected to a battery to produce an electric field. There are insulators between plates C and other two plates. A magnetic field exists along z-axis. A charged particle of mass m and charge q passes through a hole at the middle of the plate A with velocity v and strikes at Q which is the middle of the bottom edge of plate B after passing through a hole inplate C. If $E = mv^2/qa$, what will be the speed of the particle at Q?



(1) $v\sqrt{2}$ (2) $2v$

(3) $v\sqrt{5}$ (4) $v\sqrt{3}$

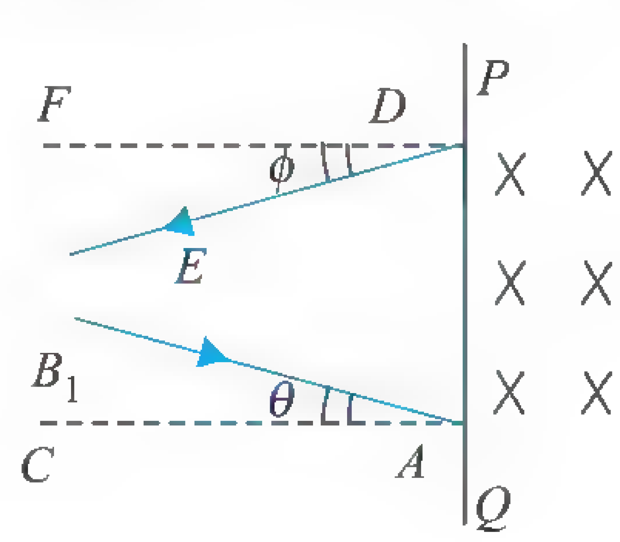
77. A magnetic field B exists between OA and OB. Inclined at an angle θ, a charged particle strikes at point A on surface OA, at a distance $(2K\cos\theta)/(qvB)$ from O, where K is kinetic energy, q is the charge and v is the velocity of the particle. At what angle with horizontal (measured from end B) will the charged particle emerge from OB?



(1) θ (2) $90° - \theta$

(3) $90° + \theta$ (4) $90°$

78. To the right of line PQ is a uniform magnetic field $\vec{B}$. B_1A is the line of incidence of a charged particle, which comes out of the field along DE; CA and DF are the normals at A and D. $B_1AC = \theta$. Angle measured from CA in clockwise direction is taken as positive. What will be the value of θ so that angle subtended by the part of the circle (along which charged particle moves in the field) at its centre and facing the circle is less than π?



(1) θ is positive

(2) $\theta = 0$

(3) θ is negative

(4) θ depends upon $\vec{B}$ and charge

79. In the previous problem, if O is the point on AD and OO_1 is the perpendicular from the centre of the circle, O_{-1}, then OA/OD will be

(1) 1 (2) > 1
(3) < 1 (4) depend upon $\vec{B}$ andcharge

80. In the previous problem, if ϕ is the angle between line of emergence DE and normal DF at point D, ratio of ϕ/θ for positive value of θ will be

(1) 1 (2) >1
(3) < 1 (4) depends upon $\vec{B}$ and charge

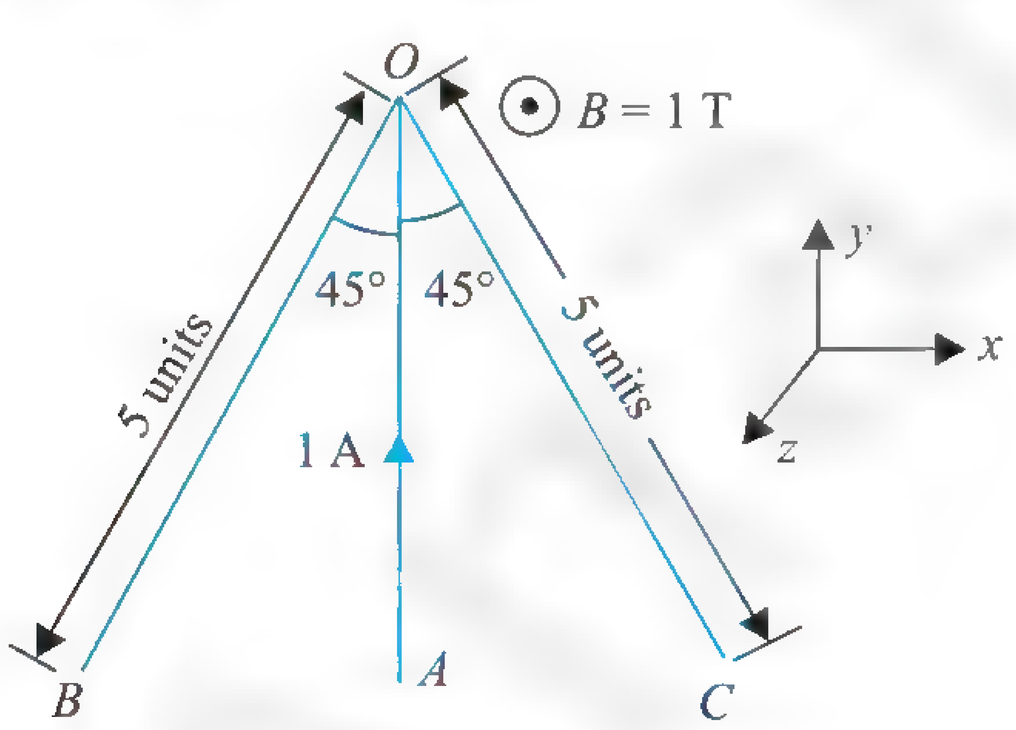
81. In the previous problem, if time period, $T = 2\pi \dfrac{m}{BQ}$ where Q is the charge of the particle and m is its mass, the ratio of time spent by the particle in the field when θ is positive to when θ is negative is given by

(1) $\left(\dfrac{\pi/2+\theta}{\pi/2-\theta}\right)$ (2) $\left(\dfrac{\pi+\theta}{\pi-\theta}\right)$
(3) $\left(\dfrac{\pi-\theta}{\pi+\theta}\right)$ (4) $\left(\dfrac{\pi/2-\theta}{\pi/2+\theta}\right)$

82. In the previous problem, the maximum range of movement of the centre of the part of the circle from line AD in which charged particle of charge Q moves with a velocity v when θ is positive to when θ is negative is given by

(1) $\pm\dfrac{mv}{2QB}$ (2) $\pm\dfrac{mv}{QB}$
(3) $\pm\dfrac{2mv}{QB}$ (4) $\pm\dfrac{2mv}{3QB}$

83. A triangular system of mass 100 g consisting of 3 wires of lengths, as shown in figure and length of AO as 4 units, are placed in the magnetic field of 1T. The current of 1A flows through wire AO. The wires are of same material and cross-sectional area. In which direction will the system move?



(1) at $\tan^{-1}(3/4)$ with $(-x)$ axis.
(2) along x-axis
(3) along y-axis
(4) along $(-x)$ axis

84. In the previous problem, what will be force with which system moves? (Use $\sqrt{2} = 1.4$)

(1) 3.05 N (2) 4.0 N
(3) 0.5 N (4) 1.0 N

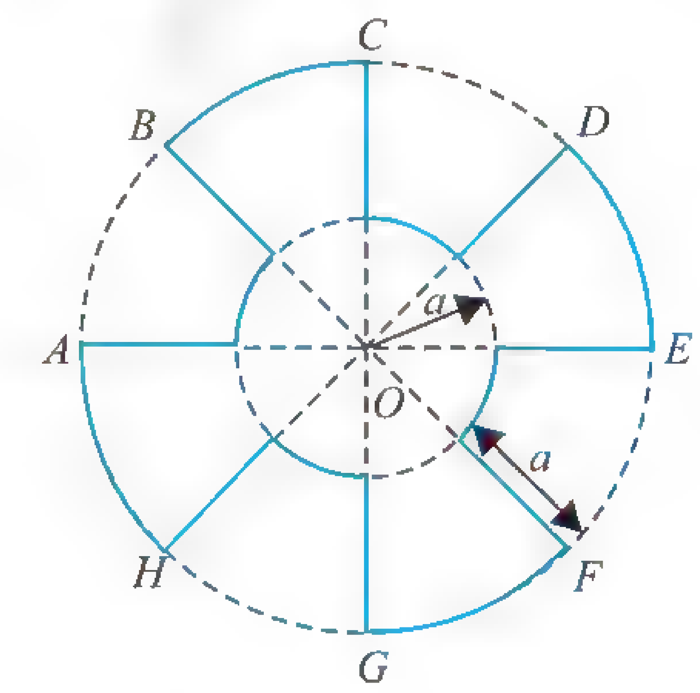
85. In the previous problem, instead of being stationary, the system enters the magnetic field with a velocity 0.5 m/sec along y-direction. Along which direction the system will move now?

(1) along $a\hat{i}+b\hat{j}$
(2) along a circular path
(3) system will become stationary
(4) along a direction in x-y plane

86. In the previous problem, what will be acceleration of the system (in m/sec²)?

(1) $5\sqrt{2}$ (2) $0.5\sqrt{2}$
(3) zero (4) 2.5

87. Current I flows through the circuit, as shown in figure. Find the magnetic moment of the figure, if $AB = BC = CD = DE = EF = FG = GH = HA$.



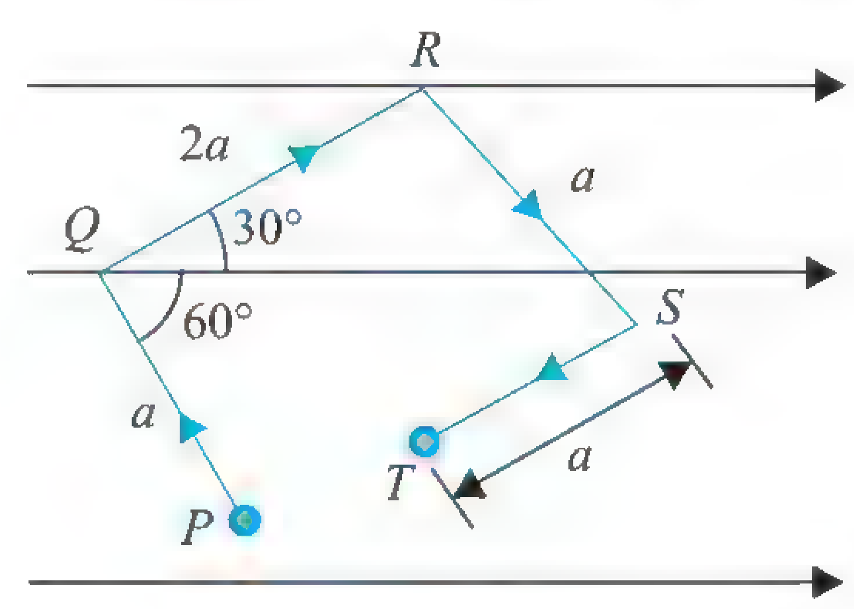
(1) $\dfrac{7}{2} I\pi a^2$

(2) $\dfrac{5}{2} I\pi a^2$

(3) $4 I\pi a^2$

(4) $\dfrac{5}{3} I\pi a^2$

88. Find the force acting on rectangular loop $PQRST$.



(1) $\dfrac{IaB}{2}$ (2) $\dfrac{IaB}{\sqrt{2}}$
(3) $\dfrac{IaB\sqrt{3}}{2}$ (4) $IaB\sqrt{3}$

Multiple Correct Answers Type

1. A charged particle goes undeflected in a region containing electric and magnetic fields. It is possible that

(1) $\vec{E} \parallel \vec{B}, \vec{v} \parallel \vec{E}$
(2) $\vec{E}$ is not parallel to $\vec{B}$
(3) $\vec{v} \parallel \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$
(4) $\vec{E} \parallel \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$

2. If a charged particle goes unaccelerated in a region containing electric and magnetic fields, then

(1) $\vec{E}$ must be perpendicular to $\vec{B}$
(2) $\vec{v}$ must be perpendicular to $\vec{E}$
(3) $\vec{v}$ must be perpendicular to $\vec{B}$
(4) E must be equal to vB

3. The force $\vec{F}$ experienced by a particle of charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by

$\vec{F} = q(\vec{v} \times \vec{B})$. Which pairs of vectors are at right angles to each other?

(1) $\vec{F}$ and $\vec{v}$ (2) $\vec{F}$ and $\vec{B}$
(3) $\vec{B}$ and $\vec{v}$ (4) $\vec{F}$ and $(\vec{v} \times \vec{B})$

4. Which of the following statements is correct?
(1) If a moving charged particle enters into a region of magnetic field from outside, it does not complete a circular path.
(2) If a moving charged particle traces a helical path in a uniform magnetic field, the axis of the helix is parallel to the magnetic field.
(3) The power associated with the force exerted by a magnetic field on a moving charged particle is always equal to zero.
(4) If in a region a uniform magnetic field and a uniform electric field both exist, a charged particle moving in this region cannot trace a circular path.

5. An electron is moving along the positive x-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative x-axis. This can be done by applying the magnetic field along
(1) y-axis (2) z-axis
(3) y-axis only (4) z-axis only

6. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. During its motion along a helical path, the particle will
(1) never move parallel to the x-axis
(2) move parallel to the x-axis once during every rotation for all values of θ
(3) move parallel to the x-axis at least once during every rotation if $\theta = 45°$
(4) never move perpendicular to the x-direction

7. In previous problem, if the pitch of the helical path is equal to the maximum distance of the particle from the x-axis, then which of the following are not correct?
(1) $\cos\theta = \frac{1}{\pi}$ (2) $\sin\theta = \frac{1}{\pi}$
(3) $\tan\theta = \frac{1}{\pi}$ (4) $\tan\theta = \pi$

8. A proton is fired from origin with velocity $\vec{v} = v_0\hat{j} + v_0\hat{k}$ in a uniform magnetic field $\vec{B} = B_0\hat{j}$. In the subsequent motion of the proton
(1) its z-coordinate can never be negative
(2) its x-coordinate can never be positive
(3) its x- and z-coordinates cannot be zero at the same time
(4) its y-coordinate will be proportional to its time of flight

9. Velocity and acceleration vector of a charged particle moving in a magnetic field at some instant are $\vec{v} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = 2\hat{i} + x\hat{j}$. Select the correct options:
(1) $x = -1.5$
(2) $x = 3$
(3) Magnetic field is along z-direction
(4) Kinetic energy of the particle is constant

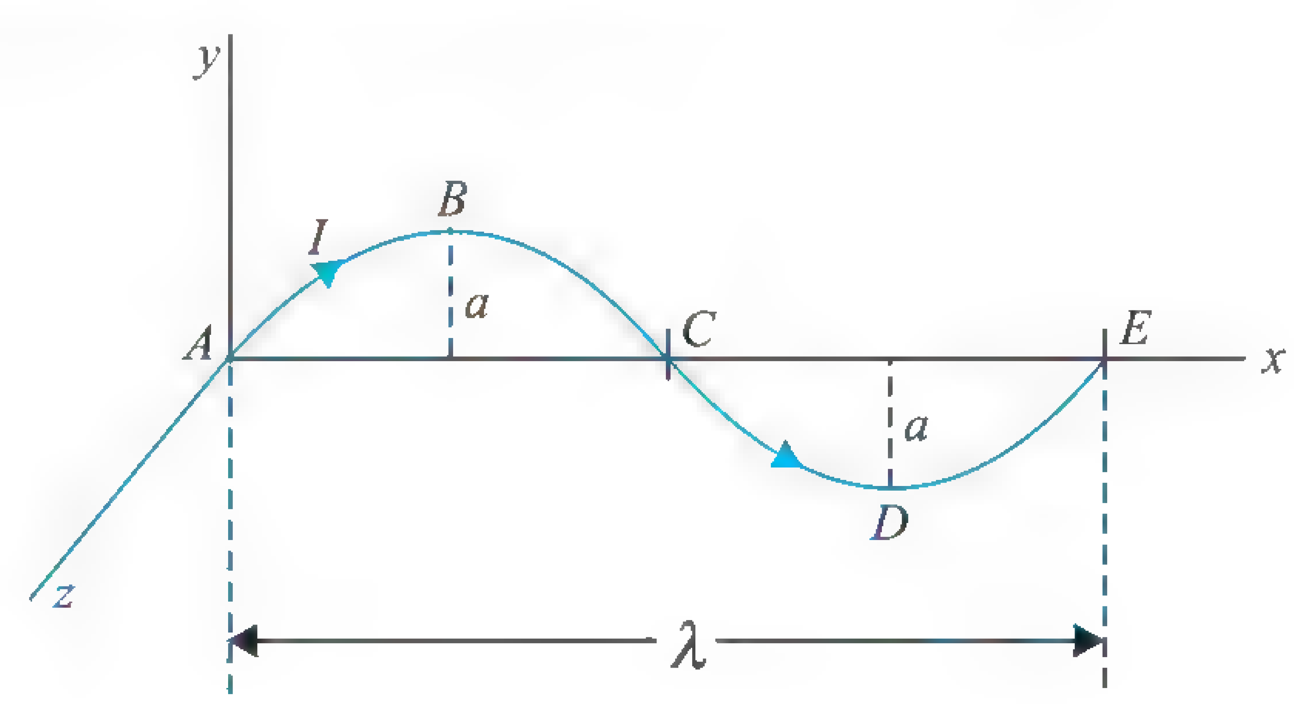
10. Let $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity along a straight line enters the region and is found to pass through it undeflected. Indicate which of the following statements are consistent with the obserations:
(1) $\vec{E} = 0$ and $\vec{B} = 0$
(2) $\vec{E} \neq 0$ and $\vec{B} = 0$
(3) $\vec{E} \neq 0$ and $\vec{B} \neq 0$ and both $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$
(4) $\vec{E}$ is parallel to $\vec{v}$ but $\vec{B}$ is perpendicular to $\vec{v}$

11. When a current carrying coil is placed in a uniform magnetic field with its magnetic moment anti-parallel to the field.
(1) torque on it is maximum
(2) torque on it is zero
(3) potential energy is maximum
(4) dipole is in unstable equilibrium

12. A charged particle P leaves the origin with speed $v = v_0$ at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. P will also strike T if
(1) $B = 2B_0$, $v = 2v_0$ (2) $B = 2B_0$, $v = v_0$
(3) $B = B_0$, $v = 2v_0$ (4) $B = \frac{B_0}{2}$, $v = 2v_0$

13. A particle having a mass of 0.5 g carries a charge of 2.5×10^{-8} C. The particle is given an initial horizontal velocity of 6×10^4 ms^{-1}. To keep the particle moving in a horizontal direction
(1) the magnetic field may be perpendicular to the direction of the velocity
(2) the magnetic field should be along the direction of the velocity
(3) magnetic field should have a minimum value of 3.27 T
(4) no magnetic field is required

14. A conductor $ABCDE$, shaped as shown, carries current I. It is placed in the x–y plane with the ends A and E on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be



(1) zero, if B is in the x-direction
(2) λBI in the z-direction, if B is in the y-direction
(3) λBI in the negative y-direction, if B is in the z-direction
(4) λaBI, if B is in the x-direction

15. In previous problem, if the current is I and the magnetic field at D has magnitude B, then
(1) $B = \frac{\mu_0 I}{2\sqrt{2}\pi}$

(2) $B = \dfrac{\mu_0 I}{2\sqrt{3}\,\pi}$

(3) B is parallel to the z-axis

(4) B makes an angle of 45° with the x–y plane

16. A charged particle moves in a gravity free space where an electric field of strength E and a magnetic field of induction B exist. Which of the following statement is/are correct?

(1) If $E \neq 0$ and $B \neq 0$, velocity of the particle may remain constant

(2) If $E = 0$, the particle cannot trace a circular path

(3) If $E = 0$, kinetic energy of the particle remains constant

(4) None of these.

17. A charged particle of unit mass and unit charge moves with velocity $\vec{v} = (8\hat{i} + 6\hat{j})$ m s^{-1} in a magnetic field of $\vec{B} = 2\hat{k}$ T. Choose the correct altenative(s).

(1) The path of the particle may be $x^2 + y^2 - 4x - 21 = 0$

(2) The path of the particle may be $x^2 + y^2 = 25$

(3) The path of the particle may be $y^2 + z^2 = 25$

(4) The time period of the particle will be 3.14 s

18. A charged particle of specific charge α moves with a velocity $\vec{v} = v_0\hat{i}$ in a magnetic field $\vec{B} = \dfrac{B_0}{\sqrt{2}}\left(\hat{j} + \hat{k}\right)$.

Then (specific charge = charge per unit mass)

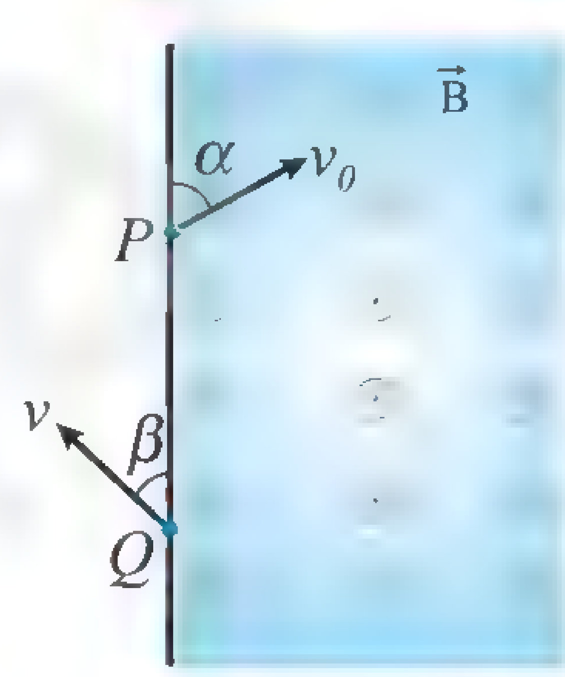
(1) path of the particle is a helix

(2) path of the particle is circle

(3) distance moved by the particle in time $t = \dfrac{\pi}{B_0\alpha}$ is $\dfrac{\pi v_0}{B_0\alpha}$

(4) velocity of the particle after time $t = \dfrac{\pi}{B_0\alpha}$ is $\left(\dfrac{v_0}{2}\hat{i} + \dfrac{v_0}{2}\hat{j}\right)$

19. A particle of charge $-q$ and mass m enters a uniform magnetic field $\vec{B}$ (perpendicular to paper inward) at P with a velocity v_0 at an angle α and leaves the field at Q with velocity v at angle β as shown in figure.



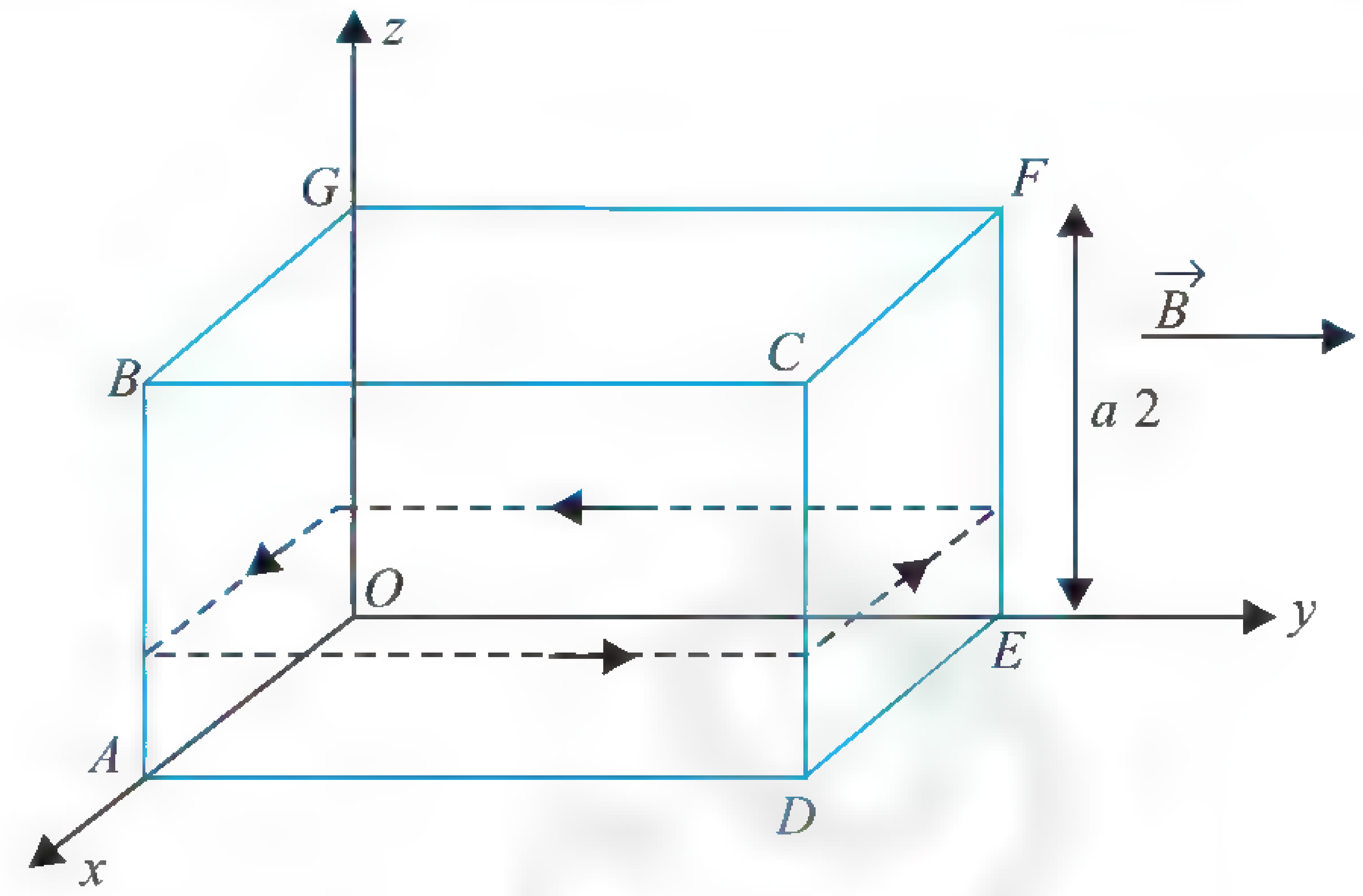
(1) $\alpha = \beta$

(2) $v = v_0$

(3) $PQ = \dfrac{2mv_0 \sin\alpha}{Bq}$

(4) The particle remains in field for time $t = \dfrac{2m(\pi - \alpha)}{Bq}$

20. A wooden cubical block $ABCDEFG$ of mass m and side a is wrapped by a square wire loop of perimeter $4a$, carrying current I. The whole system is placed at frictionless horizontal surface in a uniform magnetic field $\vec{B} = B_0\hat{j}$ as shown in figure. In this situation, normal force between horizontal surface and block passes through a point at a distance x from center. Choose correct statement(s).



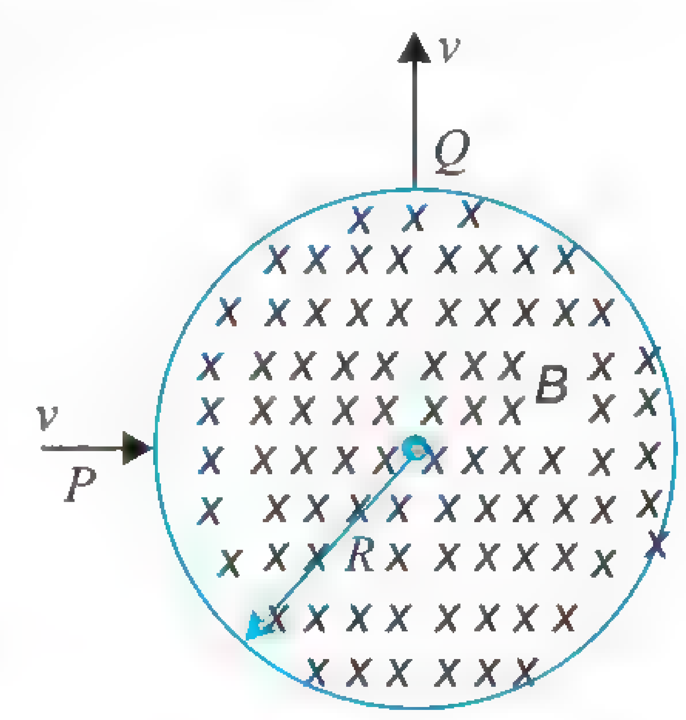
(1) The block must not topple if $I < \dfrac{mg}{aB_0}$

(2) The block must not topple if $I < \dfrac{mg}{2aB_0}$

(3) $x = \dfrac{a}{4}$ if $I = \dfrac{mg}{2aB_0}$

(4) $x = \dfrac{a}{4}$ if $I = \dfrac{mg}{4aB_0}$

21. A particle of charge q and mass m enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in the region of radius R and is uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are correct?



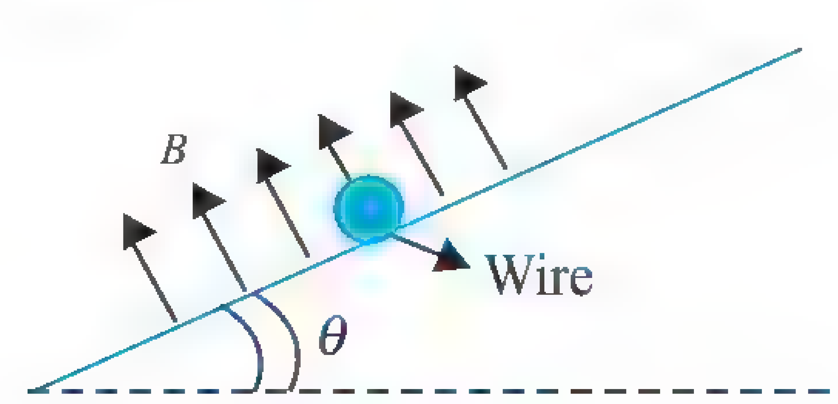
(1) $B = \dfrac{mv}{qR}$

(2) $T = \dfrac{\pi R}{2v}$

(3) $T = \dfrac{\pi m}{2qB}$

(4) None of these

22. A wire of mass m and length l is placed on a smooth incline making an angle θ with the horizontal, whose front view is shown in figure. When a finite amount of charge is passed through it in an infinitesimal time, the wire immediately acquires some velocity and then ascends the incline by a distance s. For this small duration, we can neglect the gravitational force because the current can be considered very large due to small time duration. The amount of charge passed through the wire is



(1) $\dfrac{m\sqrt{2gs\sin\theta}}{Bl}$ (2) $\dfrac{mv}{Bl}$

(3) $\dfrac{m\sqrt{2gs\sin\theta}}{Bl\cos\theta}$ (4) information insufficient

23. A particle is released from the origin with a velocity $v\hat{i}$. The electric field in the region is $E\hat{i}$ and magnetic field is $B\hat{k}$. Then

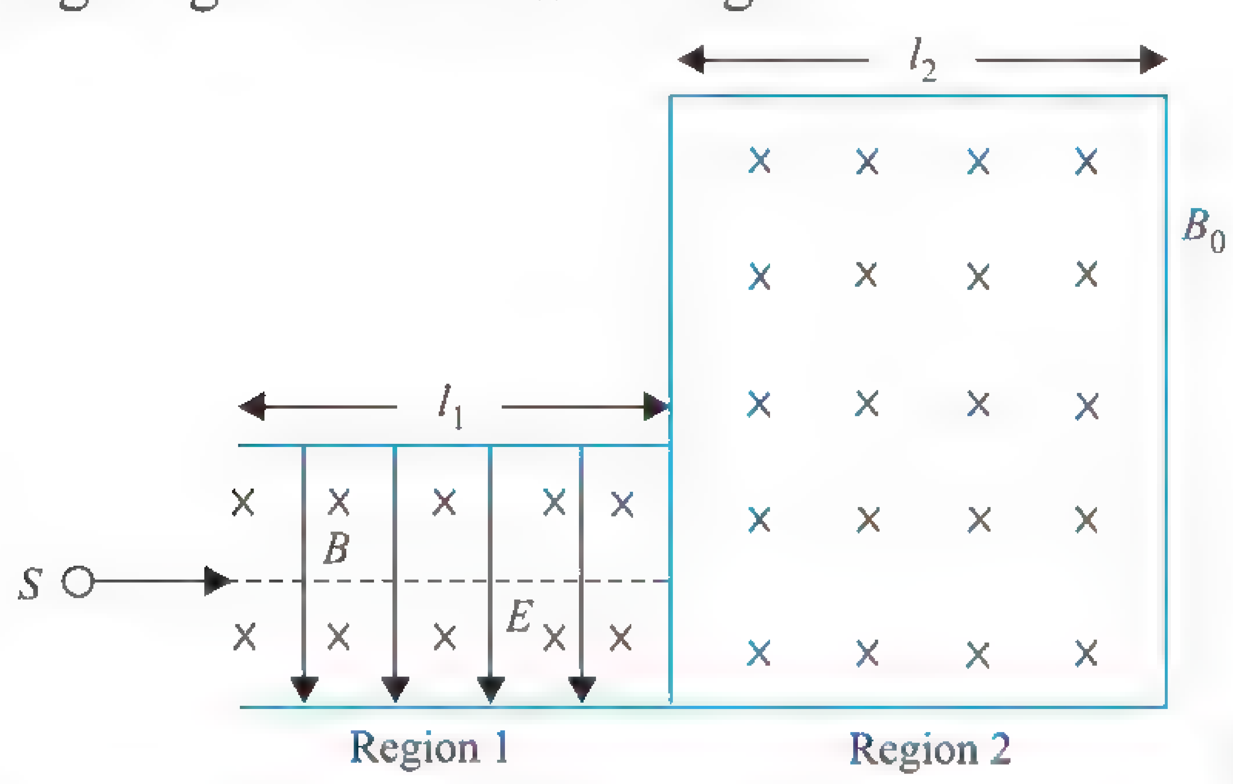
(1) If $v = 0$ and $E = 0$ then particle will execute a circular path.

(2) If $v = 0$, $E \neq 0$ and particle is positively charged then it executes clockwise circle as seen from $+z$ direction.

(3) If $v = 0$ and $E \neq 0$ then the particle will execute a cycloidal motion in the $x - y$ plane.

(4) If $v > 0$ and $E = 0$ and the particle is positively charged and $B > 0$ then the particle will execute anticlockwise circle as seen from $+z$ direction.

24. A charged particle of specific charge s passes undeviated through region 1 as shown in figure.



(1) Velocity of particle in region 1 is $v = E/B$

(2) Work done to move the charged particle in region 1 and region 2 is zero

(3) The radius of the trajectory of the charged particle in Region 2 is (E/sBB_0)

(4) The particle emerges from region 2 with a velocity $\vec{v}'$ where $\vec{v}' = -\vec{v}$ for $l_2 > \dfrac{E}{sBB_0}$

25. A particle of specific charge 'α' is projected from origin at $t = 0$ with a velocity $\vec{v} = v_0(\hat{i} + \hat{k})$ in a magnetic field $\vec{B} = -B_0\hat{k}$. Then: (Mass of particle = 1 unit)

(1) At $t = \dfrac{\pi}{4\alpha B_0}$, speed of the particle is v_0

(2) At $t = \dfrac{\pi}{\alpha B_0}$, velocity of the particle is $-v_0(\hat{i} - \hat{k})$

(3) At $t = \dfrac{2\pi}{\alpha B_0}$, distance travelled by the particle is less than $\dfrac{2\sqrt{2}\pi v_0}{\alpha B_0}$

(4) At $t = \dfrac{2\pi}{\alpha B_0}$, magnitude of displacement of the particle is more than $\dfrac{2v_0}{\alpha B_0}$

Linked Comprehension Type

For Problems 1–3

A particle of mass m and charge q is accelerated by a potential difference V volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field of induction is B.

1. What would be the speed of second particle so that both particles meet again and again after a regular interval of time, which should be minimum?

(1) $\sqrt{\dfrac{qV}{m}}\cos\theta$ (2) $\sqrt{\dfrac{2qV}{m}}\cos\theta$

(3) $\sqrt{\dfrac{qV}{m}}\sin\theta$ (4) $\sqrt{\dfrac{qV}{2m}}\cos\theta$

2. Find the time interval after which they meet.

(1) $\dfrac{2\pi m}{qB}$ (2) $\dfrac{\pi m}{2qB}$

(3) $\dfrac{\pi m}{qB}$ (4) $\dfrac{3\pi m}{2qB}$

3. Find the distance travelled by the second particle during that interval mentioned in the above problem.

(1) $\sqrt{\dfrac{Vm}{q}}\dfrac{2\pi}{B}\cos\theta$ (2) $\sqrt{\dfrac{2Vm}{3q}}\dfrac{2\pi}{B}\cos\theta$

(3) $\sqrt{\dfrac{2Vm}{q}}\dfrac{2\pi}{B}\cos\theta$ (4) $\dfrac{2}{3}\sqrt{\dfrac{Vm}{q}}\dfrac{\pi}{m}\cos\theta$

For Problems 4–5

A charged particle carrying charge q = 10 μC moves with velocity $v_1 = 10^6$ m s^{-1} at angle 45° with x-axis in the xy plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6$ m s^{-1} along the z-axis, it experiences a force F_2 in y-direction.

4. Find the magnetic field $\vec{B}$.

(1) $(10^{-3}\,\text{T})(\hat{i} + \hat{j})$ (2) $(2 \times 10^{-3}\,\text{T})\hat{i}$

(3) $(10^{-3}\,\text{T})\hat{i}$ (4) $(2 \times 10^{-3}\,\text{T})(\hat{i} + \hat{j})$

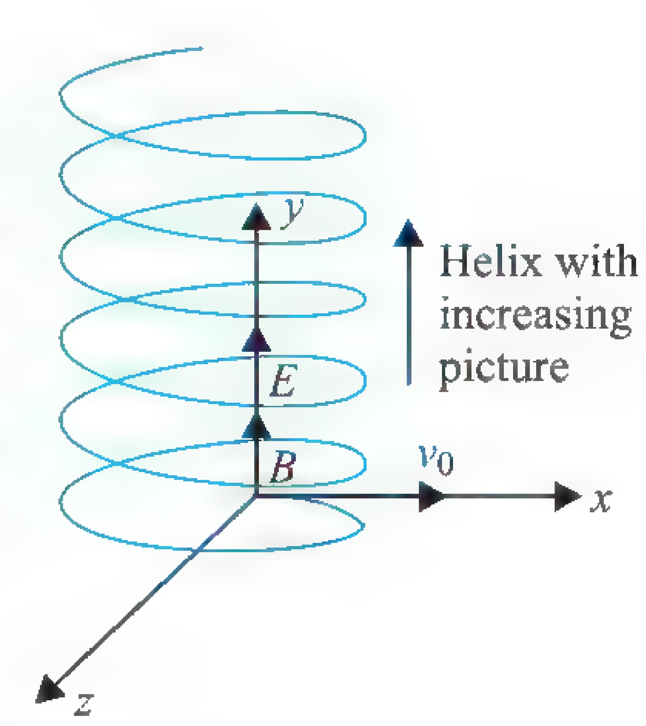
5. Find the magnitude of the force F_2.

(1) 10^{-2} N (2) 10^{-3} N

(3) 10^{-4} N (4) 10^{-5} N

For Problems 6–7

Uniform electric and magnetic fields with strength E and induction B, respectively, are along y-axis as shown in figure. A particle with specific charge q/m leaves the origin O in the direction of x-axis with an initial non-relativistic velocity v_0.



6. The coordinate y_n of the particle when it crosses the y-axis for the n^{th} time is

(1) $\dfrac{2\pi^2 mn^2 E}{qB^2}$ (2) $\dfrac{\pi^2 mn^2 E}{qB^2}$

(3) $\dfrac{2\pi^2 mn^2 E}{3qB^2}$ (4) $\dfrac{\sqrt{3}\pi^2 mn^2 E}{qB^2}$

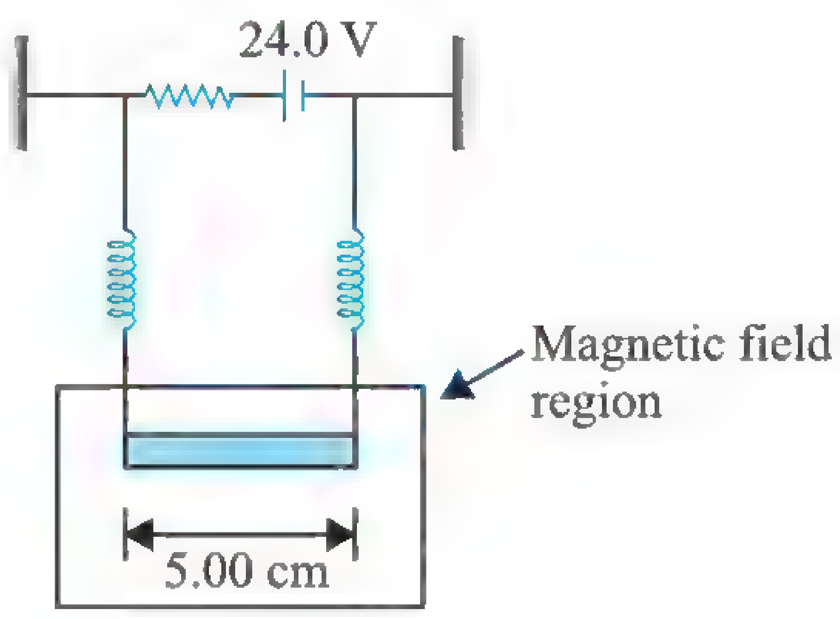
7. The angle α between the particle's velocity vector and y-axis at that moment is

(1) $\tan^{-1}\left(\dfrac{3v_0B}{2\pi nE}\right)$ (2) $\tan^{-1}\left(\dfrac{v_0B}{\pi nE}\right)$

(3) $\tan^{-1}\left(\dfrac{v_0B}{\sqrt{2}\pi nE}\right)$ (4) $\tan^{-1}\left(\dfrac{v_0B}{2\pi nE}\right)$

For Problems 8–9

The circuit in figure consists of wires at the top and bottom and identical metal springs at the left and right sides. The wire at the bottom has a mass of 10.0 g and is 5.00 cm long. The wire is hanging as shown in the figure. The springs stretch 0.500 cm under the weight of the wire, and the circuit has a total resistance of 12.0 Ω. When a magnetic field is turned on, the springs stretch an additional 0.300 cm.

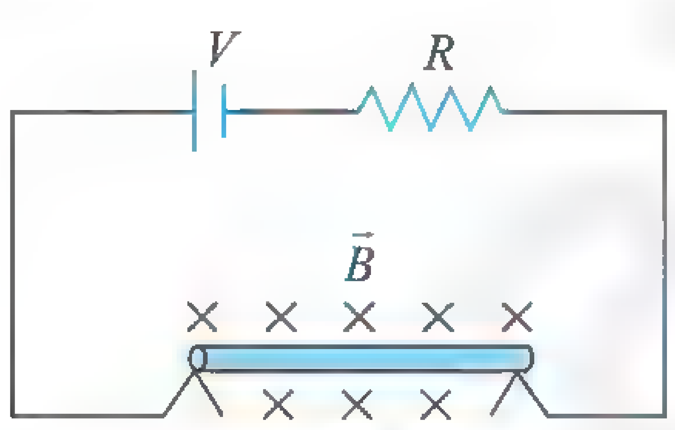


8. From the above statements we can conclude that
(1) the magnetic field is directed into the plane of page
(2) the magnetic field is directed out of the page
(3) the magnetic field is toward left in the plane of page
(4) the magnetic field is toward right in the plane of page

9. The magnitude of magnetic field is
(1) 1.2 T (2) 6 T
(3) 0.6 T (4) 12 T

For Problems 10–11

A thin, 50 cm long metal bar with mass 750 g rests on, but is not atta-ched to, two metallic supports in a uniform 0.450 T magnetic field, as shown in figure. A battery and a 25 Ω resistor in series are connected to the supports.

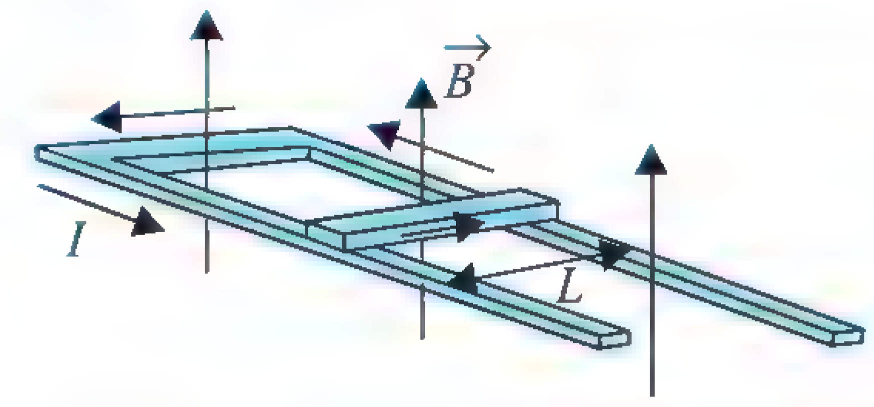


10. What is the largest voltage the battery can have without breaking the circuit at the supports?
(1) 817 V (2) 412 V
(3) 325 V (4) 160 V

11. The battery voltage has the maximum value calculated in above question. If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2 Ω, find the initial acceleration of the bar.
(1) 113 m s^{-2} (2) 55 m s^{-2}
(3) 180 m s^{-2} (4) 12.4 m s^{-2}

For Problems 12–13

A conducting bar with mass m and length L slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current I in the rails and bar, and a constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails (as shown in figure).



12. Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance and electrical resistance.
(1) ILB, to the right. (2) ILB, to the left.
(3) $2ILB$, to the right. (4) $2ILB$, to the left.

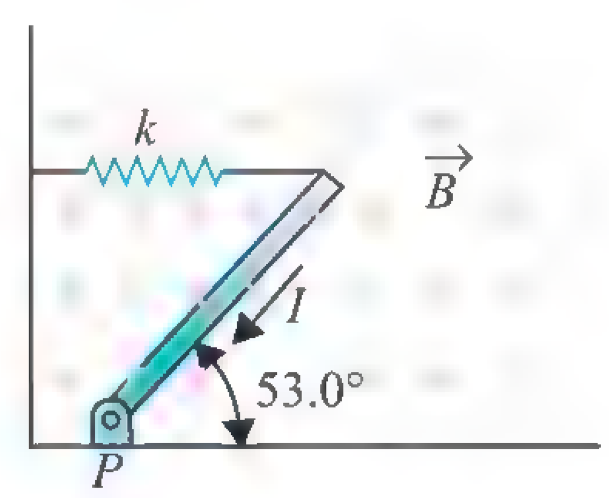
13. If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v.

(1) $\dfrac{3v^2m}{2ILB}$ (2) $\dfrac{5v^2m}{2ILB}$

(3) $\dfrac{v^2m}{ILB}$ (4) $\dfrac{v^2m}{2ILB}$

For Problems 14–15

A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point P (as shown in figure). A horizontal spring with force constant $k = 4.80$ N m^{-1} connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the figure. There is current $I = 6.50$ A in the rod, in the direction shown.

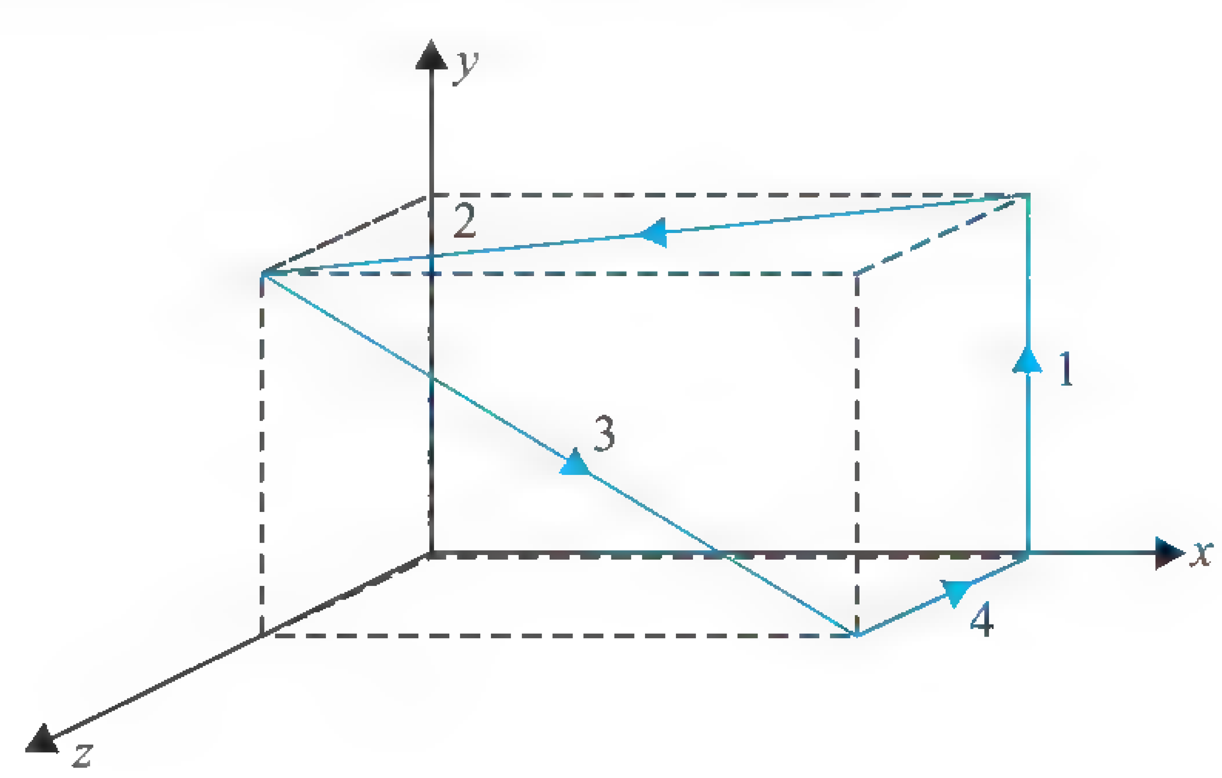


14. Calculate the torque due to the magnetic force on the rod, for an axis at P.
(1) 0.0442 N m^{-1}, clockwise
(2) 0.0442 N m^{-1}, anticlockwise
(3) 0.022 N m^{-1}, clockwise
(4) 0.022 N m^{-1}, anticlockwise

15. When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed?
(1) 0.05765 m, stretched (2) 0.05765 m, compressed
(3) 0.0242 m, stretched (4) 0.0242 m, compressed

For Problems 16–18

A wire carrying a 10 A current is bent to pass through various sides of a cube of side 10 cm as shown in figure. A magnetic field $\vec{B} = (2\hat{i} - 3\hat{j} + \hat{k})$ T is present in the region.



16. Find the net force on the loop shown.

(1) $\vec{F}_{\text{net}} = 0$ (2) $\vec{F}_{\text{net}} = (0.1\hat{i} - 0.2\hat{k})\,\text{N}$

(3) $\vec{F}_{\text{net}} = (0.3\hat{i} + 0.4\hat{k})\,\text{N}$ (4) $\vec{F}_{\text{net}} = (0.36\hat{k})\,\text{N}$

17. Find the magnetic moment vector of the loop.

(1) $(0.1\hat{i} + 0.05\hat{j} - 0.05\hat{k})\,\text{Am}^2$

(2) $(0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})\,\text{Am}^2$

(3) $(0.1\hat{i} - 0.05\hat{j} + 0.05\hat{k})\,\text{Am}^2$

(4) $(0.1\hat{i} - 0.05\hat{j} - 0.05\hat{k})\,\text{Am}^2$

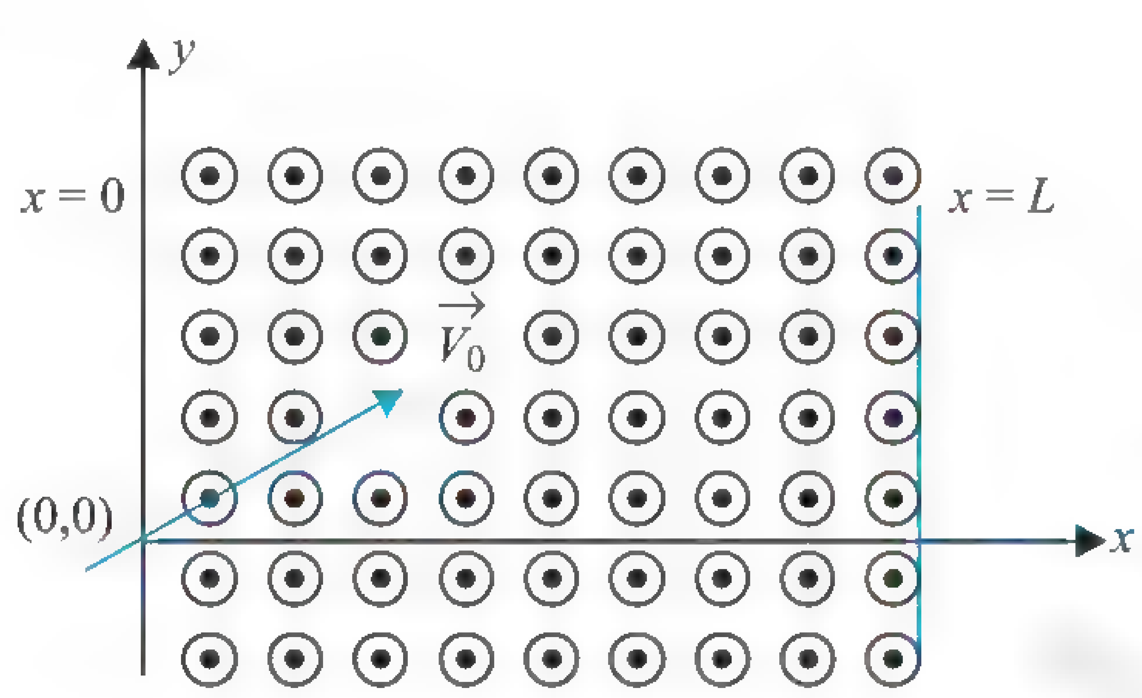
18. Find the net torque on the loop.

(1) $-0.1\hat{i} + 0.4\hat{k}$ Nm (2) $-0.1\hat{i} - 0.4\hat{k}$ Nm

(3) $0.2\hat{i} - 0.4\hat{k}$ Nm (4) $0.1\hat{i} - 0.4\hat{k}$ Nm

For Problems 19–20

The region between $x = 0$ and $x = L$ is filled with uniform constant magnetic field $25(\text{T})\hat{k}$. A particle of mass m = 50 g having positive charge q = 1 C and velocity $\vec{V}_0 = 50\sqrt{3}\,\hat{i} + 50\,\hat{j}$ m s^{-1} enters the region of the magnetic field. Neglect gravity throughout the question.



19. The value of L if the particle emerges from the region of magnetic field with its velocity $100(\text{m s}^{-1})\hat{i}$ is

(1) 20 cm (2) 10 cm

(3) 30 cm (4) None of these

20. The maximum and minimum values of L such that velocity of emerging particle makes angle $\pi/3$ with the y-axis are

(1) $L_{\text{min}} = 10$ cm, $L_{\text{max}} = \infty$

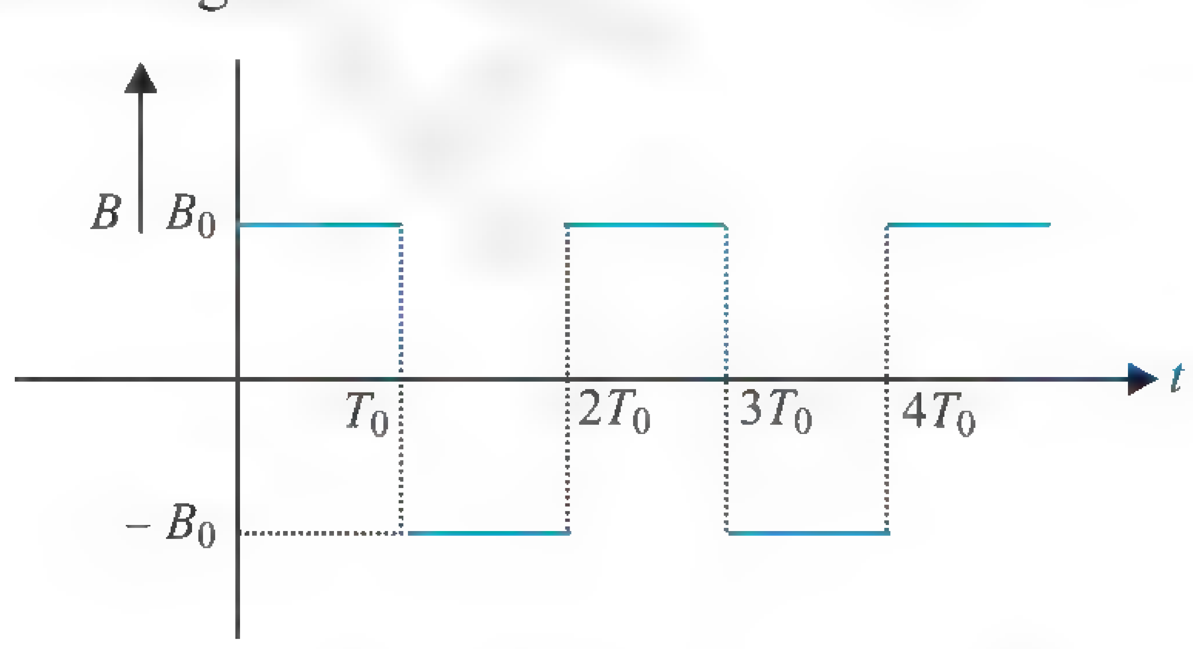
(2) $L_{\text{min}} = 20$ cm, $L_{\text{max}} = 40$ cm

(3) $L_{\text{min}} = 10$ cm, $L_{\text{max}} = 20$ cm

(4) $L_{\text{min}} = 20$ cm, $L_{\text{max}} = \infty$

For Problems 21–22

In a region, magnetic field along x-axis changes according to the graph given in figure:



If time period, pitch and radius of helix path are T_0, P_0 and R_0, respectively, and if the particle is projected at an angle θ_0 with the positive x-axis toward positive y-axis in x–y plane, then

21. Select the correct statement:

(1) At $t = \dfrac{T_0}{2}$, coordinates of charge are $\left(\dfrac{P_0}{2}, 0, -2R_0\right)$

(2) At $t = \dfrac{3T_0}{2}$, coordinates of charge are $\left(\dfrac{3P_0}{2}, 0, 2R_0\right)$

(3) At $t = \dfrac{T_0}{2}$, coordinates of charge are $(P_0, 0, -2R_0)$

(4) At $t = \dfrac{3T_0}{2}$, coordinates of charge are $(3P_0, 0, 2R_0)$

22. Select the correct statement:

(1) Two extremes (positions of charge particle during the motion) from x-axis are at a distance $2R_0$ from each other

(2) Two extremes from x-axis are at a distance $4R_0$ from each other

(3) Two extremes (positions of charge particle during the motion) from x-axis are at a distance R_0 from each other

(4) Two extremes from x-axis are at a distance $3R_0$ from each other

For Problems 23–25

In a certain region of space, there exists a uniform and constant electric field of magnitude E along the positive y-axis of a coordinate system. A charged particle of mass m and charge $-q$ ($q > 0$) is projected from the origin with speed $2v$ at an angle of 60° with the positive x-axis in x–y plane. When the x-coordinate of particle becomes $\sqrt{3}\,mv^2/qE$ a uniform and constant magnetic field of strength B is also switched on along positive y-axis.

23. Velocity of the particle just before the magnetic field is switched on is

(1) $v\hat{i}$ (2) $v\hat{i} + \dfrac{\sqrt{3}v}{2}\hat{j}$

(3) $v\hat{i} - \dfrac{\sqrt{3}v}{2}\hat{j}$ (4) $2v\hat{i} - \dfrac{\sqrt{3}v}{2}\hat{j}$

24. x-coordinate of the particle as a function of time after the magnetic field is switched on is

(1) $\dfrac{\sqrt{3}mv^2}{qE} - \dfrac{mv}{qB}\sin\left(\dfrac{qB}{m}t\right)$

(2) $\dfrac{\sqrt{3}mv^2}{qE} + \dfrac{mv}{qB}\sin\left(\dfrac{qB}{m}t\right)$

(3) $\dfrac{\sqrt{3}mv^2}{qE} - \dfrac{mv}{qB}\cos\left(\dfrac{qB}{m}t\right)$

(4) $\dfrac{\sqrt{3}mv^2}{qE} + \dfrac{mv}{qB}\cos\left(\dfrac{qB}{m}t\right)$

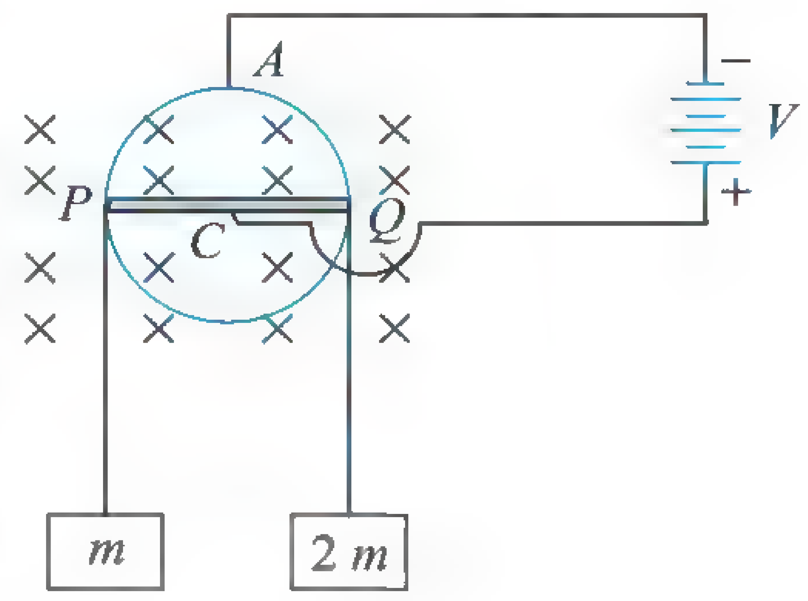
25. z-coordinate of the particle as a function of time after the magnetic field is switched on is

(1) $\dfrac{mv}{qB}\left[1 - \cos\left(\dfrac{qB}{m}t\right)\right]$ (2) $-\dfrac{mv}{qB}\left[1 + \cos\left(\dfrac{qB}{m}t\right)\right]$

(3) $-\dfrac{mv}{qB}\left[1 - \cos\left(\dfrac{qB}{m}t\right)\right]$ (4) $\dfrac{mv}{qB}\left[1 + \cos\left(\dfrac{qB}{m}t\right)\right]$

For Problems 26–28

A conducting ring of mass m and radius r has a weightless conducting rod PQ of length $2r$ and resistance $2R$ attached to it along its diameter. It is pivoted at its center C with its plane vertical, and two blocks of mass m and $2m$ are suspended by means of a light inextensible string passing over it as shown in figure. The ring is free to rotate about C and the system is placed in a magnetic field B (into the plane of the ring). A circuit is now completed by connecting the ring at A and C to a battery of emf V. It is found that for certain value of V, the system remains static.



26. In static condition, find the current through rod PC.

(1) V/R (2) $V/2R$
(3) $4V/R$ (4) $2V/R$

27. Net torque applied by the tension in strings can be related as

(1) $\dfrac{3BVr^2}{R}$ (2) $\dfrac{BVr^2}{R}$

(3) $\dfrac{BVr^2}{3R}$ (4) $\dfrac{BVr^2}{2R}$

28. The value of V can be related with m, B and r as

(1) $2mgR/Br$ (2) mgR/Br
(3) $mgR/2Br$ (4) $3mgR/Br$

For Problems 29–31

A charged particle with charge to mass ratio $\left(\dfrac{q}{m}\right) = \dfrac{10^3}{19}$ C kg^{-1} enters a uniform magnetic field $\vec{B} = 20\hat{i} + 30\hat{j} + 50\hat{k}$ T at time $t = 0$ with velocity $\vec{V} = (20\hat{i} + 50\hat{j} + 30\hat{k})$ m/s. Assume that magnetic field exists in large space.

29. During the further motion of the particle in the magnetic field, the angle between the magnetic field and velocity of the particle

(1) remains constant
(2) increases
(3) decreases
(4) may increase or decrease

30. The frequency (in Hz) of the revolution of the particle in cycles per second will be

(1) $\dfrac{10^3}{\pi\sqrt{38}}$ (2) $\dfrac{10^4}{\pi\sqrt{38}}$

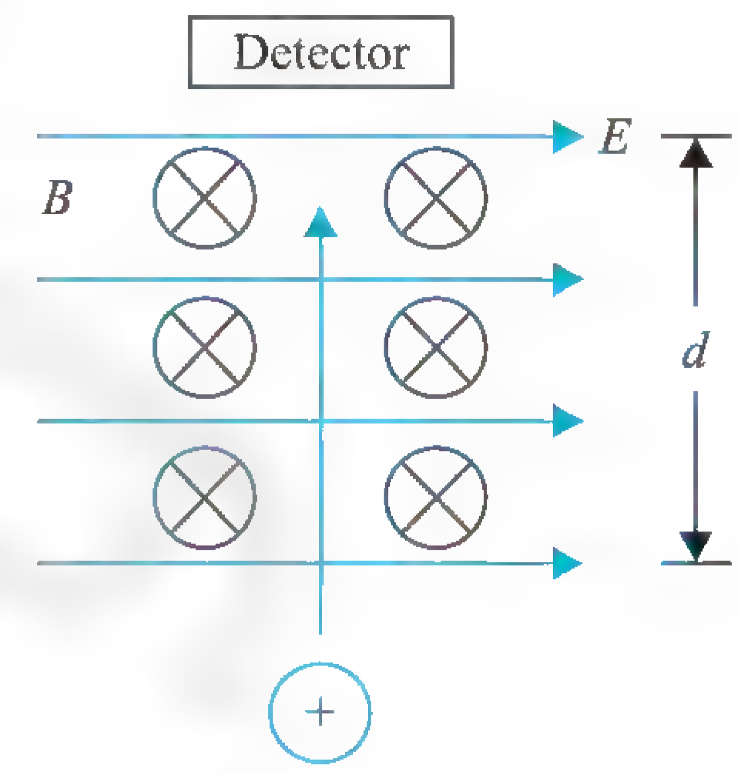
(3) $\dfrac{10^4}{\pi\sqrt{19}}$ (4) $\dfrac{10^4}{2\pi\sqrt{19}}$

31. The pitch of the helical path of the motion of the particle will be

(1) $\pi/100$ m (2) $\pi/125$ m
(3) $\pi/215$ m (4) $\pi/250$ m

For Problems 32–34

A velocity filter uses the properties of electric and magnetic field to select particles that are moving with a specific velocity. Charged particles with varying speeds are directed into the filter as shown. The filter consists of an electric field E and a magnetic field B, each of constant magnitude, directed perpendicular to each other as shown. The charge particles will experience a force due to electric field given by $F = qE$. If positively charged particles are used, this force is towards right.



The moving particle will also experience a force due to magnetic field given by $F = qvB$. When forces due to the two fields are of equal magnitude, the net force on the particle will be zero, the particle will pass through centre with its path unaltered. The electric and magnetic fields can be adjusted to choose the specific velocity to be filtered. The effect of gravity can be neglected.

32. The electric and magnetic fields are adjusted to detect particles with positive charge q of certain speed v_0. Which of the following expressions is equal to this speed?

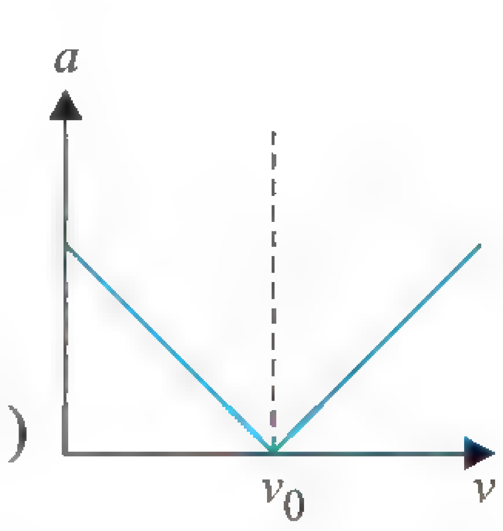
(1) $\dfrac{B}{q^2E}$ (2) $\left(\dfrac{E}{q^2B}\right)$

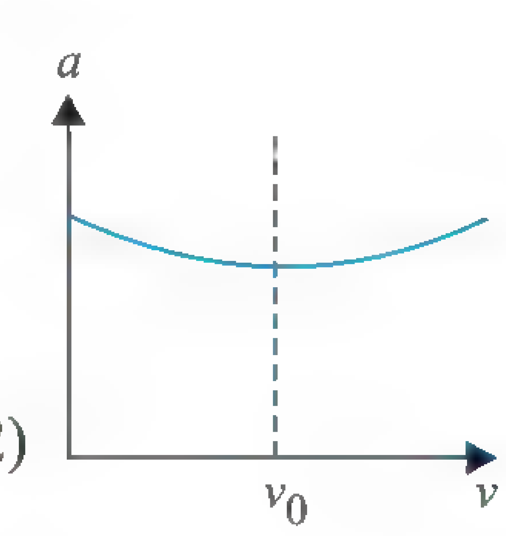
(3) $\left(\dfrac{B}{E}\right)$ (4) $\left(\dfrac{E}{B}\right)$

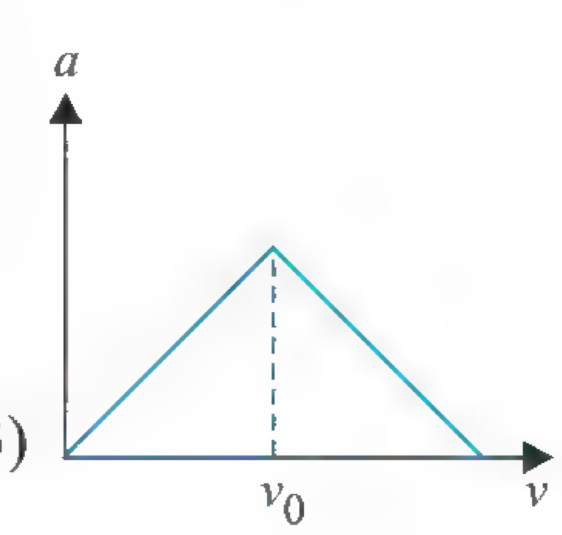
33. Which of the following is true about the velocity filter shown in figure?

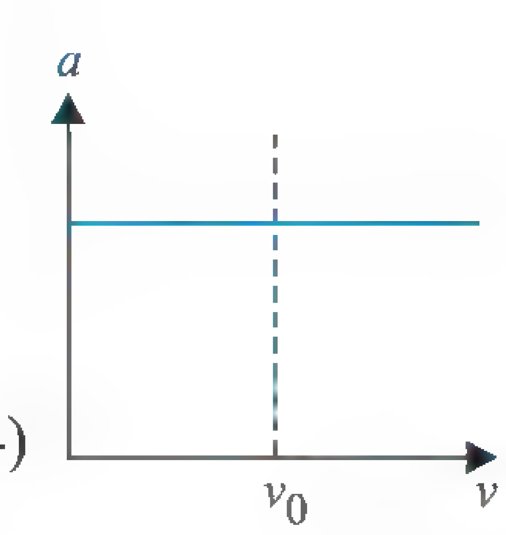
(1) it would not work with negatively charged particles.
(2) wider the detector entrance, the narrower the range of speed detected.
(3) greater the distance d, the narrower the range of speed detected.
(4) The detector may not detect a charged particle with desired filter speed if its charge is too high.

34. If the filter is set to detect particles of speed v_0, which one of the following diagrams best illustrates how the magnitude of the particle acceleration (a) depends on velocity v?

(1) 

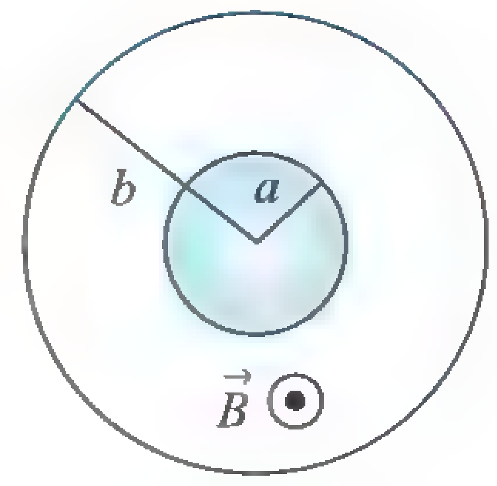
(2) 

(3) 

(4) 

For Problems 35–37

In the given arrangement, the space between a pair of co-axial cylindrical conductors is evacuated. The outer cylinder, called anode, may be given a positive potential V relative to the inner cylinder.



A static homogeneous magnetic field $\vec{B}$ parallel to the cylinder axis, directed out of plane of figure is also present. Induced charges in the conductors are neglected.

We study the dynamics of electrons with rest mass m and charge e. The electrons are released at the surface of inner cylinder. Consider the following two cases:

Case 1: Firstly the potential V is turned on, but $\vec{B}=0$. An electron with negligible velocity is ejected at the surface of inner cylinder. It is found to hit the anode.

Case 2: Now $V=0$ but $\vec{B}$ is present. An electron starts out with an initial velocity $\vec{v}_0$ in radial direction. For magnetic field larger than critical value B_c the electron will not reach the anode.

35. Considering the case 1, the trajectory of electron will be
(1) Straight line (2) Circular
(3) Parabolic (4) Helical

36. Considering case 2, the trajectory of electron will be
(1) Straight line (2) Circular
(3) Parabolic (4) Helical

37. The critical magnetic field B_C is given by
(1) $\frac{2mv_0}{eb}$ (2) $\frac{mv_0}{eb}$
(3) $\frac{2bmv_0}{e(b^2-a^2)}$ (4) $\frac{2amv_0}{e(b^2-a^2)}$

For Problems 38–40

Magnetic force on a charged particle is given by $\vec{F}_m = q(\vec{v}\times\vec{B})$ and electrostatic force $\vec{F}_e = q\vec{E}$. A particle having charge $q = 1C$ and mass 1 kg is released from rest at origin. There are electric and magnetic fields given by $\vec{E} = (10\hat{i})\text{N/C}$ for x = 1.8 m and $\vec{B} = -(5\hat{k})\text{T}$ for $1.8 \text{ m} \le x \le 2.4$ m.

A screen is placed parallel to $y-z$ plane at $x = 3$ m. Neglect gravity forces.

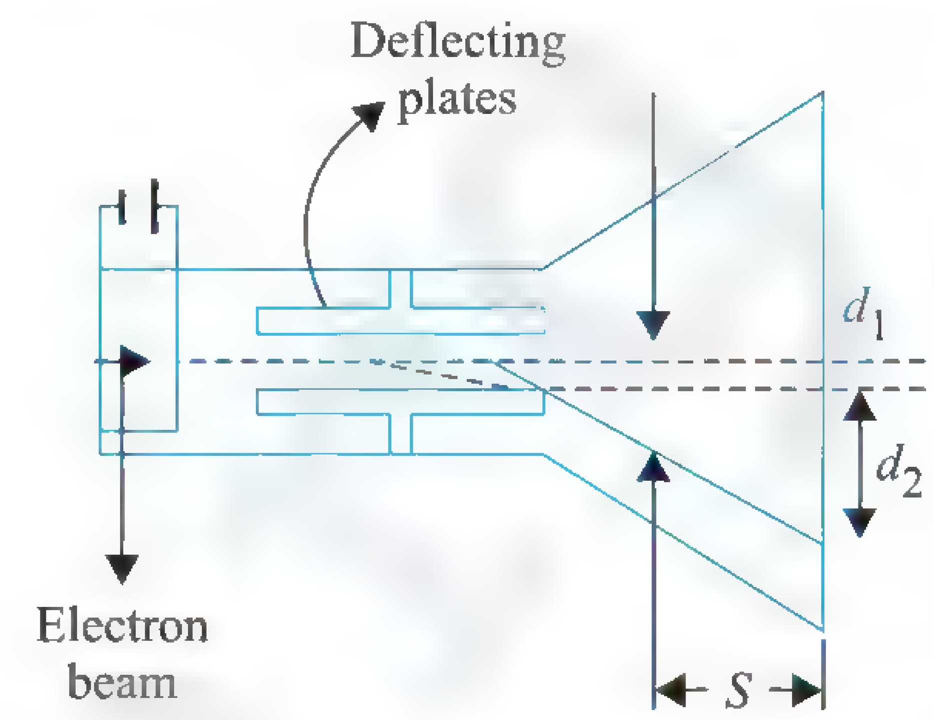
38. The speed with which the particle will collide the screen is
(1) 3 m s^{-1} (2) 6 m/s^{-1}
(3) 9 m/s^{-1} (4) 12 m/s^{-1}

39. y-coordinate of particle where it collides with screen (in meters) is
(1) $\frac{0.6(\sqrt{3}-1)}{\sqrt{3}}$ (2) $\frac{0.6(\sqrt{3}+1)}{\sqrt{3}}$
(3) $1.2(\sqrt{3}+1)$ (4) $\frac{1.2(\sqrt{3}-1)}{\sqrt{3}}$

40. Time after which the particle will collide the screen is (in seconds)
(1) $\frac{1}{5}\left(3+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)$ (2) $\frac{1}{5}\left(6+\frac{\pi}{3}+\sqrt{3}\right)$
(3) $\frac{1}{3}\left(5+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{1}{3}\left(6+\frac{\pi}{18}+\sqrt{3}\right)$

For Problems 41–43

Following experiment was performed by J.J. Thomson in order to measure ratio of charge e and mass m of electron.



Electrons emitted from a hot filament are accelerated by a potential difference V. As the electrons pass through deflecting plates, they encounter both electric and magnetic fields. The entire region in which electrons leave the plates they enter a field free region that extends to fluorescent screen. The entire region in which electrons travel is evacuated.

Firstly, electric and magnetic fields were made zero and position of undeflected electron beam on the screen was noted. The electric field was turned on and resulting deflection was noted. Deflection is given by $d_1 = \frac{eEL^2}{2mV^2}$ where L = length of deflecting plate and v = speed of electron.

In second part of experiment, magnetic field was adjusted so as to exactly cancel the electric force leaving the electron beam undeflected. This gives $eE = evB$. Using expression for d_1, we can find out $\frac{e}{m} = \frac{2d_1E}{B^2L^2}$

41. If the electron is deflected downward when only electric field is turned on, in what direction do the electric and magnetic fields point in second part of experiment
(1) The electric field points to the top, while the magnetic field points into the page
(2) The electric field points to the top, while the magnetic field points out of page
(3) Electric field points to the bottom, while the magnetic field points out of page
(4) Electric field points to the bottom, while the magnetic field points into the page

42. A beam of electron with velocity 3×10^7 m s^{-1} is deflected 2mm while passing through 10 cm in an electric field of 1800 V/m perpendicular to its path. e/m for electron is
(1) 1.5×10^{11} C kg^{-1} (2) 2×10^{11} C kg^{-1}
(3) 2.5×10^{11} C kg^{-1} (4) 3×10^{11} C kg^{-1}

43. If the electron speed were doubled by increasing the potential difference V, which of the following would be true in order to correctly measure e/m.
(1) Magnetic field would have to be halved.
(2) Magnetic field would have to be doubled.
(3) Length L of the plates would have to be doubled.
(4) Length L of the plates would have to be halved.

Matrix Match Type

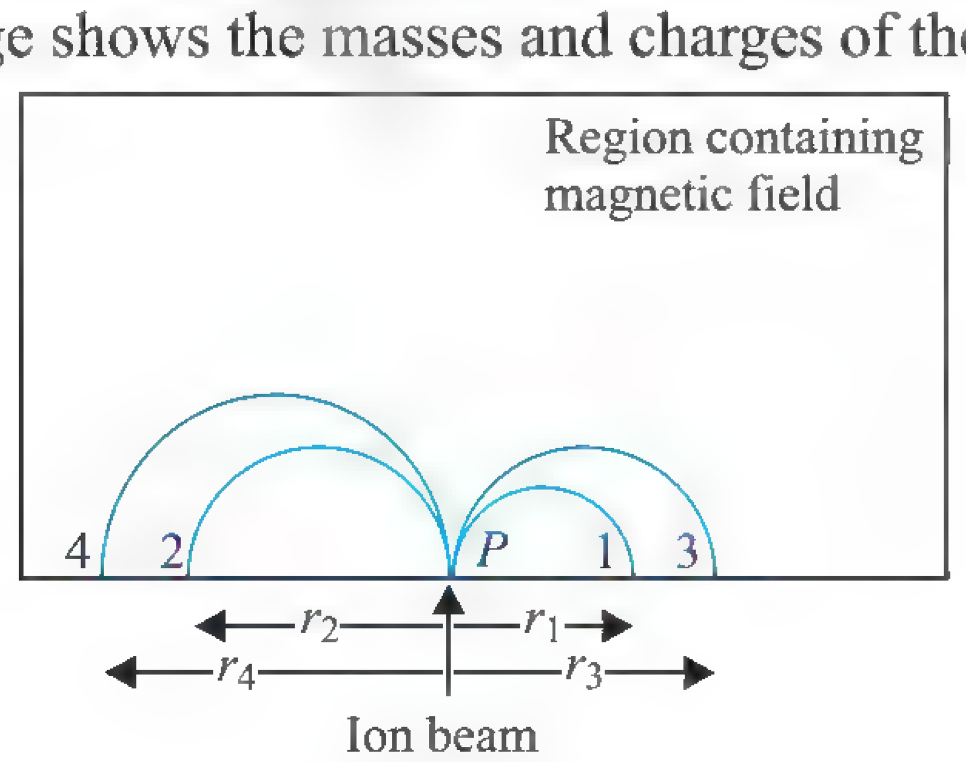
1. A charged particle passes through a region that could have electric field only or magnetic field only or both electric and magnetic fields or none of the fields. Match Column I with Column II.

Column I		Column II	
i.	Kinetic energy of the particle remains constant	**a.**	Under special conditions, this is possible when both electric and magnetic fields are present
ii.	Acceleration of the particle is zero	**b.**	The region has electric field only
iii.	Kinetic energy of the particle changes and it also suffers deflection	**c.**	The region has magnetic field only
iv.	Kinetic energy of the particle changes but it suffers no deflection	**d.**	The region contains no field

2. Match Column I with Column II.

Column I		Column II	
i.	A charge particle is moving in uniform electric and magnetic fields in gravity free space.	**a.**	Velocity of the particle may be constant.
ii.	A charge particle is moving in uniform electric, magnetic and gravitational fields.	**b.**	Path of the particle may be straight line.
iii.	A charge particle is moving in uniform magnetic and gravitational fields (where electric field is zero).	**c.**	Path of the particle may be circular.
iv.	A charge particle is moving in only uniform electric field.	**d.**	Path of the particle may be helical.

3. A beam consisting of four types of ions A, B, C, and D enters a region at P that contains a uniform magnetic field as shown in the figure. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The table on the next page shows the masses and charges of the ions.



$r_4 > r_3 = r_2 > r_1$

Ion	Mass	Charge
A	$2\,m$	e
B	$4\,m$	$-e$
C	$2\,m$	$-e$
D	m	$+e$

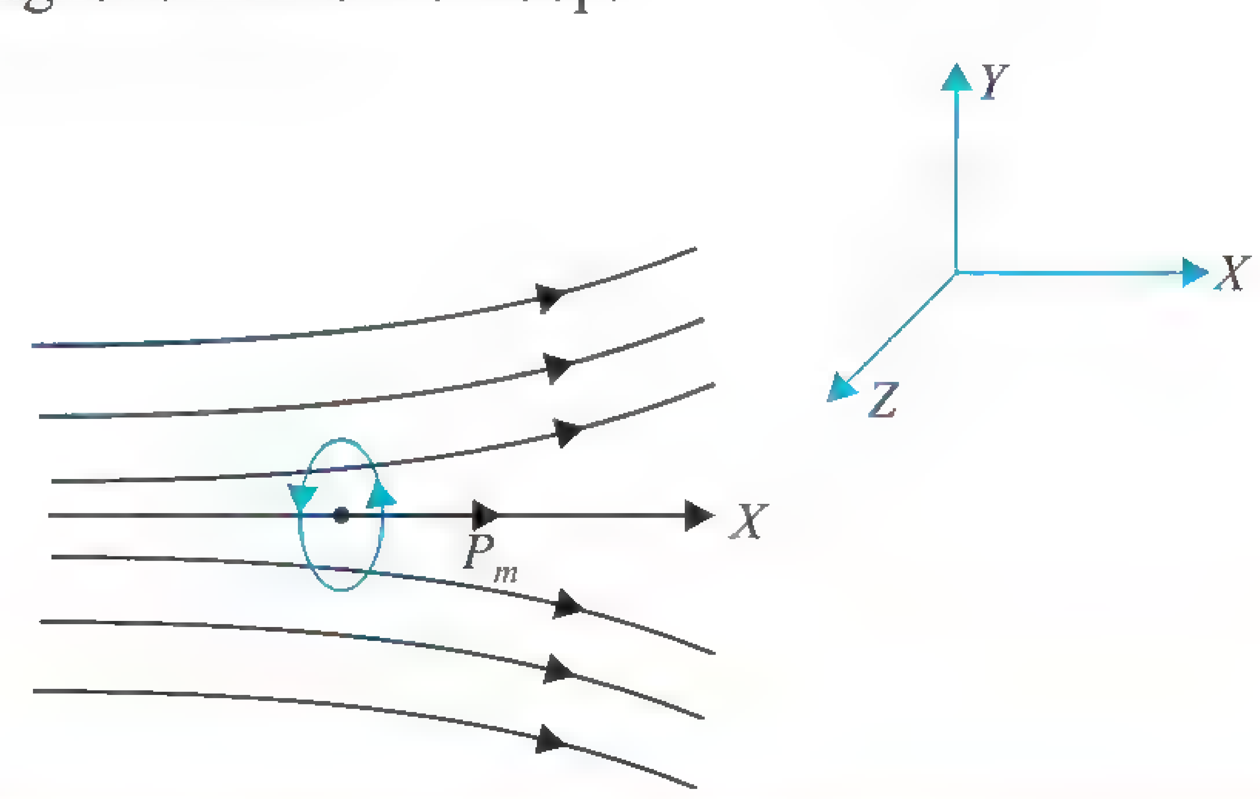
The ions fall at different positions 1, 2, 3, and 4 as shown. Correctly match the ions with their respective positions of fall.

Column I		Column II	
i.	A	**a.**	4
ii.	B	**b.**	2
iii.	C	**c.**	1
iv.	D	**d.**	3

4. A charged particle with some initial velocity is projected in a region where non-zero electric and/or magnetic fields are present. In column I, information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in Column II the probable path of the charged particle is mentioned. Match the entries of Column I with the entries of Column II.

Column I		Column II	
i.	$\vec{E}=0, \vec{B}\neq 0,$ and initial velocity may be at any angle with $\vec{B}$	**a.**	Straight line
ii.	$\vec{E}\neq 0, \vec{B}=0$ and initial velocity may be at any angle with $\vec{E}$	**b.**	Parabola
iii.	$\vec{E}\neq 0, \vec{B}\neq 0, \vec{E}\parallel\vec{B}$ and initial velocity is $\perp$ to both	**c.**	Circular
iv.	$\vec{E}\neq 0, \vec{B}\neq 0, \vec{E}$ is perpendicular to $\vec{B}$ and non-uniform pitch $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$	**d.**	Helical path

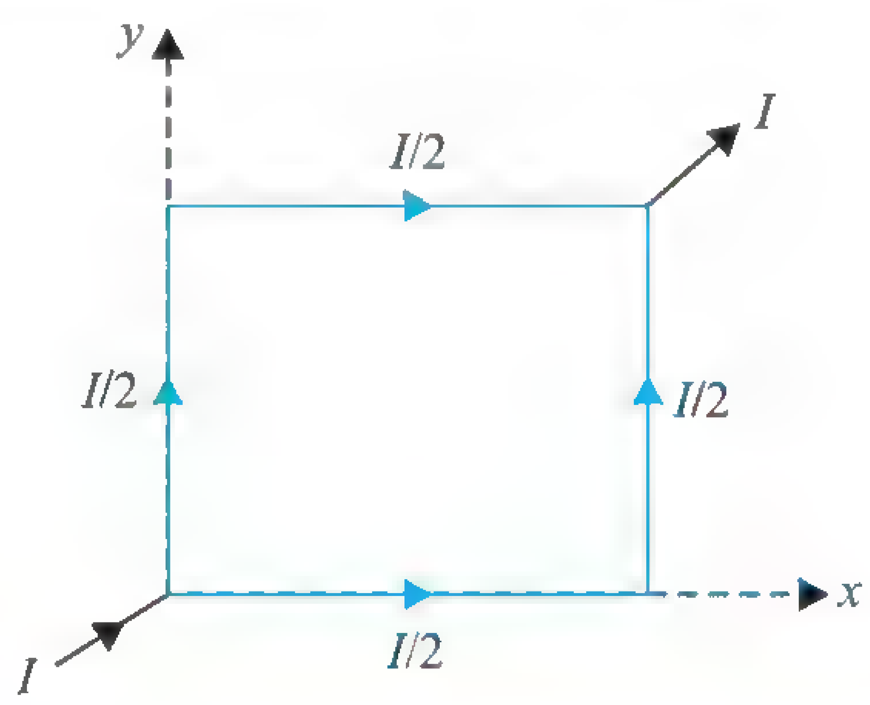
5. An elementary current loop is placed in a non-uniform magnetic field as shown in figure. In Column I, different orientations of loop are described and in Column II, the corresponding forces experienced by the loop. P_m is magnetic moment of loop.



Column I		Column II	
i.	In the given situation,	**a.**	resultant force is acting along $\vec{P}_m$

ii.	If loop is rotated such that $\vec{P}_m$ is along +ve Z direction	**b.**	resultant force is acting opposite to $\vec{P}_m$
iii.	If loop is rotated such that $\vec{P}_m$ is along –ve Z direction	**c.**	$F_x = 0, F_y = 0$
iv.	If loop is rotated such that $\vec{P}_m$ is along +ve Y direction	**d.**	$F_x = 0, F_z = 0$

6. A square loop of uniform conducting wire is as shown in figure. A current I (in amperes) enters the loop from one end and exits the loop from opposite end as shown. The length of one side of the square loop is l meter. The wire has uniform cross-section area and uniform linear mass density. In four situations of Column I, the loop is subjected to four different magnetic fields. Under the conditions of Column I, match the Column I with corresponding results of Column II (B_0 in Column I is a positive non-zero constant)



	Column I		Column II
i.	$\vec{B} = B_0\hat{i}$ in tesla	**a.**	Magnitude of net force on the loop is $\sqrt{2}\,B_0Il$ newton
ii.	$\vec{B} = B_0\hat{j}$ in tesla	**b.**	Magnitude of net force on the loop is zero
iii.	$\vec{B} = B_0(\hat{i}+\hat{j})$ in tesla	**c.**	Magnitude of net torque of the loop about its center is zero
iv.	$\vec{B} = B_0\hat{k}$ in tesla	**d.**	Magnitude of net force on the loop is B_0Il newton

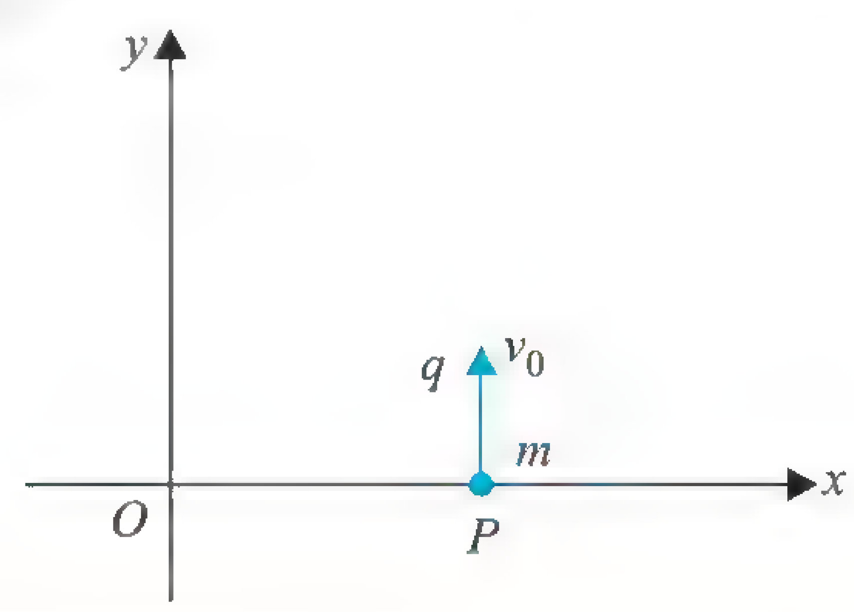
7. A charged particle having a charge q and mass m is to be subjected to a combination of constant uniform magnetic field $(\vec{B})$ and a constant uniform gravitational field $(\vec{E})$. Apart from these field forces there exists no other force. Now match the column.

	Column I		Column II
i.	The charged particle moves without change in its direction.	**a.**	It is possible that both $\vec{B}$ and $\vec{E}$ are zero.
ii.	The charged particle moves without change in its velocity.	**b.**	It is possible that both $\vec{B}$ and $\vec{E}$ are non zero.
iii.	The charged particle takes a circular path.	**c.**	It is possible that $\vec{B}$ is zero and $\vec{E}$ is not zero.
iv.	The charged particle take a parabolic path.	**d.**	It is possible that $\vec{B}$ is non-zero and $\vec{E}$ is zero.

8. Column I shows the state of motion of a charged particle. Column II shows the possible combination of electric field and magnetic field under which the path in column I is possible. Match appropriately

	Column I		Column II
i.	Charge at rest experiences a force.	**a.**	$E = 0, B = 0$
ii.	A charge in motion goes undeviated with same velocity.	**b.**	$E \neq 0, B \neq 0$
iii.	A charge in motion goes undeviated with varying speed.	**c.**	$E = 0, B \neq 0$
iv.	A charged particle undergoes helical motion.	**d.**	$E \neq 0, B = 0$

9. A negatively charged particle of mass 'm' having magnitude of charge q enters a magnetic field $\vec{B} = B_0\hat{k}$ T at point P(3 m, 0, 0) with velocity $\vec{v}_0 = 3\hat{j} + 4\hat{k}$ m/s at $t = 0$ as shown in the figure $\left[\text{Given } \dfrac{qB_0}{m} = 1 \text{ rad/s}\right]$ [No other field is present]

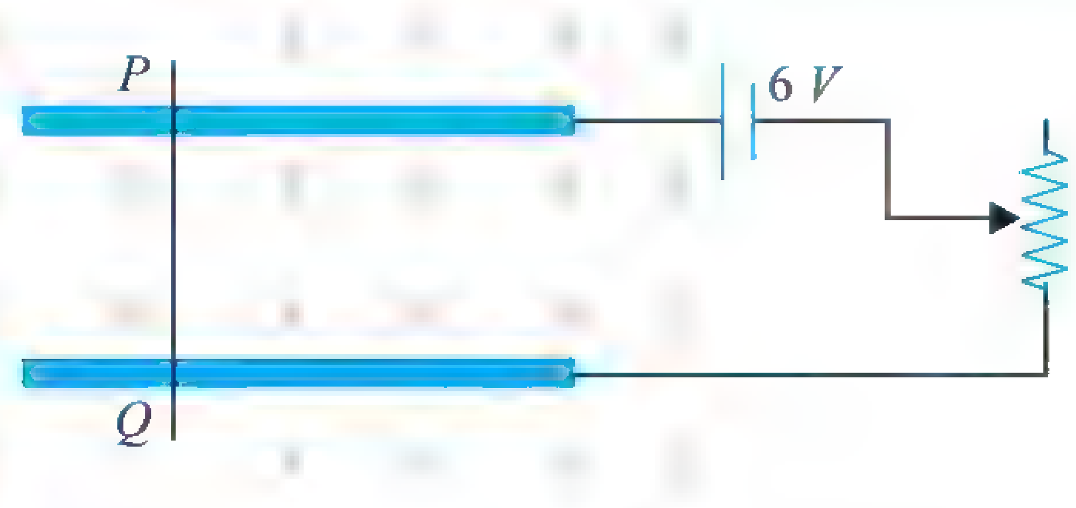


Now match the following:

	Column I		Column II
i.	Pitch of the motion of the particle	**a.**	$(-3\sin t\,\hat{i} + 3\cos t\,\hat{j})$ unit
ii.	$\dfrac{24\pi}{25} \times$ Radius of curvature of particle during motion at any time $t = t$ sec	**b.**	$(-3\cos t\,\hat{i} - 3\sin t\,\hat{j})$ unit
iii.	Velocity component of particle in x-y plane.	**c.**	8π unit
iv.	Acceleration of particle.	**d.**	Constant

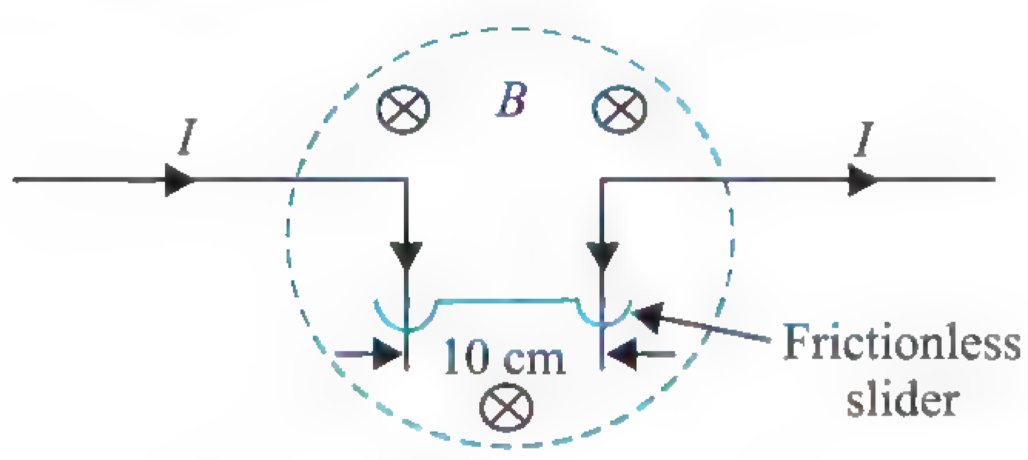
Numerical Value Type

1. A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 5 cm as shown in figure. A vertically downward magnetic field of magnitude 0.80 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20.0 Ω, the wire PQ starts sliding on the rails. The coefficient of friction between wires and rails is found to be $n/25$. Find n.



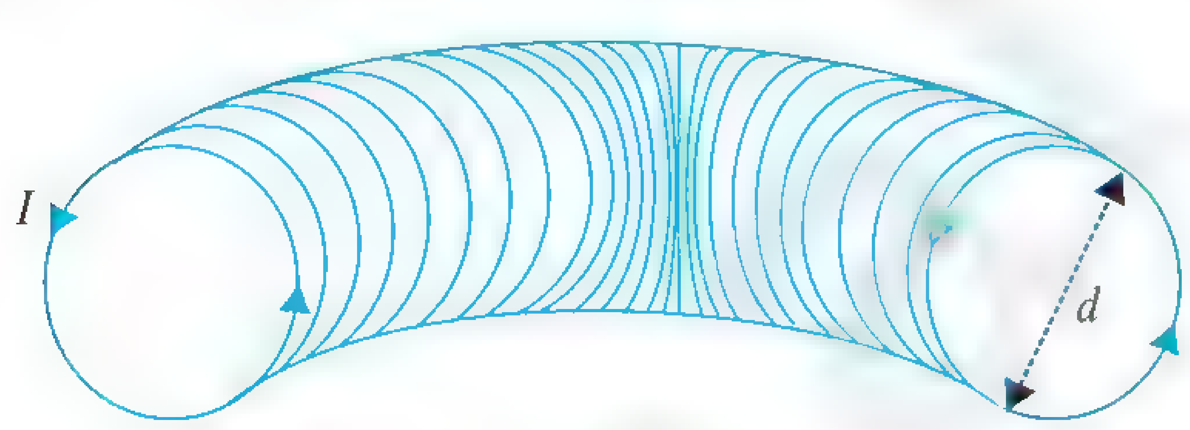
2. A current $I = 10$ A flows in a ring of radius $r_0 = 15$ cm made of a very thin wire. The tensile strength of the wire is equal to $T = 1.5$ N. The ring is placed in a magnetic field, which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find B (in T) at which the ring is broken.

3. A 10 cm length of wire with a mass of 20 g is attached frictionlessly to the vertical segments of a wire in which a current I flows. The surrounding has uniform horizontal field $B = 10^4$ G and the direction is shown in figure. What must be the current I (in A) to maintain the 10 cm wire in an equilibrium position?

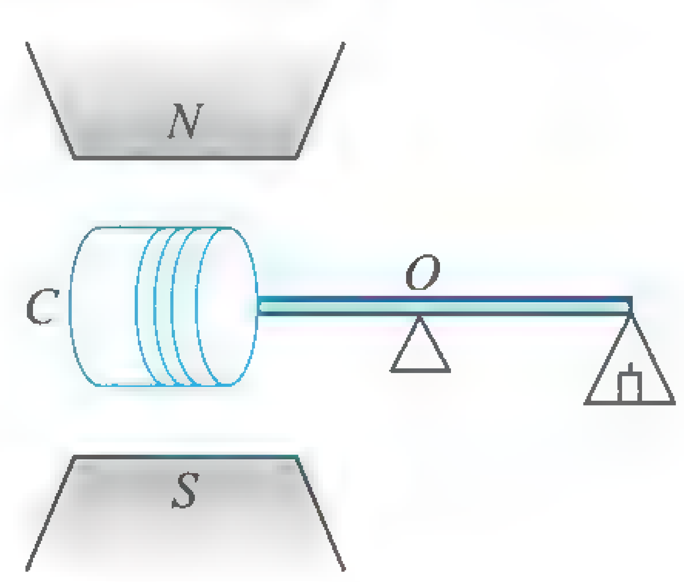


4. A current I flows in a rectangularly shaped wire whose center lies at $(x_0, 0, 0)$ and whose vertices are located at the points A $(x_0 + d, -a, -b)$, B $(x_0 - d, a, -b)$, C $(x_0 - d, a, +b)$, and D $(x_0 + d, -a, +b)$ respectively. Assume that $a, b, d << x_0$. Find the magnitude of magnetic dipole moment vector of the rectangular wire frame in J/T. (Given: $b = 10$ m, $I = 0.01$ A, $d = 4$ m, $a = 3$ m.)

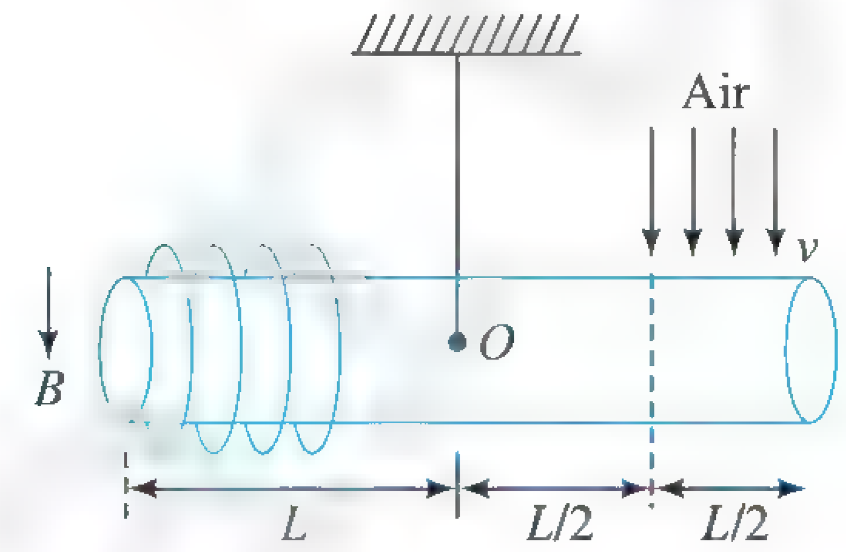
5. Calculate the magnetic moment (in Am^2) of a thin wire with a current $I = 8$ A, wound tightlly on a half a tore (see figure). The diameter of the cross section of the tore is equal to $d = 5$ cm, and the number of turns is $N = 500$.



6. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in figure. The area of the coil is $S = 1$ cm^2, the length of the right arm of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil, equilibrium is restored by putting an additional weight of mass $m = 60$ mg on the balance pan. Find the magnetic induction field (in terms of $\times 10^{-1}$ T) between the poles of the electromagnet, assuming it to be uniform.

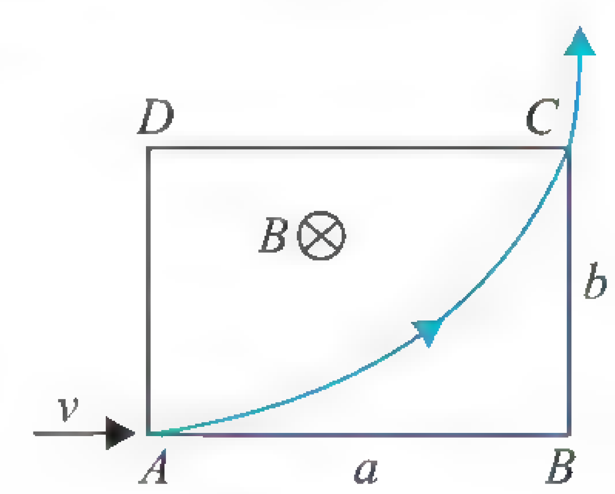


7. A non-conducting non-magnetic rod having circular cross section of radius R is suspended from a rigid support as shown in the figure. A light and small coil of 300 turns is wrapped tightly at the left end of the rod where uniform magnetic field B exists in vertically downward direction. Air of density ρ hits the half of the right part of the rod with velocity V as shown in the figure. What should be current in clockwise direction (as seen from O) in the coil so that rod remains horizontal? Give answer in mA. Given $\frac{2}{Lv}\sqrt{\frac{\pi RB}{\rho}} = \frac{1}{\sqrt{5}} A^{-1/2}$.



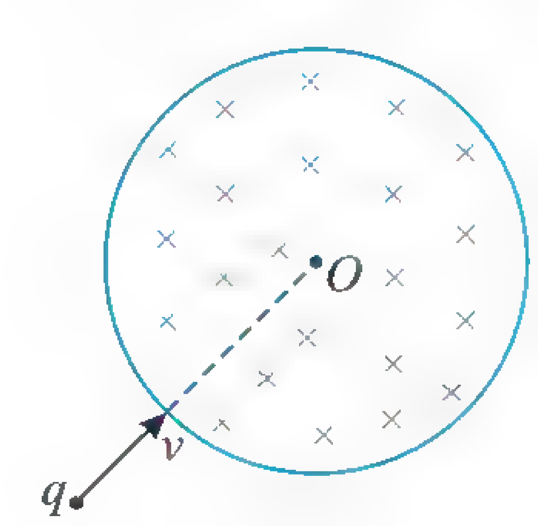
8. A charged particle enters a uniform magnetic field with velocity $v_0 = 4$ m/s perpendicular to it, the length of magnetic field is $x = \frac{\sqrt{3}}{2}R$, where R is the radius of the circular path of the particle in the field. Find the magnitude of change in velocity (in m/s) of the particle when it comes out of the field.

9. A charged particle of mass $m = 1$ mg and charge $q = 1$ μC enters along AB at point A in a uniform magnetic field $B = 1.2$ T existing in the rectangular region of size $a \times b$, where $a = 4$ m and $b = 3$ m. The particle leaves the region exactly at corner point C. What is the speed v (in m s^{-1}) of the particle?



10. A magnetic field $\vec{B} = -B_0\hat{i}$ exists within a sphere of radius $R = v_0T\sqrt{3}$ where T is the time period of one revolution of a charged particle starting its motion from origin and moving with a velocity $\vec{v}_0 = \frac{v_0}{2}\sqrt{3}\,\hat{i} - \frac{v_0}{2}\hat{j}$. Find the number of turns that the particle will take to come out of the magnetic field.

11. The figure shows a circular region of radius $R = \sqrt{3}$ m which has a uniform magnetic field $B = 0.2$ T directed into the plane of the figure. A particle having mass $m = 2$ g, speed $v = 0.3$ m/s and charge $q = 1$ mC is projected along the radius of the circular region as shown in figure. Calculate the angular deviation (in degrees) produced in the path of the particle as it comes out of the magnetic field. Neglect any other force apart from the magnetic force.



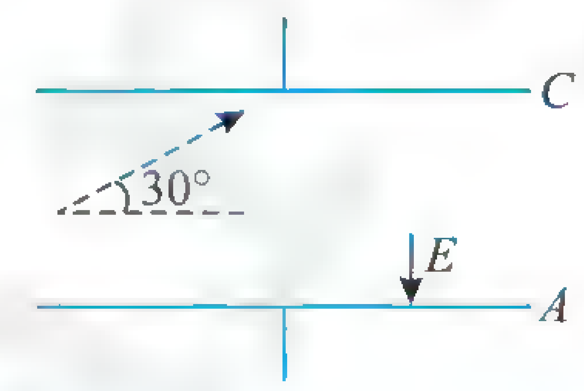
12. A charged particle carrying charge q = 1 mC moves in uniform magnetic field with velocity $v_1 = 10^6$ m/s at angle 45° with x-axis in the xy-plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6$ m/s along the z-axis, it experiences a force F_2 in y-direction. Find the magnitude of the force F_2 (in N).

13. What is the value of B (in × 10^{-8} T) that can be set up at the equator to permit a proton of speed 10^7 m/s to circulate around the earth?

$[R = 6.4 \times 10^6 \text{ m}, \ m_P = 1.67 \times 10^{-27} \text{ kg}]$

14. A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with E = 120 kV/m and B = 50 mT. Then the beam strikes a grounded target. Find the force (in $\times 10^{-5}$ N) imparted by the beam on the target if the beam current is equal to I = 0.80 A.

15. A charged particle having charge 10^{-6} C and mass of 10^{-10} kg is fired at the middle of the plate making an angle 30° with plane of the plate. Length of the plate is 0.17 m and it is separated by 0.1 m. Electric field $E = 10^{-3}$ N/C is present between the plates. Just outside the plates magnetic field is present. Find the magnitude of the magnetic field (in mT) perpendicular to the plane of the figure, if it has to graze the plate at C and A (in mT) parallel to the surface of the plate. (Neglect gravity)

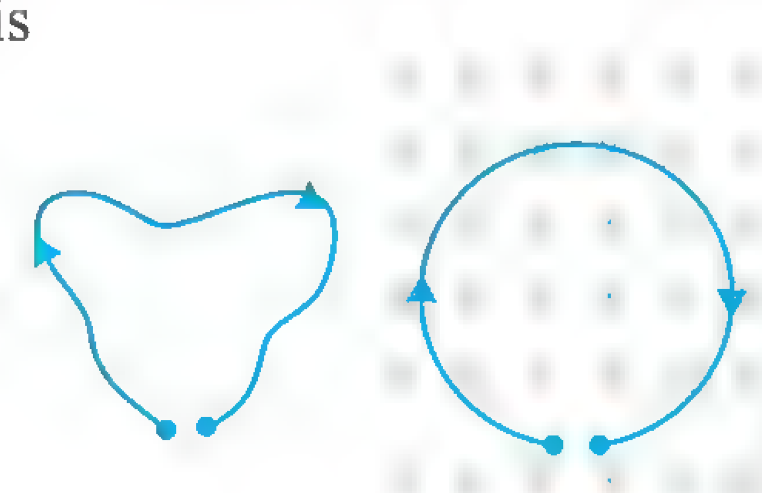


Archives

JEE Advanced

Single Correct Answer Type

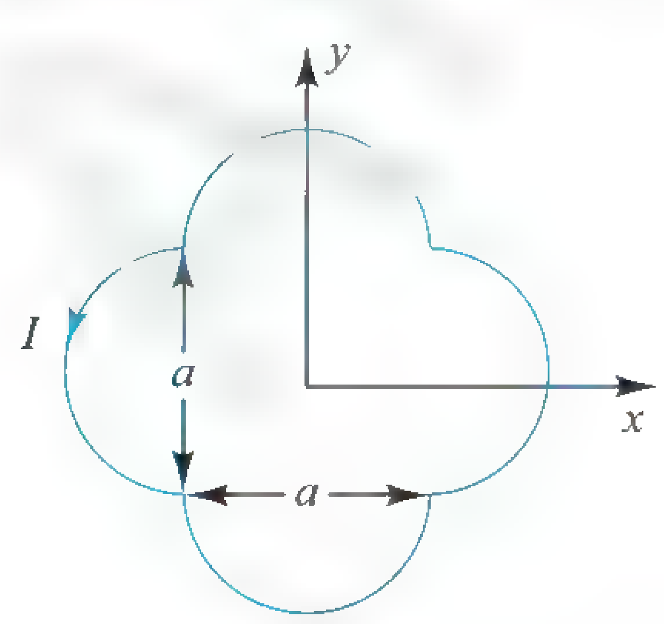
1. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(1) IBL (2) $\dfrac{IBL}{\pi}$

(3) $\dfrac{IBL}{2\pi}$ (4) $\dfrac{IBL}{4\pi}$ **(IIT-JEE 2010)**

2. A loop carrying current I lies in the x–y plane as shown in figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is



(1) $a^2 I\hat{k}$ (2) $\left(\dfrac{\pi}{2}+1\right)a^2 I\hat{k}$

(3) $-\left(\dfrac{\pi}{2}+1\right)a^2 I\hat{k}$ (4) $(2\pi+1)a^2 I\hat{k}$

(IIT-JEE 2012)

Multiple Correct Answers Type

1. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(1) They will never come out of the magnetic field region.
(2) They will come out travelling along parallel paths.
(3) They will come out at the same time.
(4) They will come out at different times.

(IIT-JEE 2011)

2. Consider the motion of a positive point charge in a region where are simultaneous uniform electric and magnetic field $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?

(1) If $\theta = 0°$, the charge moves in a circular path in the x-z plane
(2) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(3) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis
(4) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis **(JEE Advanced 2013)**

3. A particle of mass M and positive charge Q, moving with a constant velocity $u_1 = 4\hat{i}\,\text{m s}^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i} + \hat{j})\,\text{m s}^{-1}$. The correct statement(s) is (are)

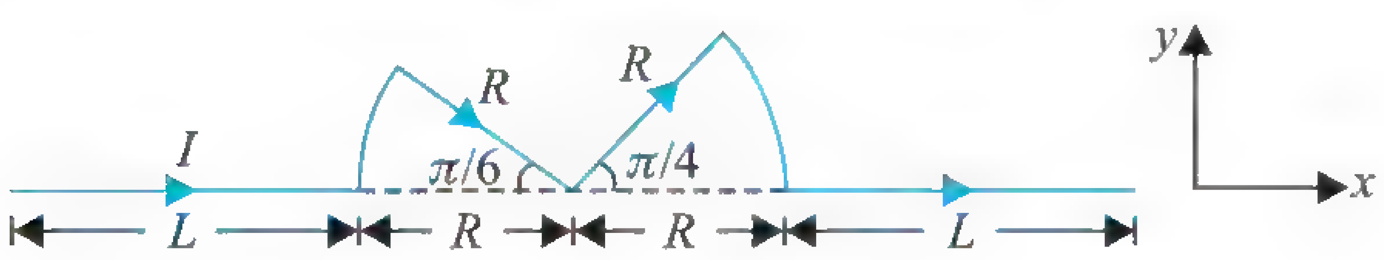
(1) The direction of the magnetic field is $-z$ direction.
(2) The direction of the magnetic field is $+z$ direction.

(3) The magnitude of the magnetic field $\dfrac{50\pi M}{3Q}$ units

(4) The magnitude of the magnetic field is $\dfrac{100\pi M}{3Q}$ units

(JEE Advanced 2013)

4. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are):



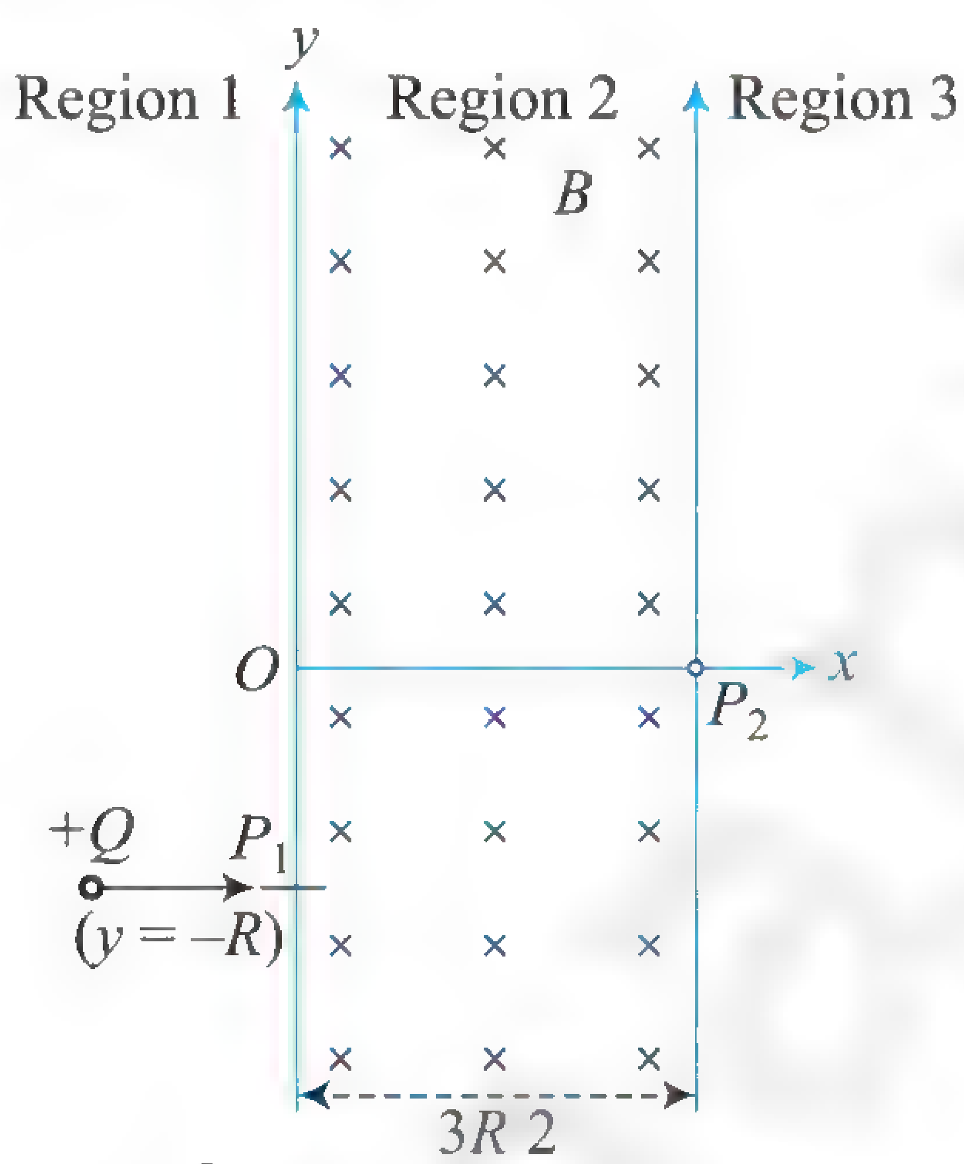
(1) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$

(2) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(3) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$

(4) If $\vec{B}$ is along $\hat{z}$, $F = 0$ **(JEE Advanced 2015)**

5. A uniform magnetic field B exists in the region between $x = 0$ and $x = \dfrac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along x-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following options(s) is/are correct?



(1) For $B = \dfrac{8}{13}\dfrac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(2) For $B > \dfrac{2}{3}\dfrac{p}{QR}$, the particle will re-enter region 1

(3) For a fixed B, particle of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

(4) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $\dfrac{p}{\sqrt{2}}$

(JEE Advanced 2017)

6. Two identical moving coil galvanometers have $10\ \Omega$ resistance and full scale deflection at $2\ \mu A$ current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with $R = 1000\ \Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct ?

(1) The measured value of R will be $978\ \Omega < R < 982\ \Omega$

(2) The resistance of voltmeter will be $100\ k\Omega$

(3) The resistance of the ammeter will be $0.02\ \Omega$ (round off to 2nd decimal place)

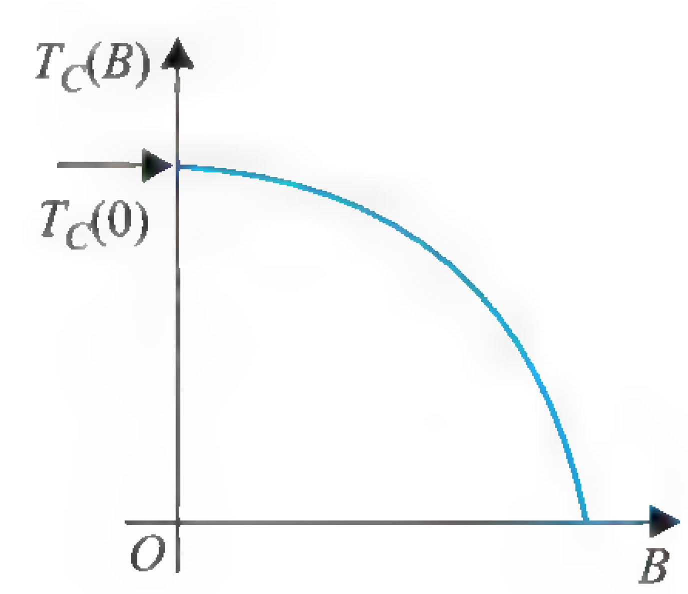
(4) If the ideal cell is replaced by a cell having internal resistance of 5Ω then the measured value of R will be more than $1000\ \Omega$

(JEE Advanced 2019)

Linked Comprehension Type

For Problems 1–2

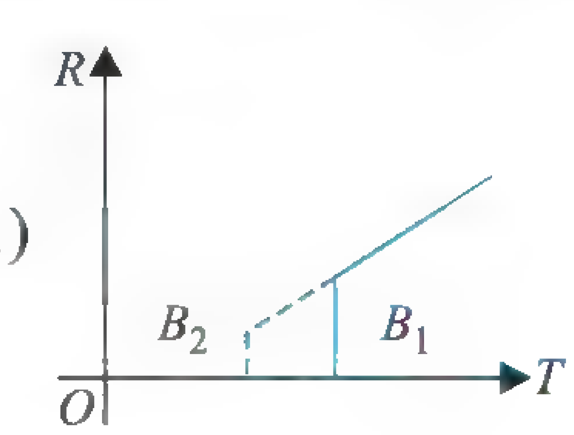
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_c(B)$ is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in figure.



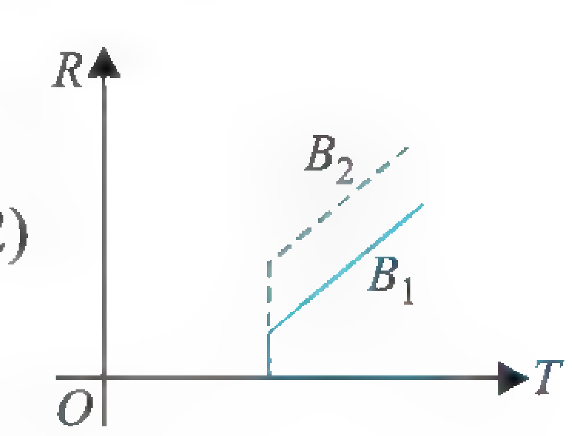
(IIT-JEE 2010)

1. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

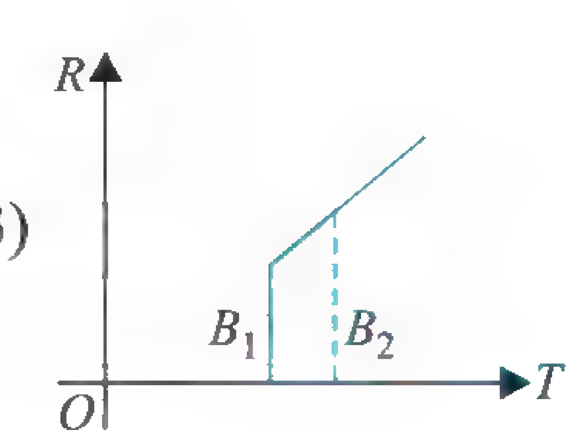
(1)



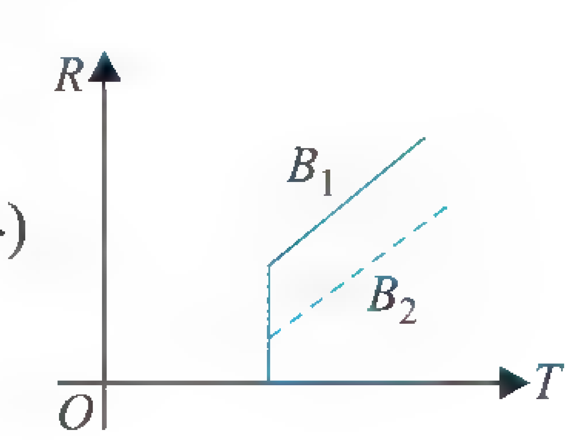
(2)



(3)



(4)



2. A superconductor has $T_c(0) = 100$ K. When a magnetic field of 7.5 T is applied, its T_c decreases to 75 K. For this material one can definitely say that when

(1) $B = 5$ T, $T_c(B) = 80$ K

(2) $B = 5$ T, $75\text{ K} < T_c(B) < 100$ K

(3) $B = 10$ T, $75\text{K} < T_c(B) < 100$ K

(4) $B = 10$ T, $T_c = 70$K

Matrix Match Type

Answer Q.1, Q.2 and Q.3 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin ($x = 0$, $y = 0$, $z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in columns I, II and III, respectively. The quantities E_0 and B_0 are positive in magnitude. **(JEE Advanced 2017)**

	Column I		Column II		Column III
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i)	$\vec{E} = E_0\hat{z}$	(P)	$\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii)	$\vec{E} = -E_0\hat{y}$	(Q)	$\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii)	$\vec{E} = -E_0\hat{x}$	(R)	$\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv)	$\vec{E} = E_0\hat{x}$	(S)	$\vec{B} = B_0\hat{z}$

1. In which case will the particle move in a straight line with constant velocity?

(1) (IV) (i) (S) (2) (III) (ii) (R)
(3) (III) (iii) (P) (4) (II) (iii) (S)

2. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$)?

(1) (III) (ii) (P) (2) (III) (ii) (R)
(3) (IV) (ii) (S) (4) (II) (iii) (Q)

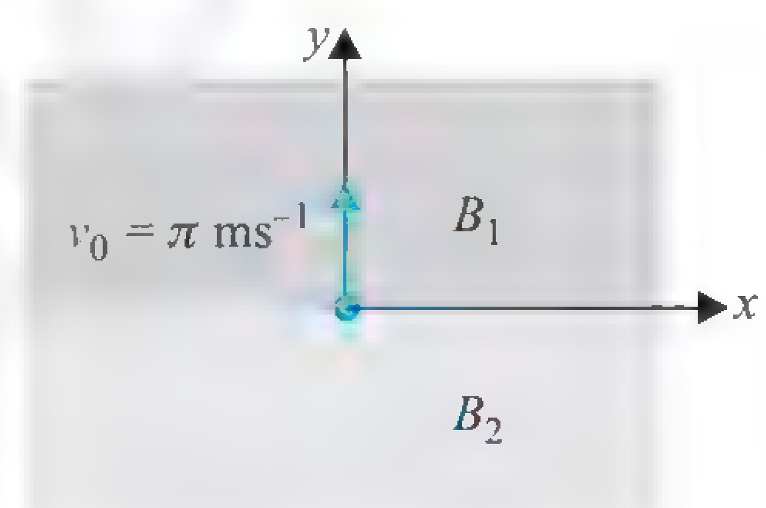
3. In which case will the particle describe a helical path with axis along the positive z-direction?

(1) (III) (iii) (P) (2) (II) (ii) (R)
(3) (IV) (ii) (R) (4) (IV) (i) (S)

Numerical Value Type

1. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is ________. **(JEE Advanced 2014)**

2. In the x-y plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi\text{ms}^{-1}$ at $t = 0$, as shown in the figure.



Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is ________.

(JEE Advanced 2018)

3. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} Nm rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0–1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is ________.

(JEE Advanced 2018)

4. An α-particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α-particle and the sulfur ion move in circular orbits of radii r_α and r_s, respectively. The ratio (r_s/r_α) is ________.

(JEE Advanced 2021)

Answers Key

EXERCISES

Single Correct Answer Type

1. (1)	**2.** (1)	**3.** (3)	**4.** (2)	**5.** (3)
6. (2)	**7.** (4)	**8.** (1)	**9.** (1)	**10.** (1)
11. (2)	**12.** (3)	**13.** (3)	**14.** (2)	**15.** (2)
16. (1)	**17.** (4)	**18.** (3)	**19.** (2)	**20.** (2)
21. (2)	**22.** (1)	**23.** (4)	**24.** (2)	**25.** (2)
26. (2)	**27.** (2)	**28.** (2)	**29.** (4)	**30.** (1)
31. (4)	**32.** (2)	**33.** (4)	**34.** (2)	**35.** (2)
36. (3)	**37.** (1)	**38.** (4)	**39.** (3)	**40.** (1)
41. (3)	**42.** (4)	**43.** (3)	**44.** (1)	**45.** (2)
46. (1)	**47.** (3)	**48.** (1)	**49.** (3)	**50.** (2)
51. (3)	**52.** (1)	**53.** (1)	**54.** (3)	**55.** (1)
56. (1)	**57.** (1)	**58.** (3)	**59.** (1)	**60.** (1)
61. (3)	**62.** (4)	**63.** (2)	**64.** (3)	**65.** (2)
66. (2)	**67.** (1)	**68.** (1)	**69.** (3)	**70.** (2)
71. (4)	**72.** (1)	**73.** (2)	**74.** (2)	**75.** (4)
76. (3)	**77.** (4)	**78.** (3)	**79.** (1)	**80.** (1)
81. (1)	**82.** (2)	**83.** (2)	**84.** (3)	**85.** (2)
86. (3)	**87.** (2)	**88.** (1)		

Multiple Correct Answers Type

1. (1),(2)	**2.** (1),(2)	**3.** (1),(2)
4. (1),(2),(3),(4)	**5.** (1),(2)	**6.** (1),(4)
7. (1),(2),(3)	**8.** (2),(4)	**9.** (1),(2),(3)
10. (1),(2),(3)	**11.** (2),(3),(4)	**12.** (1),(2)
13. (1),(3)	**14.** (1),(2),(3)	**15.** (1),(4)
16. (1),(3)	**17.** (1),(2),(4)	**18.** (2),(3)
19. (1),(2),(3),(4)	**20.** (2),(4)	**21.** (1),(2),(3)
22. (1),(2)	**23.** (3),(4)	**24.** (1),(2),(3),(4)
25. (2),(4)		

Linked Comprehension Type

1. (2)	**2.** (1)	**3.** (3)	**4.** (3)	**5.** (1)
6. (1)	**7.** (4)	**8.** (2)	**9.** (3)	**10.** (1)
11. (1)	**12.** (1)	**13.** (4)	**14.** (1)	**15.** (1)
16. (1)	**17.** (2)	**18.** (3)	**19.** (2)	**20.** (4)
21. (1),(2)	**22.** (2)	**23.** (1)	**24.** (2)	**25.** (3)
26. (1)	**27.** (2)	**28.** (2)	**29.** (1)	**30.** (2)
31. (4)	**32.** (4)	**33.** (3)	**34.** (1)	**35.** (1)
36. (2)	**37.** (3)	**38.** (2)	**39.** (4)	**40.** (1)
41. (2)	**42.** (2)	**43.** (1)		

Matrix Match Type

1. i. → a., c., d.; ii. → a., c., d.; iii. → a., b.; iv. → a., b.
2. i.→ a., b., d.; ii. → a., b., c., d.; iii. → a., b., d.; iv. → b.
3. i. → d.; ii. → a.; iii. → b.; iv. → c.
4. i. → a., c.; ii. → a., b.; iii → d.; iv. → a.
5. i. → b.; ii. → a., c.; iii. → a., c.; iv. → a., d.
6. i. → c., d.; ii. → c., d.; iii. → b., c.; iv. → a., c.
7. i. → a., b., c., d.; ii. → a., b., d.; iii. → d.; iv. → c.
8. i. → b., d.; ii. → a., b., c.; iii. → b., d.; iv. → b., c.
9. i. → c., d.; ii. → c., d.; iii. → a.; iv. → b.

Numerical Value Type

1. (3)	**2.** (1)	**3.** (2)	**4.** (2)	**5.** (5)
6. (4)	**7.** (2)	**8.** (4)	**9.** (5)	**10.** (2)
11. (60)	**12.** (0.01)	**13.** (1.6)	**14.** (2)	**15.** (3.46)

ARCHIVES

JEE Advanced

Single Correct Answer Type

1. (3) **2.** (2)

Multiple Correct Answers Type

1. (2),(4)	**2.** (3),(4)	**3.** (1),(3)
4. (1),(2),(3)	**5.** (1),(2)	**6.** (1),(3)

Linked Comprehension Type

1. (1) **2.** (2)

Matrix Match Type

1. (4) **2.** (2) **3.** (4)

Numerical Value Type

1. (3) **2.** (2) **3.** (5.55) **4.** (4)

Solutions

Chapter 1

Concept Application Exercises

Exercise 1.1

1. $\vec{F} = q[\vec{v}\times\vec{B}] = 2\times10^{-6}[(2\hat{i}-3\hat{j}+4\hat{k})\times(3\hat{j}-2\hat{k})]$

$= 2\times10^{-6}[-6\hat{i}+4\hat{j}+6\hat{k}]\text{ N}$

By Newton's law of motion,

$$\vec{a} = \frac{\vec{F}}{m} = \frac{2\times10^{-6}}{5\times10^{-6}}(-6\hat{i}+4\hat{j}+6\hat{k})$$

$$= 0.8\,(-3\hat{i}+2\hat{j}+3\hat{k})\text{ m s}^{-2}$$

2. $\vec{F}\perp\vec{B}\ \therefore\ \vec{a}\perp\vec{B}\ \therefore\ \vec{a}\cdot\vec{B}=0$

$\therefore\quad (2\hat{i}+x\hat{j})\cdot(-3\hat{i}+2\hat{j}-4\hat{k})=0\quad\Rightarrow -6+2x=0\quad\Rightarrow x=3.$

3. (a) Let velocity of the particle is $\vec{v}=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$

then $\quad\vec{F}=q\vec{v}\times\vec{B}$

$\Rightarrow\quad -3.36\times10^{-7}\,\hat{i}+7.42\times10^{-7}\,\hat{j}$

$= -5.6\times10^{-9}(v_x\hat{i}+v_y\hat{j}+v_z\hat{k})\times(-1.25)\,\hat{k}$

$\Rightarrow\quad -3.36\,\hat{i}+7.42\,\hat{j} = 7\times10^{-2}(-v_x\,\hat{j}+v_y\,\hat{i})$

$$\Rightarrow\quad v_x = -\frac{7.42\times10^{2}}{7} = -106\text{ m s}^{-1}$$

$$\text{and}\quad v_y = \frac{-3.36\times10^{2}}{7} = -48\text{ m s}^{-1}$$

(b) Because $\vec{F}\perp\vec{v}$, so $\vec{v}\cdot\vec{F}=0$. Let us check it

$\vec{v}\cdot\vec{F} = (-106\,\hat{i}-48\,\hat{j}+v_z\,\hat{k})\cdot(-3.36\times10^{-7}\,\hat{i}+7.42\times10^{-7}\,\hat{j})$

$= 106\times3.36\times10^{-7}-48\times7.42\times10^{-7}=0$

4. (a) Let $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, then $\vec{F}=q\vec{v}\times\vec{B}$

$\Rightarrow\quad 8.40\times10^{-2}\,\hat{i}-5.60\times10^{-2}\,\hat{k}$

$=7\times10^{-6}(-4\times10^{3})\,\hat{j}\times(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k})$

$\Rightarrow\quad 84\,\hat{i}-56\,\hat{k} = 28(-B_x\,\hat{k}+B_z\,\hat{i})$

$$\Rightarrow\quad B_x=\frac{-56}{28}=-2\text{ T},\ B_z=-\frac{84}{28}=-3\text{ T}$$

(b) Because $\vec{F}\perp\vec{B}$, so $\vec{B}\cdot\vec{F}=0$.

Let us check it

$\vec{B}\cdot\vec{F}=(-2\,\hat{i}+B_y\,\hat{j}-3\,\hat{k}).(8.40\times10^{-2}\,\hat{i}-5.70\times10^{-2}\,\hat{k})=0$

5. In first case, force is along $-\hat{k}$, so $\vec{B}$ should be in xy plane or z component of B should be zero. In second case, force is along x axis, so B cannot be along x-axis. Finally we find that B is along negative y direction.

Let $\quad\vec{B}=-B_0\hat{j}$ and applying $\vec{F}=q\vec{v}\times\vec{B}$

$-(1.28\times10^{-13}\hat{k})=(1.6\times10^{-19})\times[(2\hat{i}+3\hat{j})\times(-B_0\hat{j})]\times10^{6}$

$1.28=1.6\times2\times B_0$ or $B_0=0.4\text{ T}$

$\vec{B}=-(0.4\,\hat{j})\text{ T}$

6. (a) $\vec{F} = q\vec{v}\times\vec{B}=-qv[B_x(\hat{j}\times\hat{i})+B_z(\hat{j}\times\hat{k})]$

$= qvB_x\hat{k}-qvB_z\hat{i}$

(b) $B_x>0$, $B_z<0$, sign of B_y does not matter.

(c) $\vec{F} = |q|VB_z\hat{i}-|q|VB_x\hat{k}$

7. (a) $$\vec{F}=q\vec{v}\times\vec{B}=q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & v_y & v_z\\ B_x & B_y & B_z\end{vmatrix}=q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & v\\ B_x & B_y & B_z\end{vmatrix}$$

$= -qvB_y\hat{i}+qvB_x\hat{j}$

But $\quad\vec{F}=3F_0\hat{i}+4F_0\hat{j},$

So $\quad 3F_0=-qvB_y$ and $4F_0=qvB_x$

$$B_y=-\frac{3F_0}{qv},\ B_x=\frac{4F_0}{qv},\ B_z\text{ is arbitraty.}$$

(b) $$B=\frac{6F_0}{qv}=\sqrt{B_x^2+B_y^2+B_z^2}\quad\Rightarrow\quad B_z=\pm\frac{11F_0}{qv}$$

8. The force associated with the magnetic field must point in the $\hat{j}$ direction in order to cancel the force of gravity in the $-\hat{j}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{k}$ direction (since $\hat{i}\times(-\hat{k})=\hat{j}$). Note that the charge is positive; also note that we need to assume $B_y=0$. The magnitude $|B_z|$ is given by equation (with $\phi=90°$). Therefore, with $m=1.0\times10^{-2}$ kg, $v=2.0\times10^{4}$ m/s, and $q=5.0\times10^{-5}$ C, we find

$$\vec{B}=B_z\hat{k}=-\left(\frac{mg}{qv}\right)\hat{k}=-\left(\frac{1.0\times10^{-2}\times10}{5.0\times10^{-5}\times2.0\times10^{4}}\right)\hat{k}=(-0.10\text{ T})\hat{k}$$

9. The force on the electron is

$\vec{F_B}=q\vec{v}\times\vec{B}=q(v_x\hat{i}+v_y\hat{j})\times(B_x\hat{i}+B_y\hat{j})=q(v_xB_y-v_yB_x)\hat{k}$

$\Rightarrow\quad\vec{F}=q(v_x(3B_x)-v_yB_x)\hat{k}$

where we use the fact that $B_y=3B_x$. Since the force (at the instant considered) is $F_z\hat{k}$ where $F_z=6.4\times10^{-19}$ N, then we are led to the condition

$$q(3v_x-v_y)B_x=F_z\Rightarrow B_x=\frac{F_z}{q(3v_x-v_y)}$$

Substituting $v_x=2.0$ m/s, $v_y=4.0$ m/s, and $q=-1.6\times10^{-19}$ C, we obtain

$$B_x=\frac{F_z}{q(3v_x-v_y)}=\frac{6.4\times10^{-19}}{(-1.6\times10^{-19})[3(2.0)-4.0]}=-2.0\text{ T}$$

10. The magnetic force has a magnitude of $F=|q|\,vB\sin\theta$. The field B and the directional angle θ are the same for each particle. Particle 1, however, travels faster than particle 2. By itself, a faster speed v would lead to a greater force magnitude F. But the force on each particle is the same. Therefore, particle 1 must have a smaller charge to counteract the effect of its greater speed.

We have $\underbrace{F=|q_1|v_1B\sin\theta}_{\text{Particle 1}}$ and $\underbrace{F=|q_2|v_2B\sin\theta}_{\text{Particle 2}}$

Dividing the equation for particle 1 by the equation for particle 2 and remembering that $v_1=3v_2$ gives

$$\frac{F}{F}=\frac{|q_1|v_1B\sin\theta}{|q_2|v_2B\sin\theta}\quad\text{or}\quad 1=\frac{|q_1|v_1}{|q_2|v_2}\quad\text{or}\quad\frac{|q_1|}{|q_2|}=\frac{v_2}{v_1}=\frac{v_2}{3v_2}=\frac{1}{3}$$

11. The positive plate has a charge q and is moving downward with a speed v at right angles to a magnetic field of magnitude B. The magnitude F of the magnetic force exerted on the positive plate is $F = |q|vB \sin 90°$. The charge on the positive plate is related to the magnitude E of the electric field that exists between the plates by $|q| = \varepsilon_0 AE$, where A is the area of the positive plate. Substituting this expression for $|q|$ into $F = |q|\, vB \sin 90°$ gives the answer in terms of known quantities.

$$F = (\varepsilon_0 AE)vB$$

$$= [8.85\times10^{12}](5.0\times10^{-4})(200)\times(20)(5.0) = 8.85\times10^{13}\,\text{N}$$

An application of right-hand rule shows that the magnetic force is perpendicular to the plane of the page and directed out of the page, toward the reader.

12. (a) According to right hand rule, if you extend your right hand so that your fingers point along the direction of the magnetic field B and your thumb points in the direction of the velocity v of a positive charge, your palm will face in the direction of the force F on the positive charge.

For the electron in question, the fingers of the right hand should be oriented downward (direction of B) with the thumb pointing to the east (direction of v). The palm of the right-hand points due north (the direction of F on a positive charge). Since the electron is negatively charged, it will be deflected due south.

(b) The acceleration of an electron is given by Newton's second law, where the net force is the magnetic force. Thus,

$$a = \frac{F}{m} = \frac{|q_0|vB\sin\theta}{m}$$

Since the kinetic energy is $\text{KE} = \frac{1}{2}mv^2$, the speed of the electron is $v = \sqrt{2(\text{KE})/m}$. Thus, the acceleration of the electron is

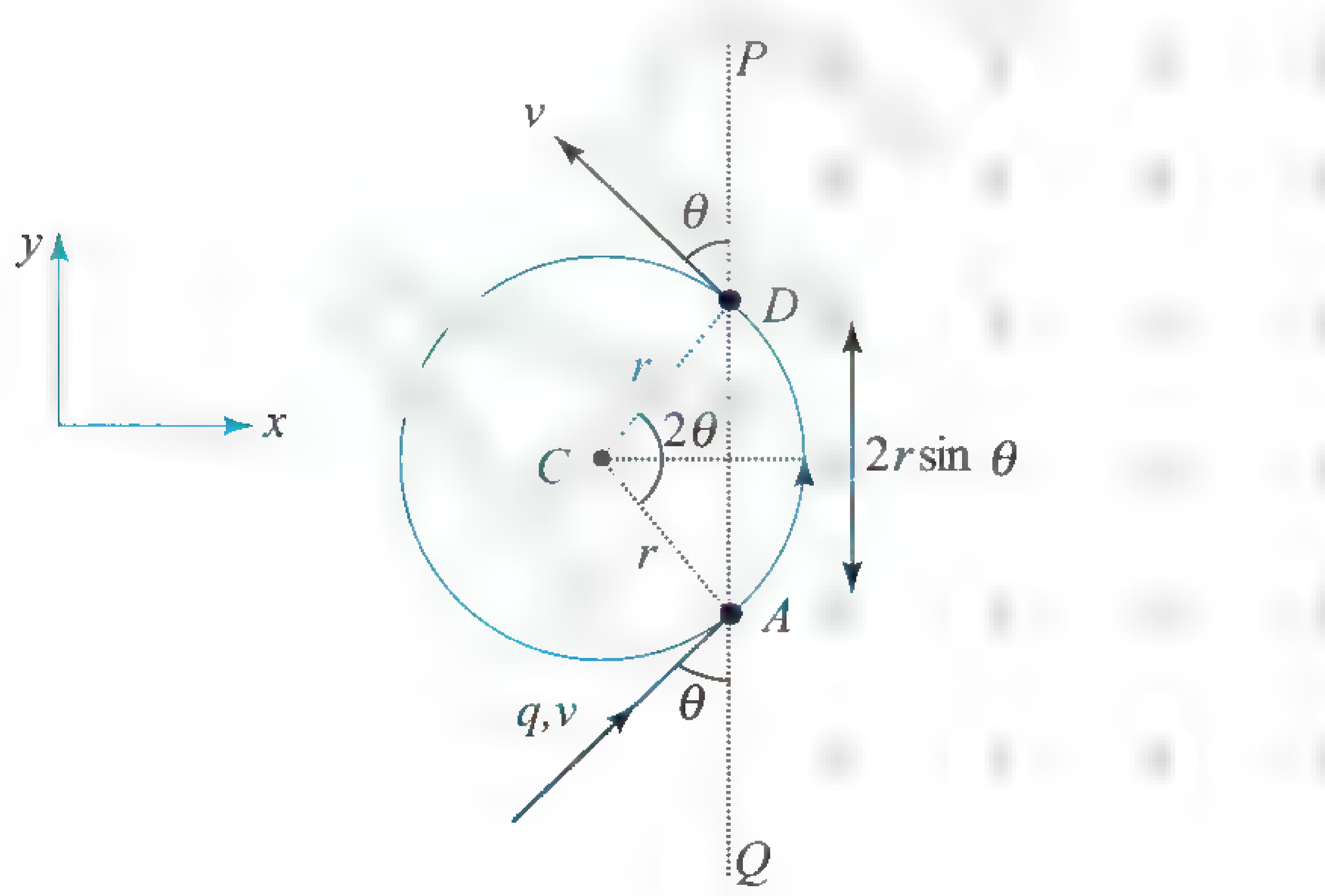
$$a = \frac{|q_0|vB\sin\theta}{m} = \frac{|q_0|\sqrt{\frac{2(\text{KE})}{m}}B\sin\theta}{m}$$

$$= \frac{(1.60\times10^{-19}\ \text{C})\sqrt{\frac{2(4.5\times10^{-15}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}}\times(2.25\times10^{-5}\ \text{T})\sin 90°}{9.0\times10^{-31}\ \text{kg}}$$

$= 4.0 \times 10^{14}$ m/s^2

Exercise 1.2

1. The particle will move in the field as shown in figure.



Angle subtended by the arc at the centre $= 2\theta$

(a) Time spent by the charge in magnetic field

$$\omega t = 2\theta \;\Rightarrow\; \frac{qB}{m}t = 2\theta \;\Rightarrow\; t = \frac{m2\theta}{qB}$$

(b) Distance travelled by the charge in magnetic field

$$= r(2\theta) = \frac{mv}{qB}\times 2\theta$$

(c) Impulse = change in momentum of the charge

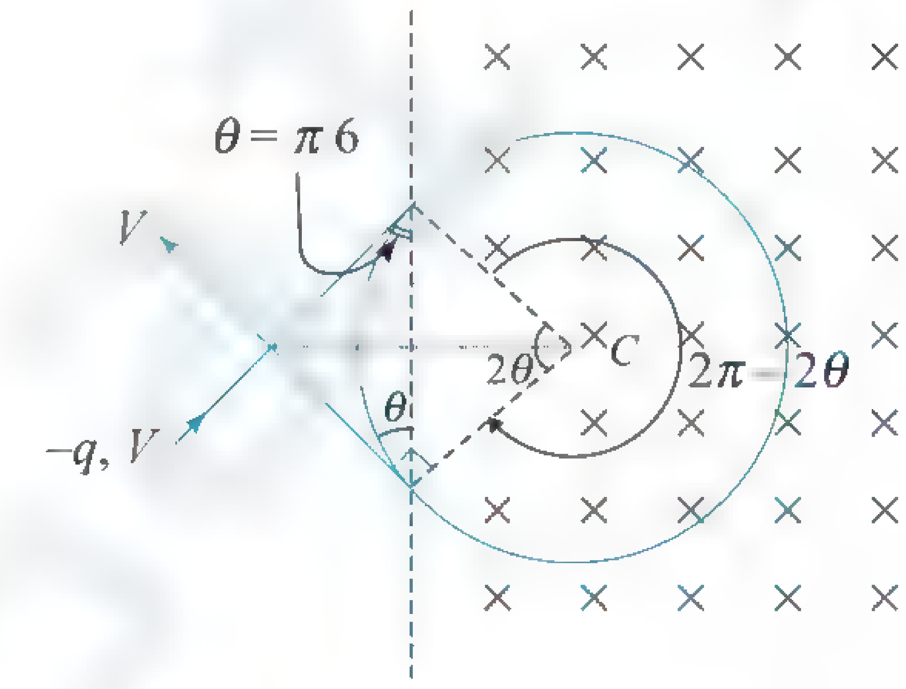
$$= (-mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) - (mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j})$$

$$= -2mv\sin\theta\,\hat{i}$$

2. (a) $2\pi - 2\theta = 2\pi - 2\times\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = \omega t = \frac{qB}{m}t$

$$\Rightarrow\quad t = \frac{5\pi m}{3qB}$$

(b) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

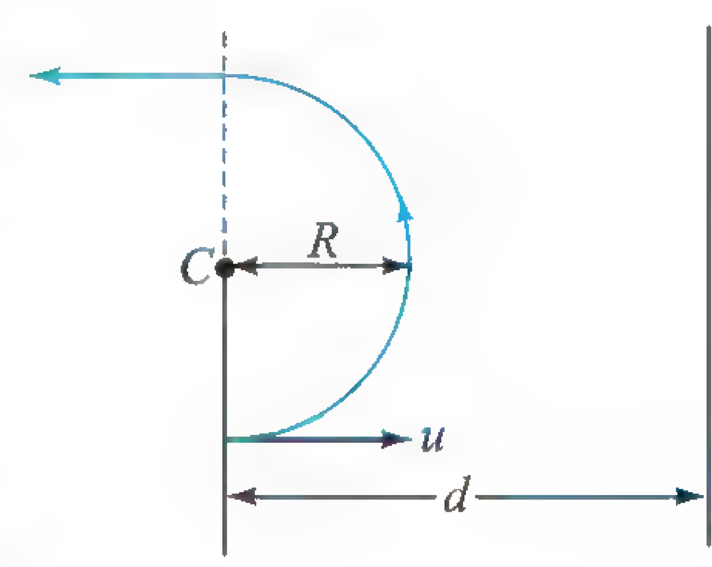


(c) Impulse = charge in linear momentum

$$= m(-v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j}) - m(v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j})$$

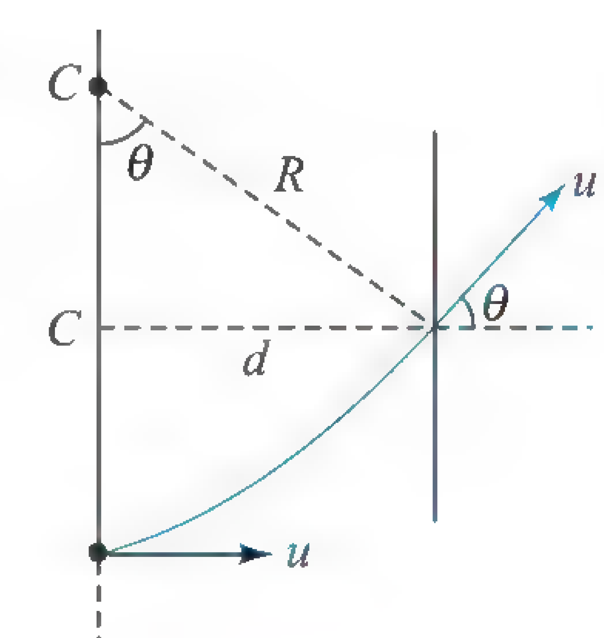
$$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\,\hat{i} = -mv\,\hat{i}$$

3. (a) $d > \frac{mu}{qB}$ means $d > R$, the particle will complete semicircle in the field.



$\therefore$ time spent $t = \frac{T}{2} = \frac{\pi m}{qB}$

(b) $d < \frac{mu}{qB} \Rightarrow q < R$, the particle will not be able to complete semicircle and it will come out from other side of the field.



$$\sin\theta = \frac{d}{R}$$

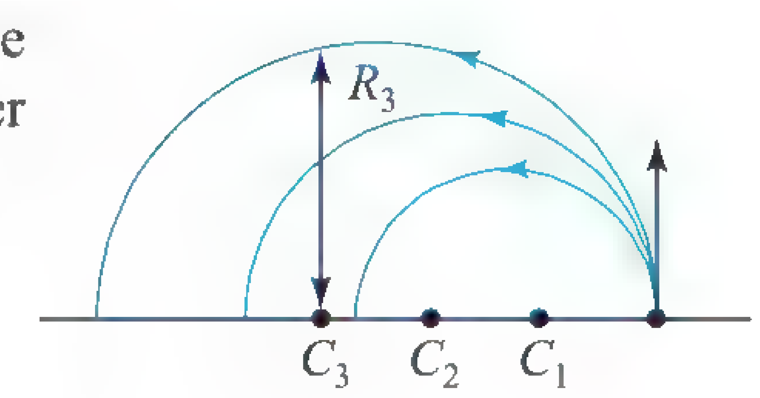
$$\Rightarrow\quad \theta = \sin^{-1}\left(\frac{d}{R}\right)$$

time spent $\omega t = \theta$

$$\Rightarrow\quad t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$

4. The path of the particle will be circular. Larger the velocity, larger will be the radius.

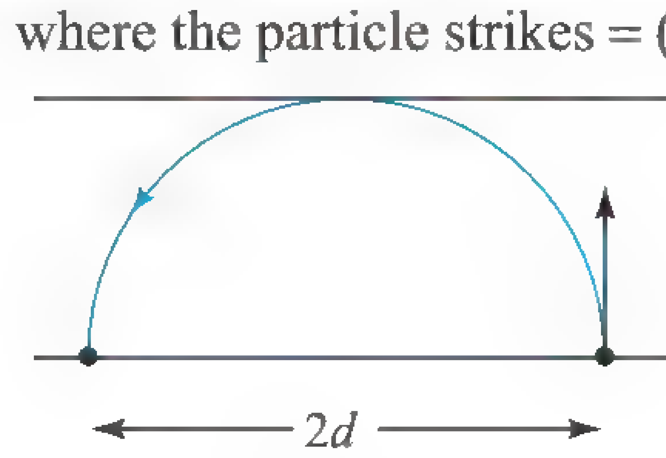
For particle not to strike, $R < d$



$$\therefore\quad \frac{mv}{qB} < d \;\Rightarrow\; v < \frac{qBd}{m}$$

For limiting case, $v = \frac{qBd}{m}$, $R = d$

$\therefore$ Coordinate where the particle strikes $= (-2d, 0, 0)$



5. (a) $p = mv = m\left(\dfrac{RqB}{m}\right) = RqB$

$= (5.0 \times 10^{-3}\text{ m})\,(6.0 \times 10^{-19}\text{ C})\,(2.0\text{ T})$

$= 6.0 \times 10^{-21}\text{ kg m s}^{-1}$

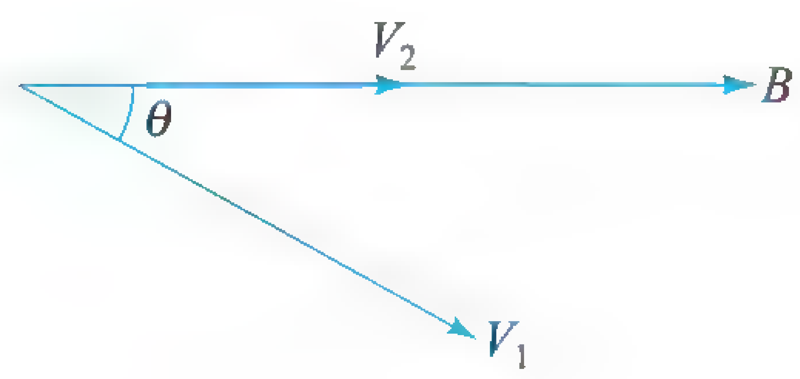
(b) $L = Rp = (5.0 \times 10^{-3}\text{ m}) \times 6.0 \times 10^{-21}$

$= 3.0 \times 10^{-23}\text{ kg m}^2\text{ s}^{-1}$

6. $qV = \dfrac{1}{2}mv_1^2 \quad \Rightarrow \quad v_1 = \sqrt{\dfrac{2qV}{m}}$

To meet them again and again:

$$v_2 = v_1\cos\theta = \sqrt{\frac{2qV}{m}}\cos\theta$$



Time interval: $T = \dfrac{2\pi m}{qB}$

Distance after which they meet = 1 pitch = v_2T

$$= \sqrt{\frac{2qV}{m}}\cos\theta \times \frac{2\pi m}{qB} = \pi\sqrt{\frac{8Vm}{qB^2}}\cos\theta$$

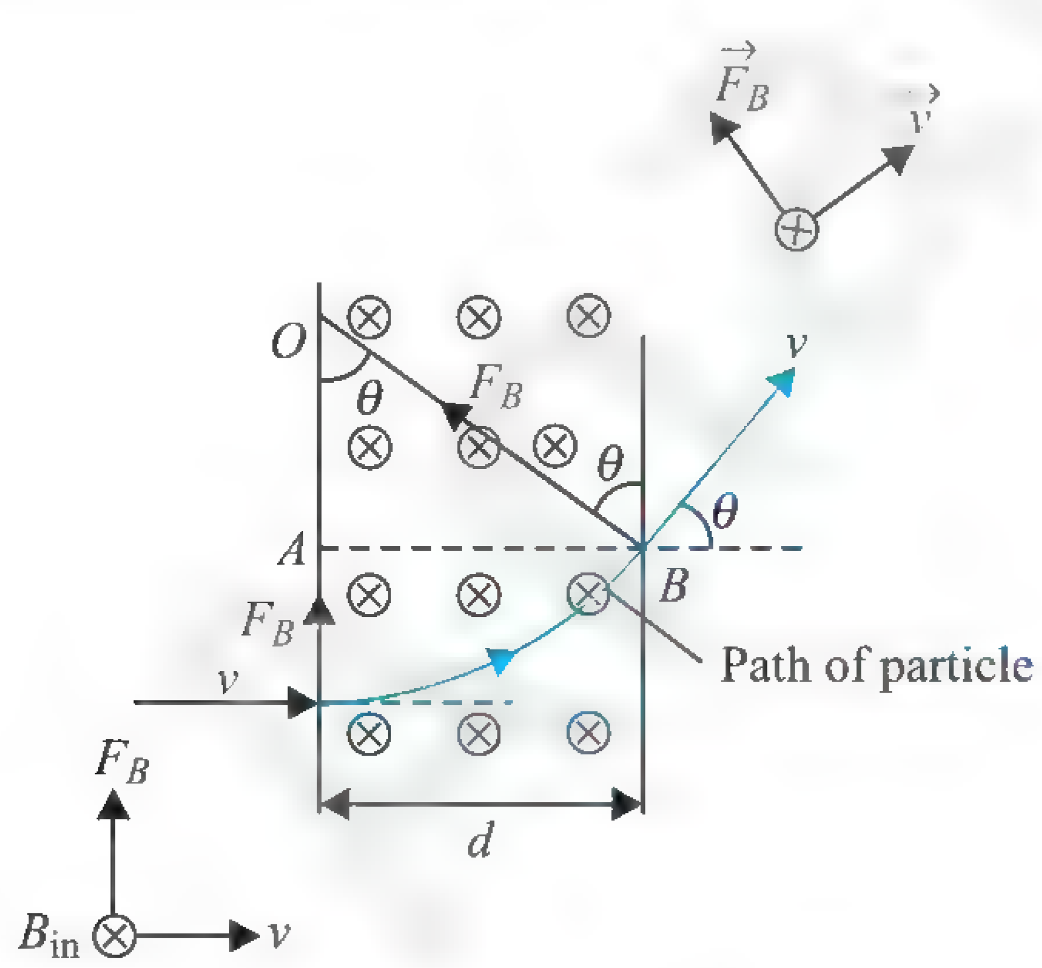
7. $qV = \dfrac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\dfrac{2qV}{m}}$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2mV}{qB^2}} \quad \Rightarrow \quad m \propto r^2$$

$$\frac{m_{\text{heavy}}}{m_{\text{light}}} = \left(\frac{l+d}{l}\right)^2$$

8. The radius of circulation is $R = \dfrac{mv}{Bq} = \dfrac{p}{Bq} = \dfrac{\sqrt{2mK}}{Bq}$.

As given, angle of deviation is θ; it is the angle between directions of final velocity and initial velocity.



In ΔOAB, $\sin\theta = \dfrac{d}{R} = \dfrac{dBq}{mv} = \dfrac{dBq}{p} = \dfrac{dBq}{\sqrt{2mK}}$

(a) $p_\alpha = p_p$, $\dfrac{\sin\theta_p}{\sin\theta_\alpha} = \dfrac{q_p}{q_\alpha} = \dfrac{1}{2}$

(b) $K_\alpha = K_p$; hence $\dfrac{\sin\theta_p}{\sin\theta_\alpha} = \dfrac{q_p}{q_\alpha} \times \sqrt{\dfrac{m_\alpha}{m_p}}$

$$= \frac{1}{2} \times \sqrt{\frac{4}{1}} = 1$$

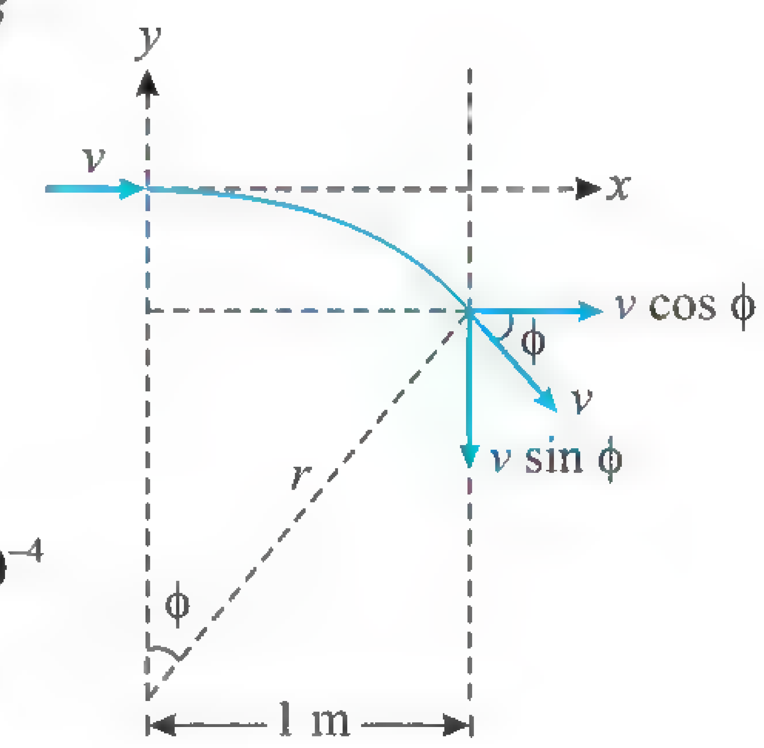
(c) When a charged particle is accelerated through a potential difference V, gain of kinetic energy is qV.

$K_p = q_\alpha V$, $K_\alpha = q_\alpha V$

Now, $\sin\theta = \dfrac{dBq}{\sqrt{2mqV}} = \dfrac{dB\sqrt{q}}{\sqrt{2mV}}$

Hence, $\dfrac{\sin\theta_p}{\sin\theta_\alpha} = \sqrt{\dfrac{q_p}{q_\alpha}} \times \sqrt{\dfrac{m_\alpha}{m_p}} = \sqrt{\dfrac{1}{2}} \times \sqrt{\dfrac{4}{1}} = \sqrt{2}$

9. (a) Radius of circular path $r = \dfrac{mv}{eB}$



From figure, $\sin\phi = \dfrac{1}{r} = \dfrac{eB}{mv}$

$\Rightarrow \quad mv\sin\phi = eB = p_y$

Hence y-component of linear momentum of proton

$p_y = eB = 1.6 \times 10^{-19} \times 5 \times 10^{-4}$

$= 8.0 \times 10^{-23}$ kg m/s

(b) Kinetic energy of the proton

$K = 50\text{ MeV} = 50 \times 1.6 \times 10^{-19}$ J

Hence $\dfrac{1}{2}mv^2 = 50 \times 1.6 \times 10^{-19}$ J

$$v = \sqrt{\frac{2 \times 50 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27}}} = 10^5\text{ m/s}$$

Here $\sin\phi = \dfrac{eB}{mv}$

$$\sin\phi = \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4}}{1.6 \times 10^{-27} \times 10^5} = \frac{1}{2}$$

$\Rightarrow \quad \phi = 30°$

10. $d = n\,(v\cos\phi)\,T = (v\cos\phi)\dfrac{2\pi mn}{qB}$, where $n = 1, 2, 3, \ldots\ldots$

where $e\Delta V = \dfrac{1}{2}mv^2$

$$v = \sqrt{\frac{2\Delta Ve}{m}}$$

Putting the value, we get $B = \dfrac{n}{d}\dfrac{2\pi m}{q}\left(\sqrt{\dfrac{2\Delta Ve}{m}}\right)\cos\phi$

For min B, $n = 1$

11. (a) The particle will move in circular paths, as velocity vector is perpendicular to magnetic field. Time period of both the particles is same $\left(T = \dfrac{2\pi m}{Bq}\right)$.

So, for collision not to take place,

$r_1 + r_2 < d$

$\dfrac{mv}{Bq} + \dfrac{2mv}{2Bq} < d \quad \text{or} \quad v < \dfrac{Bqd}{2m}$

Therefore, maximum speed should be $\dfrac{Bqd}{2m}$

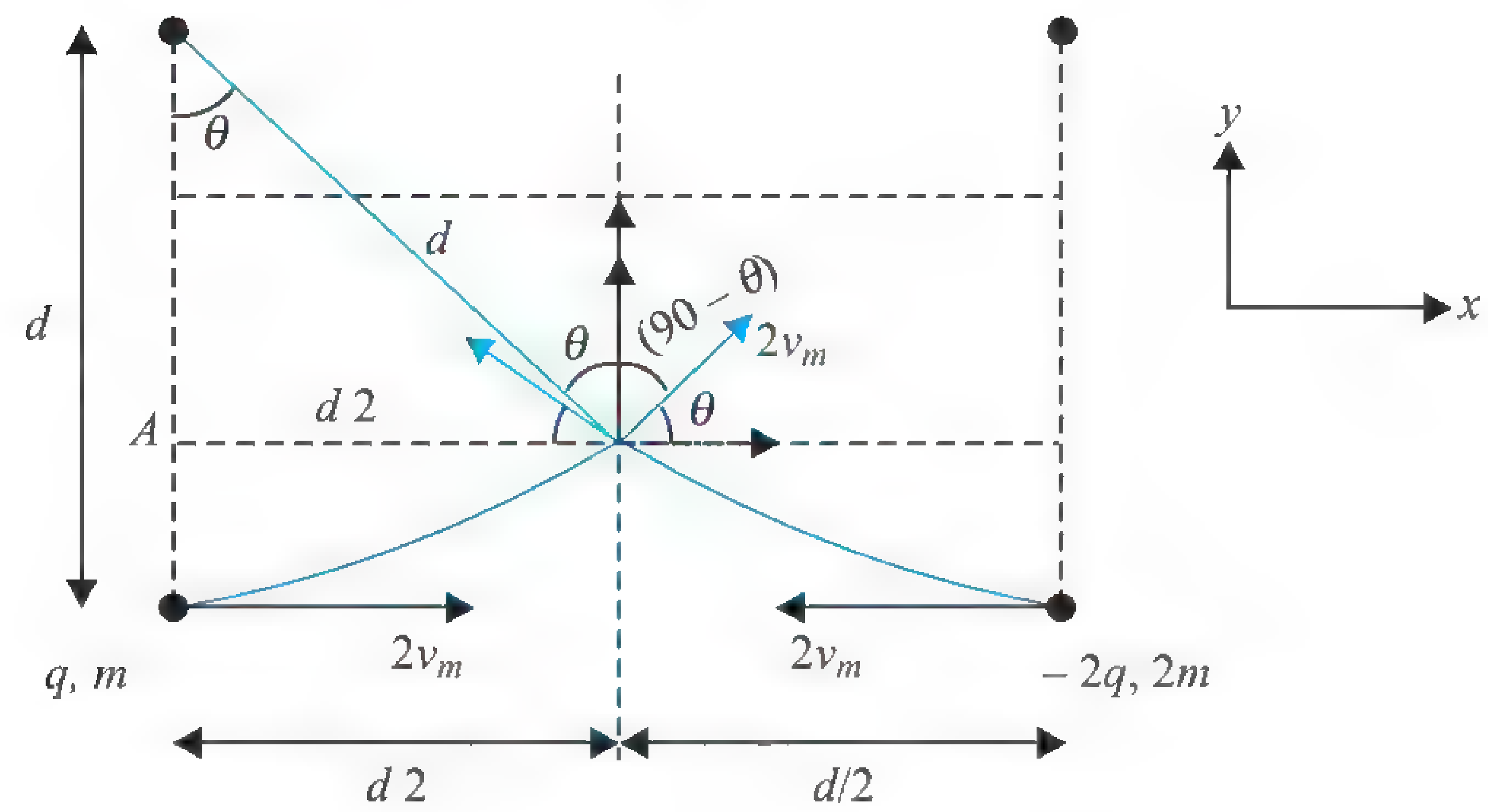
i.e., $v_m < \dfrac{Bqd}{2m}$

(b) From symmetry, it can be concluded that collision occurs at $d/2$ if

$$v = 2v_m = \frac{qBd}{m}$$

$$r = \frac{mv}{qB} = d, \sin\theta = \frac{d/2}{d} = \frac{1}{2}; \theta = \frac{\pi}{6}$$

$$t = T\left(\frac{\theta}{2\pi}\right) = \frac{2\pi m}{qB}\left(\frac{\pi/6}{2\pi}\right) = \frac{\pi m}{6qB}$$



(c) After collision, charge on the combined particle $= -q$,
Mass $= 3m$
The combined particle will have velocity in y direction just after collision. Using conservation of linear momentum

$$(mv\cos\theta\, i + mv\sin\theta\, j) + (-2mv\cos\theta\, i + 2mv\sin\theta\, j) = 3m\vec{v}$$

$$3m\vec{v} = -mv\cos\theta\, i + 3mv\sin\theta\, j$$

$$\vec{v} = \left(-\frac{v}{2\sqrt{3}}\hat{i} + \frac{v}{2}\hat{j}\right)$$

$$|\vec{v}| = v\sqrt{\left(\frac{1}{4\times 3} + \frac{1}{4}\right)} = v\sqrt{\frac{1}{4}\left(\frac{4}{3}\right)} = \frac{v}{\sqrt{3}}$$

$$|v| = \frac{1}{\sqrt{3}} \times 2 \times \frac{qBd}{2m} = \frac{qBd}{\sqrt{3}m}$$

$$\therefore \quad r = 3m \times \frac{qBd}{\sqrt{3}m \times qB} = \sqrt{3}\,d$$

12. Just before striking the plate, the electric force on the charged particle is $F_E = qE = \dfrac{qV}{d}$, in the upward direction. Since the kinetic energy of the electron is $K = \dfrac{1}{2}mv^2 = qV$, $v = \sqrt{\dfrac{2qV}{m}}$. On the other hand, the magnetic force is

$$F_B = qvB = qB\sqrt{\frac{2qV}{m}}$$

in the downward direction. To prevent the electron from striking the plate, we require $F_B = F_E$, or

$$qB\sqrt{\frac{2qV}{m}} > \frac{qV}{d} \Rightarrow B > \frac{V}{d}\sqrt{\frac{m}{2qV}} = \sqrt{\frac{mV}{2qd^2}}$$

Exercise 1.3

1. Magnetic force on wire $abcdef$ in uniform magnetic field is $\vec{F}_m = I(\vec{L}\times\vec{B})$, $\vec{L}$ is displacement between free ends of the conductor from initial to final point. $\vec{L} = (l)\hat{i}$ and

$$\vec{B} = (B)\hat{j};\ F_m = I(\vec{L}\times\vec{B}) = BIL(\hat{i}\times\hat{j}) = BIl(\hat{k})$$

2. Net force on the loop $= 3(\vec{F}_{AD})$
Force on wire ACD = Force on AD = Force on AED

$$\Rightarrow \quad F_{\text{net}} = 3(i)(AD)(B) = (3)(3.0)(2\sqrt{2})(2.0)\,\text{N}$$

$$= 36\sqrt{2}\ \text{N}.$$ Direction of this force is towards EC.

3. The net force from A to B is $d\vec{F} = I(d\vec{L}\times\vec{B})$

$$\int_A^B d\vec{F} = \int_A^P I[d\vec{L}_1\times\vec{B}] + \int_P^Q I[d\vec{L}_2\times\vec{B}] + \int_Q^R I[d\vec{L}_3\times\vec{B}]$$

$$+\int_R^T I[d\vec{L}_4\times\vec{B}] + \int_T^B I[d\vec{L}_5\times\vec{B}]$$

The entire path can be broken down into elemental vectors joined to each other in sequence. We know, from polygon law of addition of vectors, that vector joining the tail of the first vector to the head of the last vector is the resultant.

$$\vec{F} = I(\vec{L}\times\vec{B}),$$

where $|\vec{L}| = a + \sqrt{c^2 - b^2} + 2r + d$

$$F_{\text{net}} = IB(a + \sqrt{c^2 - b^2} + 2r + d)$$

and its direction is upward on the plane of paper.

4. We have for mid-air suspension,

$$mg = ilB$$

$$\Rightarrow \quad B = \frac{mg}{il} = \frac{0.2\times 10.0}{2\times 1.0} = 1.0\ \text{T}$$

5. $\vec{F} = I\vec{l}\times\vec{B}$

$$F = IlB\sin\theta$$

(a) When the current is flowing from east to west,

$$\theta = 90°$$

Hence $F = IlB = (1\text{A})(1\text{m})(3\times 10^{-5}\ \text{T}) = 3\times 10^{-5}$ N

The direction of the force is downwards. This direction may be obtained by either Fleming's left hand rule or the directional property of cross product of vectors.

(b) When the current is flowing from south to north,

$$\theta = 0° \quad \Rightarrow \quad F = 0$$

Hence, there is no force per unit length on the conductor.

6. In order to use equation $\vec{F} = I\vec{l}\times\vec{B}$, we would need to find the angle between $\vec{l}$ and $\vec{B}$. We avoid this task by using unit vector notation. From the problem figure, we see that

$$\vec{l} = a\hat{i} - a\hat{j} + a\hat{k}$$

Thus, the force is $\vec{F} = I\vec{l}\times\vec{B} = IaB(\hat{i} - \hat{j} + \hat{k})\times(\hat{j})$

$$= IaB(-\hat{i} + \hat{k})$$

The force lies in the xz plane and has a magnitude

$$F = \sqrt{2}\,IaB = 10\ \text{N}$$

7. (a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{l}\times\vec{B} \quad \text{or} \quad F = ilB$$

It acts towards right in the figure provided. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0\, mg$. Thus,

$$\mu_0 mg = ilB \text{ or } \mu_0 = \frac{ilB}{mg}$$

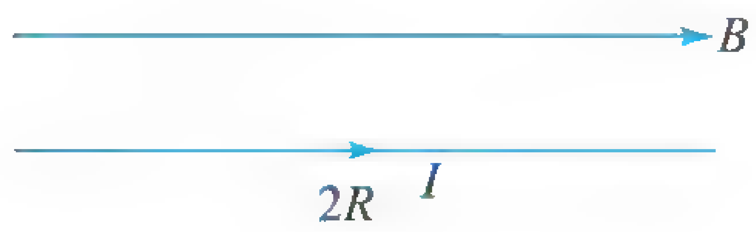
(b) If the friction coefficient is $\mu = \dfrac{\mu_0}{2} = \dfrac{ilB}{2mg}$, the wire will

slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{ilB}{2} \text{ towards left}$$

The resultant force is $ilB - \frac{ilB}{2} = \frac{ilB}{2}$ towards right. The acceleration will be $a = \frac{ilB}{2m}$. The wire will slide towards right with this acceleration.

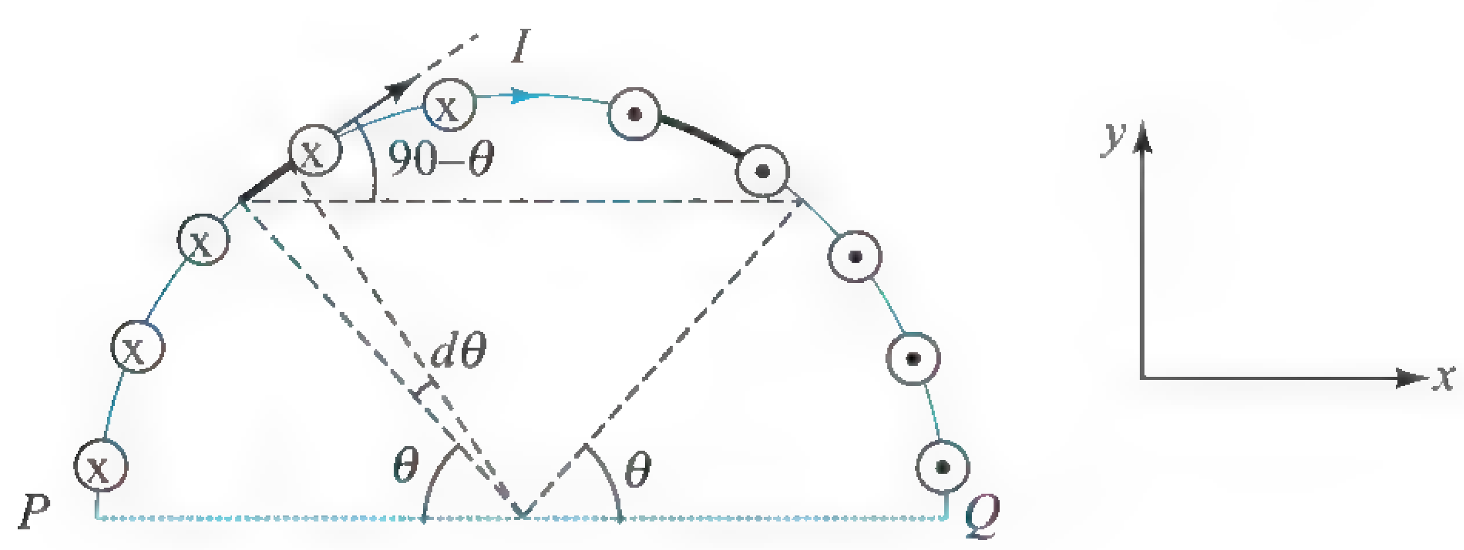
8. The wire is equivalent to a straight wire as shown in the figure shown below.



As $F = ilB \sin\theta$

$\because \quad \theta = 0$, therefore, $F_{res} = 0$

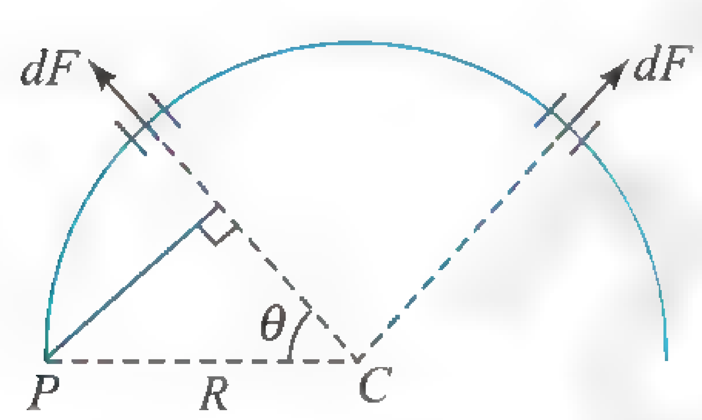
Forces on individual parts are marked in figure by ⊗ and ⊙. By symmetry, there will be pair of forces forming couples.



$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta)2R\cos\theta$$

$$= \frac{i\pi R^2}{2}B \quad \text{or} \quad \vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$

9. $\vec{F}_{net} = I\,2R\,B$. The wire is equivalent to



Force on each element is radially outward, $\tau_C = 0$

Torque about point $P = \tau_p = \int_0^{\pi}[I(Rd\theta)B\sin 90°]R\sin\theta$

$= 2\,IBR^2$

10. Magnetic force is along the positive x-axis. If motion is to occur along the incline, $\sum F = 0$: $IL\,|\,B_y\,|\cos 37° = mg \sin 37°$ from which

$$I = \frac{mg\tan 37°}{L\,|\,B_y\,|} = \frac{(0.050)(10)(0.75)}{(0.50)(0.02)} = 37.5\text{ A}$$

11. Initially, the rod will be in equlibrium if

$2T_0 = mg$ with $T_0 = kx_0$ ...(i)

When current is passed through the rod, it experiences a force $F = Bil$ vertically upward. Now for equilibrium, in this situation

$2T + Bil = mg$ with $T = kx$ ...(ii)

From equations (i) and (ii), we get

$$\frac{T}{T_0} = \frac{mg - Bil}{mg} \quad \text{or} \quad \frac{x}{x_0} = 1 - \frac{Bil}{mg}$$

Solve to get $B = \frac{mg(x_0 - x)}{ilx_0} = \frac{(1\times10^{-3})(10)(3\times10^{-2})}{(20)(25\times10^{-2})(4\times10^{-2})}$

$= 1.5 \times 10^{-3}$ T

12. By impulse of magnetic force, if the wire attains a velocity v in upward direction then we use

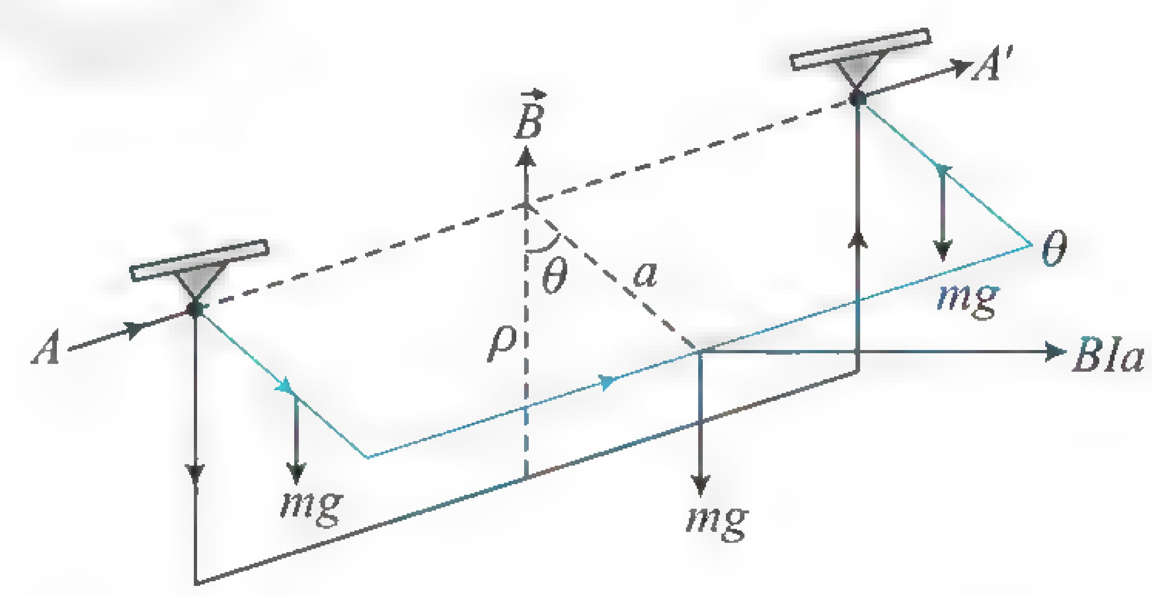
$$BIl\delta t = mv \Rightarrow v = \frac{Bql}{m}$$

Maximum height attained by wire MN is given as

$$h = \frac{v^2}{2g} = \frac{B^2q^2l^2}{2m^2g}$$

13. Let the side of square wire frame is a. The horizontal wire of the frame will experience a rightward force BIl and due to this force it experiences a torque about the axis AA' and the frame will tilt. Torque on frame about AA' is given as

$$\tau = BIa \times a\cos\theta = BIa^2\cos\theta$$



This is the deflecting torque and at equilibrium. This torque is balanced by the torque on the frame due the weight mg of the three sides of the square acting at centre of the wires as shown in figure. so at equilibrium, balancing of torques gives

$$BIa^2\cos\theta = \frac{Mg}{3} \times a\sin\theta + 2\left(\frac{Mg}{3}\right)\times\left(\frac{a}{2}\right)\sin\theta$$

$$\Rightarrow \quad BIa^2\cos\theta = \frac{2}{3}Mga\sin\theta$$

$$\Rightarrow \quad B = \frac{2Mg}{3Ia}\tan\theta$$

Mass of square wire frame can be written in terms of its density, length and cross sectional area as

$M = 3aS\rho$

$$\Rightarrow \quad B = \frac{2(3aS\rho)g}{3Ia}\tan\theta \Rightarrow B = \frac{2aS\rho g}{Ia}\tan\theta$$

Exercise 1.4

1. (a) $\phi = 90°$: $\tau = NIAB \sin(90°) = NIAB$

Direction: $\hat{k} \times \hat{j} = -\hat{i}$, $U = -MB\cos\phi = 0$

(b) $\phi = 0°$: $\tau = NIAB \sin(0) = 0$, no direction,

$M = NIA$, $U = -MB\cos\phi = -NIAB$

(c) $\phi = 90°$: $\tau = NIAB \sin(90°) = NIAB$

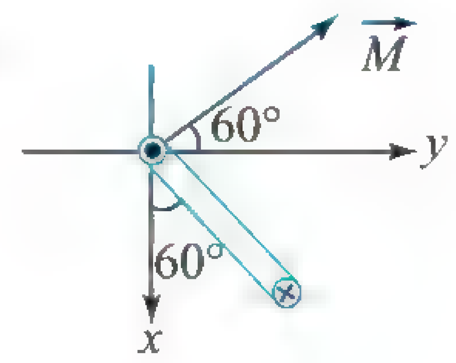
Direction: $-\hat{k} \times \hat{j} = \hat{i}$, $U = -MB\cos\phi = 0$

(d) $\phi = 180°$: $\tau = NIAB \sin(180°) = 0$, no direction,

$U = -MB\cos\phi = NIAB$

2. Magnitude of dipole moment $|\vec{M}| = IA = Il^2$

Direction of magnetic moment is found using right hand rule.



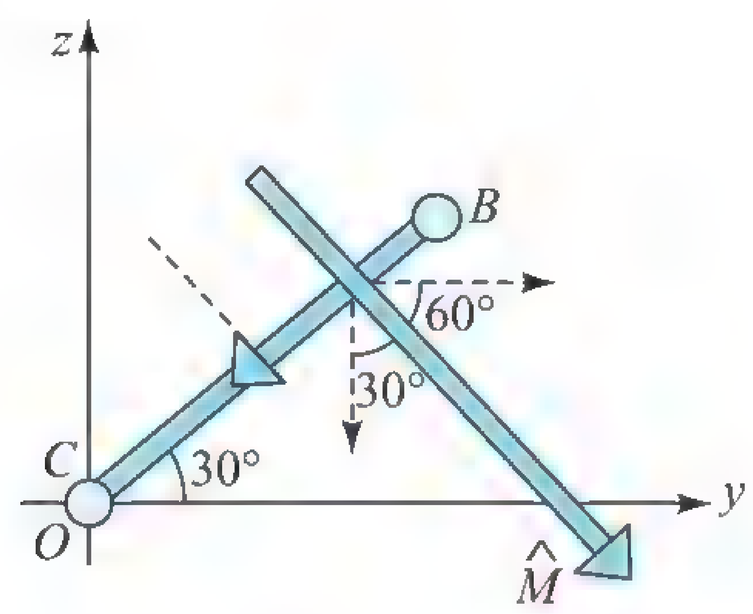
$$\vec{M} = \left(\cos 30°(-\hat{i}) + \sin 30°(\hat{j})\right)M$$

$$= \frac{Il^2}{2}(-\sqrt{3}\,\hat{i} + \hat{j})$$

3. Magnetic moment of the loop

$$|\vec{M}| = iA = 4.0 \times 20 \times 10 \times 10^{-4} = 8 \times 10^{-2}\ \text{A m}^2$$

For direction:



Unit vector in the direction of

$$\widehat{M} = \cos 60°\,\hat{j} - \cos 30\,\hat{k} = \frac{\hat{j}}{2} - \frac{\sqrt{3}}{2}\,\hat{k}$$

$$\therefore \quad \vec{M} = |\vec{M}|\,\widehat{M} = 4 \times 10^{-2}(\hat{j} - \sqrt{3}\,\hat{k})\ \text{Am}^2$$

4. The magnetic dipole moment of the current carrying coil is given by $\vec{M} = NIA\hat{n}$

$$= 100 \times 0.5 \times 0.08 \times 0.04\,\hat{i} = 0.16\,\hat{i}\ \text{A m}^2$$

The torque acting on the coil is

$$\vec{\tau} = \vec{M} \times \vec{B} = M(\hat{i} \times \hat{j})B = 0.16 \times \frac{0.05}{\sqrt{2}}\,\hat{k}$$

$$= 4\sqrt{2} \times 10^{-3}\ (\text{Nm})\,\hat{k}$$

5. The magnetic moment of the loop is in the positive z-direction (right hand thumb rule).

(a) The magnetic moment of the loop is given by

$$\vec{M} = NIA\hat{k} = (10)(5.0)(0.40)^2\hat{k} = 8.0\ \text{Am}^2\,\hat{k}$$

(b) The torque on the current loop is given by

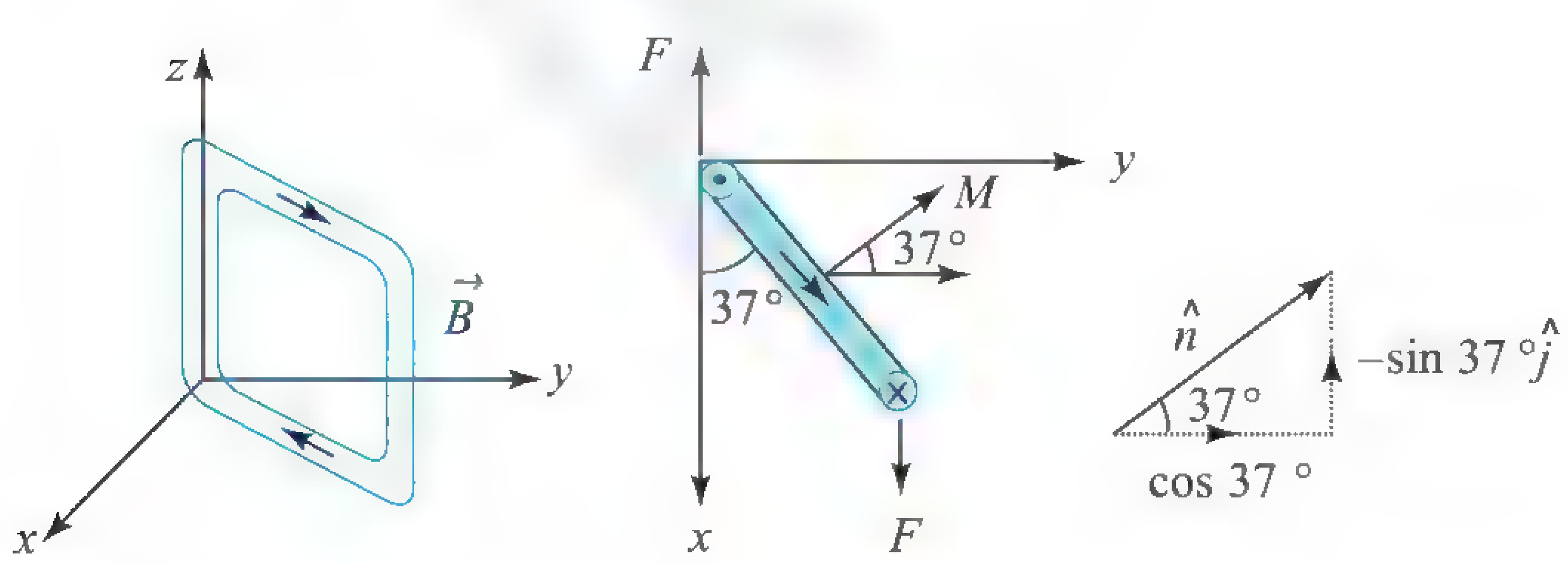
$$\vec{\tau} = \vec{M} \times \vec{B} = (8.0\hat{k}) \times (0.3\hat{i} + 0.4\hat{k}) = 2.4\ \text{Nm}\,\hat{j}$$

(c) The potential energy is the negative dot product of $\vec{M}$ and $\vec{B}$:

$$U = -\vec{M} \cdot \vec{B} = -(5.76\hat{k}) \cdot (0.3\hat{i} + 0.4\hat{k}) = -2.30\ \text{J}$$

6. (a) From Fig. (b), we see that the unit vector normal to loop

$$\hat{n} = -\sin 37°\,\hat{i} + \cos 37°\,\hat{j} = -0.6\hat{i} + 0.8\hat{j}$$



(a) (b) (c)

The magnetic moment is

$$\vec{M} = NIA\hat{n} = (5)(2)(0.2)^2(-0.6\hat{i} + 0.8\hat{j})$$

$$= -0.24\hat{i} + 0.32\hat{j}\ \text{A m}^2$$

(b) The torque, $\vec{\tau} = \vec{M} \times \vec{B} = (-0.24\hat{i} + 0.32\hat{j}) \times (0.5\hat{j})$

$$= -0.12\hat{k}\ \text{N m}$$

(c) The potential energy of the loop is $U = -MB\cos\theta$ where $M = NIA = 0.4\ \text{Am}^2$ and the position of minimum energy is $\theta = 0$.

Thus, the external work, $W_{ext} = +\Delta U$, needed to rotate it to the given orientation, is given by

$$W_{ext} = U_f - U_i = (-MB\cos 37°) - (-MB\cos 0°)$$

$$= (0.4)(0.5)(1 - 0.8) = 0.04\ \text{J}$$

The external work is positive since the dipole moment is rotated away from alignment with the field.

7. The normal to the loop, OP, makes an angle $\theta = 37°$ with the $+x$ direction, the field direction. Hence,

$$\tau = NIAB\sin\theta = (1)(10\ \text{A})(\pi \times 25 \times 10^{-4}\ \text{m}^2)(0.04\ \text{T})\sin 37°$$

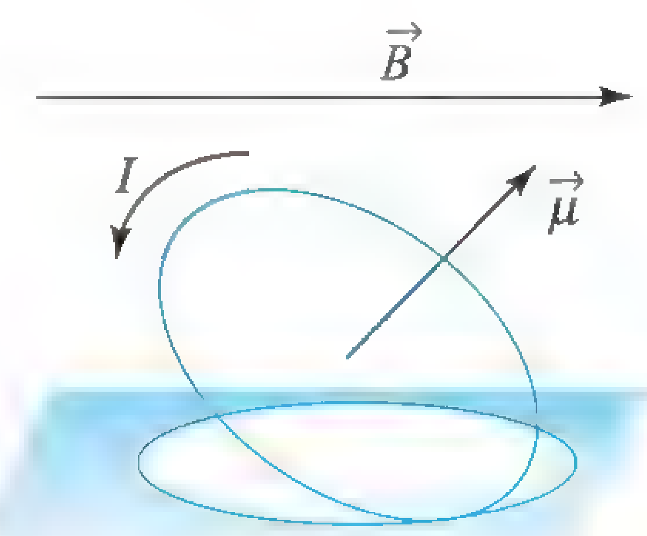
$$= 6\pi \times 10^{-4}\ \text{Nm}$$

The right hand rule shows that the loop will rotate about the y-axis, so as to decrease the angle labelled 37°.

8. $\theta = \dfrac{NBAI}{k}$, $\theta' = \dfrac{10NB(A/16)I}{k}$

$$\Rightarrow \quad \theta' = \frac{5}{8}\theta$$

9. The loop will start to lift off when the magnetic torque equals the gravitational torque as shown in figure



The magnetic torque acting on the loop, $\tau_m = MB = I\pi R^2 B$.

The gravitational torque exerted on the loop, $\tau_g = mgR$.

So $I\pi R^2 B = mgR \quad \Rightarrow \quad I = \dfrac{mg}{\pi RB}$

10. $\tau_{max} = MB = IAB = I\pi r^2 B = 5 \times \pi \times (0.05)^2 \times 3 \times 10^{-3}$

$$= 375\,\pi \times 10^{-7}\ \text{Nm}$$

$$U_{max} = +MB,\ U_{min} = -MB$$

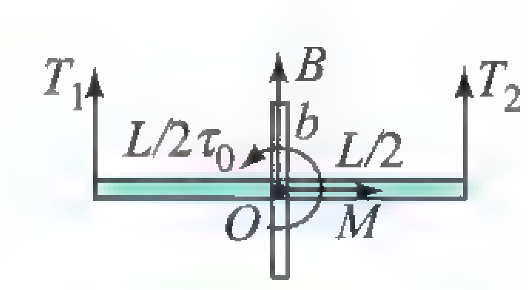
11. Let mass of the hanging system is m, then

$2T_0 = mg \rightarrow$ in the absence of magnetic field

(a) when magnatic field is applied:

$\tau_0 = MB = \pi b^2 IB$ ..(i)

$T_1 + T_2 = mg$ ...(ii)



Taking torque about O:

$$\tau_0 + T_2\frac{L}{2} = T_1\frac{L}{2} \quad \text{...(iii)}$$

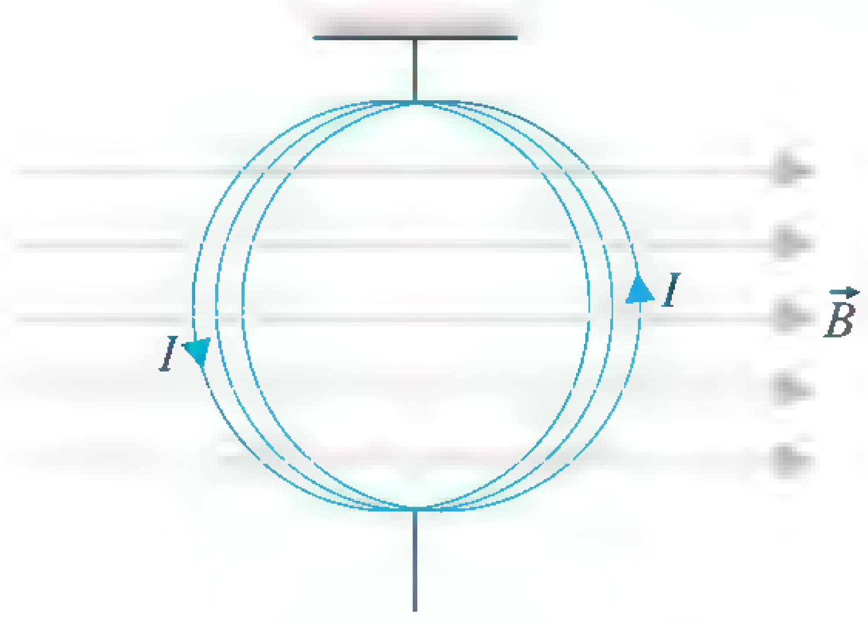
Solve equations (i), (ii) and (iii) to get

$$T_1 = \frac{mg}{2} + \frac{\pi b^2 BI}{L} \quad \text{and} \quad T_2 = \frac{mg}{2} - \frac{\pi b^2 BI}{L}$$

(b) In this case, magnetic field will produce no torque, so tension will remain same, $T_0 = mg/2$.

12. Magnetic force $F_m = I\left(\oint d\vec{l} \times \vec{B}\right)$

For coil or close loop $\oint d\vec{l} = 0$ so $\vec{F}_m = 0$



The torque $\vec{\tau}$ on a coil of any shape having N turns and Current I in a magnetic field B is given by $\tau = NIAB\sin\theta$

$$\tau = 100\times 10\times \pi \times (5\times 10^{-2})^2 \times 2.0\times \sin 90° = 5\pi\,\text{N m}$$

The direction of $\vec{\tau}$ is vertically upwards. To prevent the coil from turning, an equal and opposite torque must be applied.

13. We assume equal and opposite currents in wires PQ and RS. Then we split the given loop in three independent square loops and find magnetic moment vector of the loop which is only due to the square coil $PQRS$ as top and bottom square coils magnetic moments are equal and opposite, so will cancel each other. Thus magnetic moment of the given loop is given as

$$\vec{M} = Ia^2\hat{j} = 2\times(2)^2\hat{j} = 8\hat{j}$$

The magnetic induction vector in space is given as

$$\vec{B} = 2\hat{j}$$

The torque on the given loop is given as

$$\vec{\tau} = \vec{M}\times\vec{B} = 0$$

14. The average current in the loop is $i = \dfrac{q}{T} = \dfrac{q}{2\pi r/v} = \dfrac{qv}{2\pi r}$ and its magnetic dipole moment is

$$\vec{M} = iA = \left(\frac{qv}{2\pi r}\right)(\pi r^2) = \frac{1}{2}qvr$$

With $\vec{\tau} = \vec{M}\times\vec{B}$, we find the maximum torque exerted on the loop by a uniform magnetic field to be

$$\tau_{max} = MB = \frac{1}{2}qvrB$$

Exercises

Single Correct Answer Type

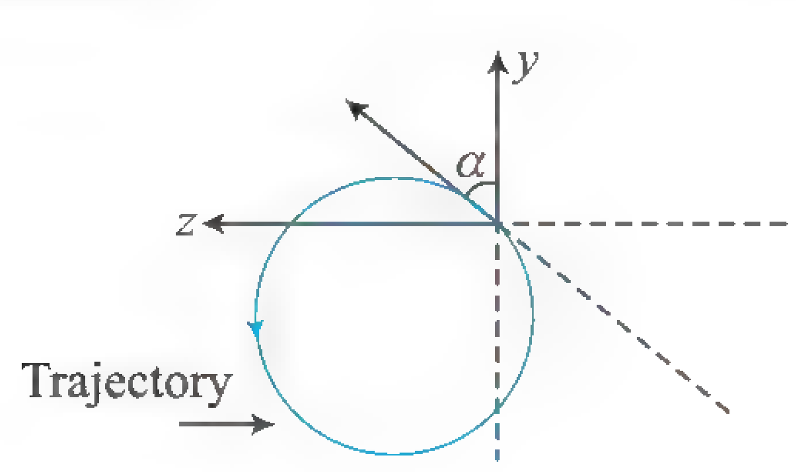
1. (1) It is clear from Right Palm Rule that the charges are positively charged.

As $r = \dfrac{mv}{qB}$

The radius of path is not the only function of either m or q. For (2), (3) and (4) we cannot make clear statement, but statement (1) is certainly true.

2. (1) $r = \dfrac{mv}{qB}$ $\Rightarrow$ $r \propto m$ (for v same for both sense)

3. (3) We can see from trajectory that both y- and z-coordinates can become negative.



This trajectory lies in y–z plane. So, x-coordinate is always zero.

4. (2) For the particle to move along anticlockwise path, force should be along $\hat{j}$. Velocity is along $\hat{i}$.

Now, $\vec{F}_m = -e(\vec{v}\times\vec{B})$

In terms of unit vectors only,

$$\hat{j} = -(\hat{i}\times\hat{?}) \quad \text{or} \quad \hat{j} = \hat{?}\times\hat{i}$$

Clearly, $\hat{?}$ is $\hat{k}$.

5. (3) $F = Bqv$

But $\dfrac{1}{2}mv^2 = eV$ or $v = \sqrt{\dfrac{2eV}{m}}$

$$\therefore \quad F = Bq\sqrt{\frac{2eV}{m}}$$

$$\Rightarrow \quad F \propto \sqrt{V} \text{ and}$$

$$F' \propto \sqrt{2V}$$

$$\frac{F'}{F} = \sqrt{2} \quad \text{or} \quad F' = \sqrt{2}F$$

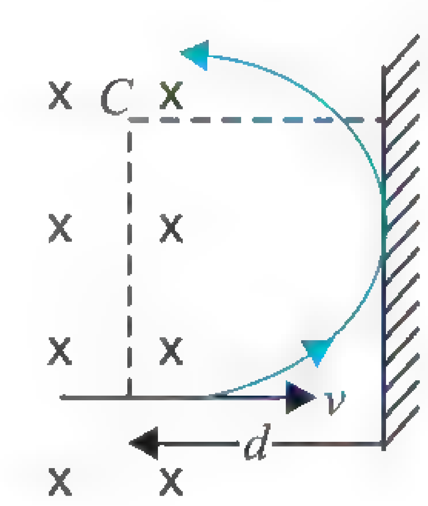
6. (2) The charged particle has circular path in the case when only magnetic field is present.

7. (4) Depending on the direction of magnetic field, tension may increase or decrease.

8. (1) Time interval in which $\vec{v}$ returns to its initial value is same as time period of the particle, hence the required time $= \dfrac{2\pi m}{eB}$

9. (1) $\vec{F} \propto \left(\vec{v}\times\vec{B}\right) = \hat{k}\left[aD - dA\right]$

10. (1) The particle moves in a circular path with radius d if it is to just miss the wall.



$$\Rightarrow \quad mv = Bqr,\ r = d$$

or $B = \dfrac{v}{(q/m)d} = \dfrac{v}{sd}$

11. (2) For a particle moving in any combination of electric and magnetic fields, work is done only by the electric field.

Energy of the particle = work done by the electric field = electric field × displacement in the direction of the electric field.

12. (3) $T = \dfrac{2\pi m}{Bq}$ or $T \propto \dfrac{m}{q}$

$$\frac{T_\alpha}{T_p} = \frac{4m}{2q}\times\frac{q}{m} = 2$$

or $T_\alpha = 2\left[\dfrac{25}{5}\right]\mu\text{s} = 10\ \mu\text{s}$

13. (3) $\dfrac{1}{2}mv^2 = qV$ or $v = \sqrt{\dfrac{2qV}{m}}$

Centripetal force, $\dfrac{mv^2}{R} = qvB$

$$\therefore \quad v = \left(\frac{qB}{m}\right)R$$

Hence, $\sqrt{\dfrac{2qV}{m}} = \left(\dfrac{qB}{m}\right)R$

or $R=\left(\frac{2mV}{q}\right)^{1/2}\times\frac{1}{B}$

Here, V, q and B are constant. Hence, $m \propto R^2$

So, $\frac{m_1}{m_2}=\left(\frac{R_1}{R_2}\right)^2$

14. (2) $Bqv=\frac{mv^2}{r}$ or $Bqr=mv$

For electron as well as proton B is the same, r is the same and numerically charge q is same; therefore mv is constant.

$$m_e v_e = m_p v_p \text{ or } v_p=\left(\frac{m_e}{m_p}\right)v_e$$

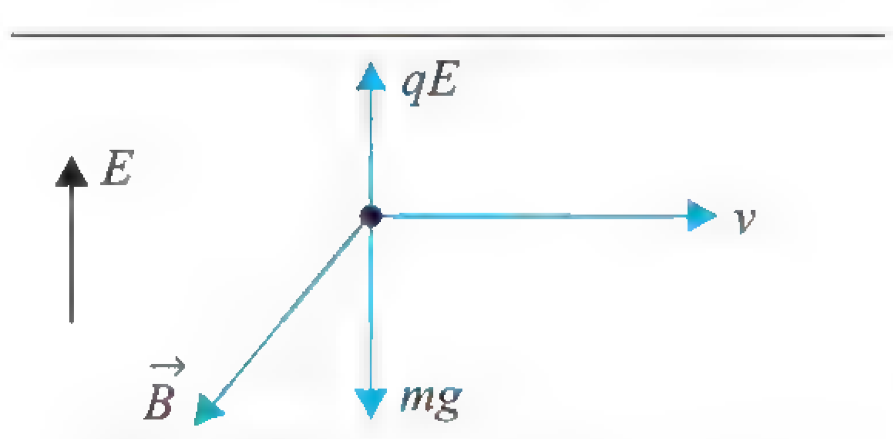
or $v_p=\left(\frac{0.90\times10^{-30}}{1.8\times10^{-27}}\right)\left(3.0\times10^{6}\right)=1.5\times10^3$ m/s

15. (2) Since the charged particle passes straight without deflection, therefore

$$Bqv=mg$$

or $v=\frac{mg}{Bq}=\frac{10^{-3}\times10}{1\times10^{-5}}=10^3$ ms^{-1}

16. (1) Net force on the particle should be zero.



$qE=10^{-6}\times200=2\times10^{-4}$ N

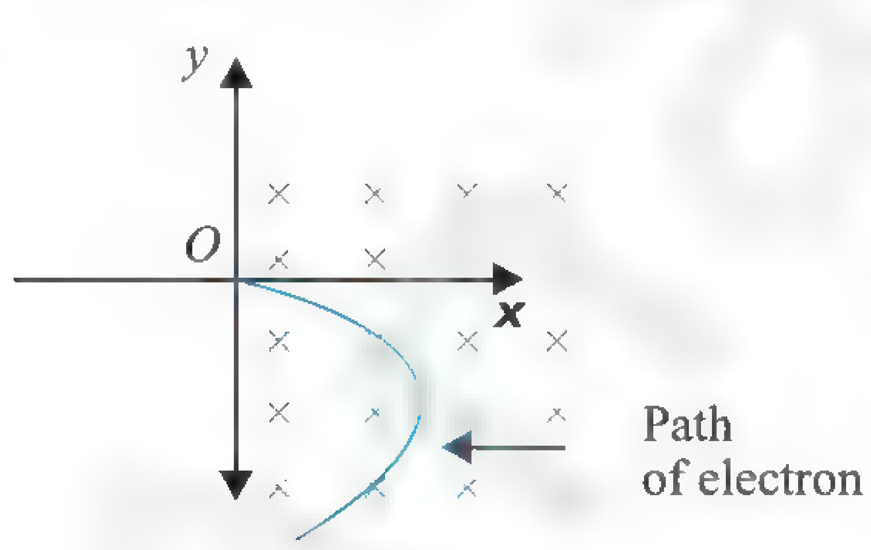
$mg=2\times10^{-5}\times9.8=1.96\times10^{-4}$ N

Since $qE>mg$, so magnetic force qvB should act downward to balance the forces

$qE=mg+qvB \Rightarrow 2\times10^{-4}=1.96\times10^{-4}+10^{-6}v\times2$

$\Rightarrow v=2$ m/s

17. (4) y will be less than zero. Speed will remain same. The trajectory will be as shown in figure.



18. (3) Knowledge based.

19. (2) Knowledge based.

20. (2) Due to linear scale, $i\propto\theta$.

21. (2) As the loop is placed in horizontal plane, so area vector is along vertical direction. From $\vec{\tau}=I\left(\vec{A}\times\vec{B}\right)$, as $\vec{A}$ is in vertical direction, $\vec{\tau}$ would be the plane of loop only. So, option (1) is wrong because for rotation of loop about its own axis $\vec{\tau}$ must be along vertical direction. (2) is correct because we can produce torque in the plane of the loop and due to this the loop can tip over.

22. (1) See the direction of torque about centre.

23. (4) Net force on a current carrying loop in a uniform magnetic field is zero. So, magnetic force cannot balance its weight.

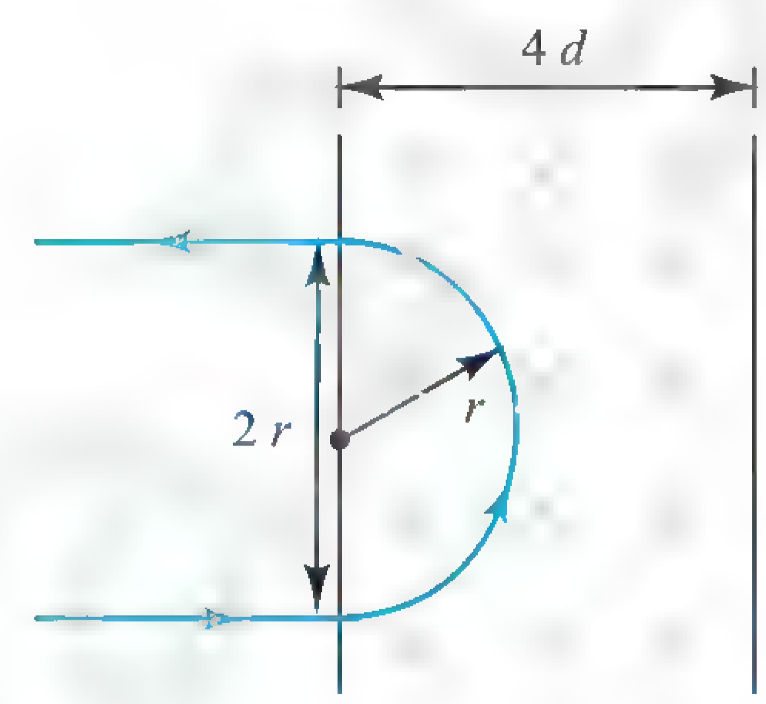
24. (2) As we know that

$$\vec{F}=\oint I d\vec{l}\times\vec{B} \quad\text{and}\quad \vec{\tau}=\vec{M}\times\vec{B}$$

25. (2) $T=2\pi m/qB$, $\left(\frac{m}{q}\right)_{\text{He}}>\left(\frac{m}{q}\right)_{\text{p}}>\left(\frac{m}{q}\right)_{\text{e}}$

$\therefore \quad t_{\text{He}}>t_{\text{p}}>t_{\text{e}}$

26. (2) $r=\frac{mv}{qB}=\frac{v}{B\alpha}=\frac{(2\alpha d)(B)}{(B\alpha)}=2d$



i.e., the electron will move out after travelling on a semicircular path of radius $r=2d$.

27. (2) $r=\frac{mv}{qB}=\frac{\sqrt{2mE}}{qB}=\frac{\sqrt{2mqV}}{qB}=\sqrt{\frac{m}{q}}\times\frac{\sqrt{2V}}{B}$

$\Rightarrow \frac{r_2}{r_1}=\sqrt{\frac{4}{1}}\times\sqrt{\frac{1}{2}}=\sqrt{2} \Rightarrow r_2=\sqrt{2}r_1=5\sqrt{2}$ cm

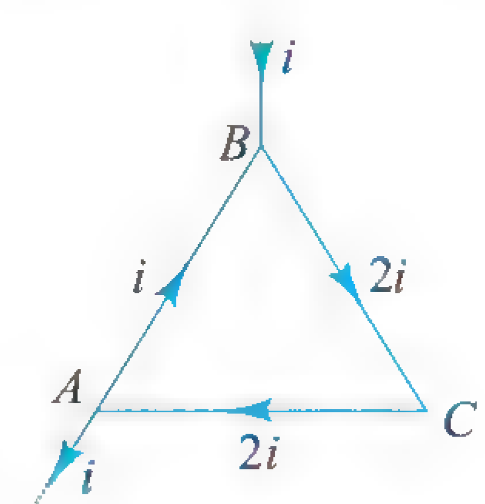
28. (2) $r=\frac{mv\sin\theta}{qB}$, Pitch: $p=v\cos\theta T=v\cos\theta\frac{2\pi m}{qB}$

Now, $p=r$, solve to get $\theta=\tan^{-1}2\pi$

29. (4) $R=\frac{mv}{qB}=\frac{m}{qB}\sqrt{\frac{2(\text{KE})}{m}}=\frac{\sqrt{2m}}{qB}\sqrt{\text{KE}}$

$\Rightarrow \frac{R_1}{R_2}=\frac{\sqrt{\text{KE}}}{\sqrt{\text{KE}/2}} \Rightarrow R_2=\frac{R_1}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$ cm

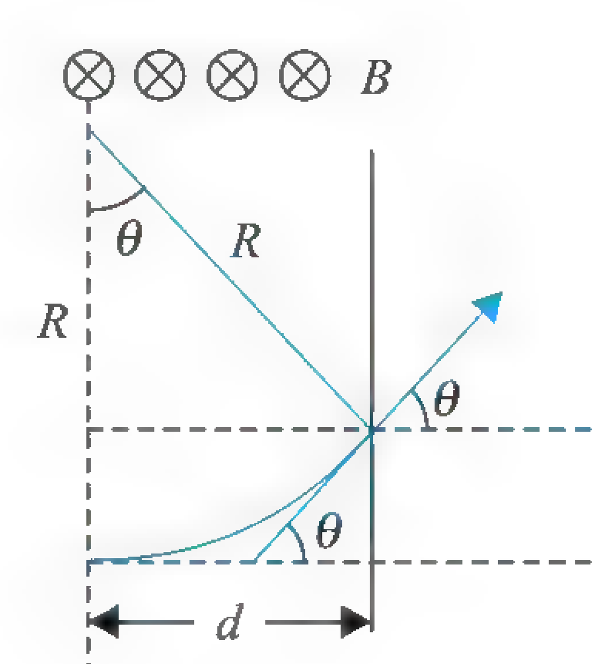
30. (1) The equivalent figure can be redrawn as shown in figure.



Force on $AB=ilB$, Force on $BCA=2ilB$ in opposite direction to that on BA.

Hence, net force $=2ilB-ilB=ilB$

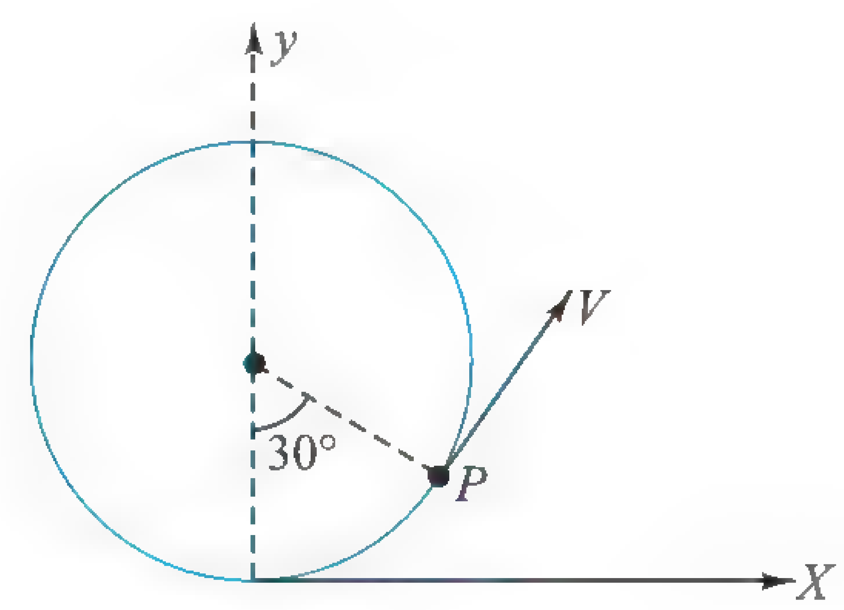
31. (4) $\sin\theta=(d/R)$



$$d = R\sin\theta = \frac{mv}{qB}\sin\theta$$

$$\Rightarrow \quad \frac{q}{m} = \frac{v\sin\theta}{Bd}$$

32. (2) Time period, $T = \frac{2\pi m}{qB} = \frac{2\pi}{\pi \times 2} = 1\,\text{s}$

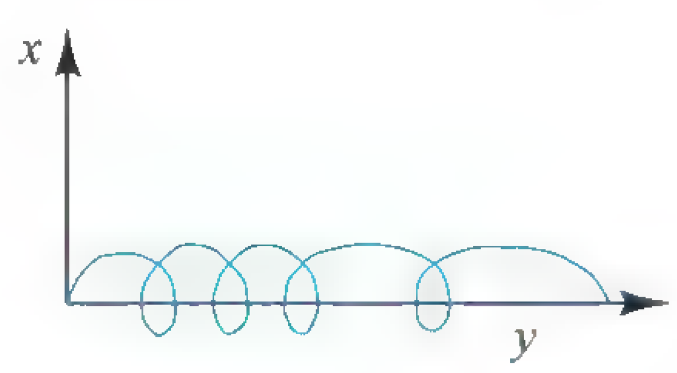


Thus, particle will be at point P after $t = \frac{1}{12}$ s

$$\vec{v} = 10[\cos 30\hat{i} + \sin 30\hat{j}]$$

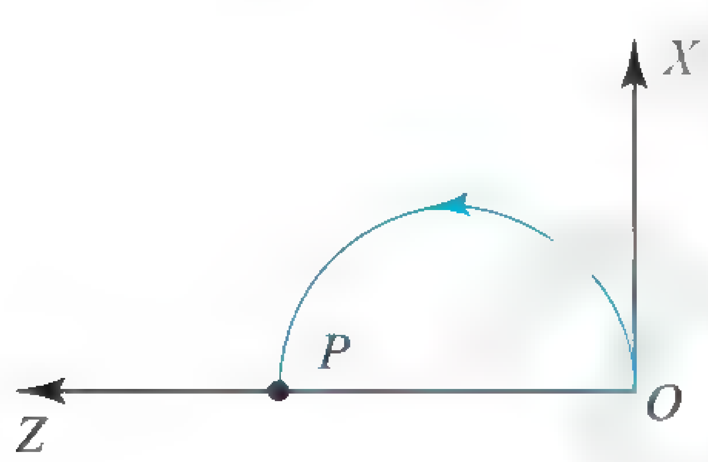
$$\vec{v} = 10\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] = 5\left[\sqrt{3}\hat{i} + \hat{j}\right]\text{ms}^{-1}$$

33. (4) The particle will move in a non-uniform helical path with increasing pitch as shown in figure.



Its time period will be: $T = \frac{2\pi m}{qB} = 2\pi$ s

Changing the view, the particle seems to move in a circular path in (X–Z) plane as shown in figure.



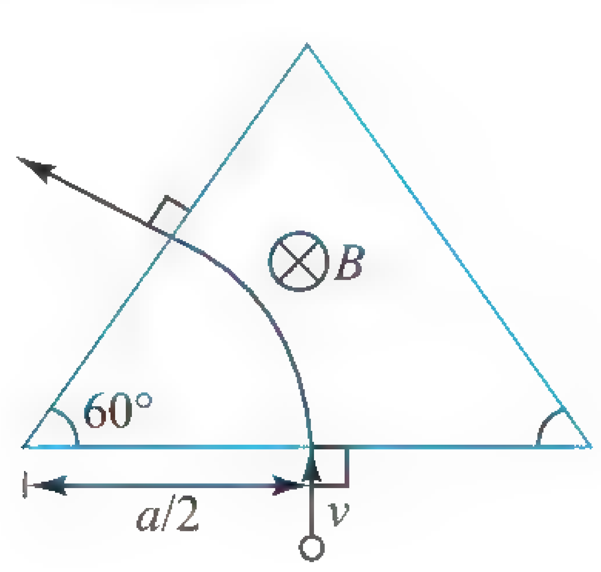
After π seconds, the particle will be at point 'P' (after completing half circle), hence X-coordinate of P will be 0.

$$y(\pi) = 0(\pi) + \frac{1}{2}\frac{Eq}{m}(\pi)^2 = \frac{\pi^2}{2},$$

$$z = 2r = 2mv/qB = 2\text{ m}$$

Hence, the coordinates of the particle are $\left(0, \frac{\pi^2}{2}, 2\right)$

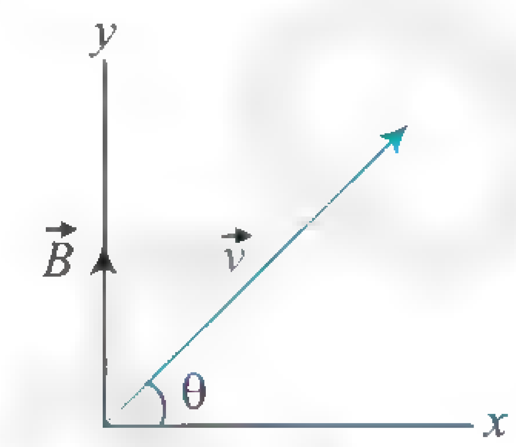
34. (2) The charged particle moves in a circle of radius $a/2$.



$$qvB = \frac{mv^2}{a/2} \quad \text{or} \quad B = \frac{2mv}{qa}$$

35. (2) Each and every pair of loop elements located symmetrically w.r.t. central line experience zero net force, so total magnetic force experienced by the loop is zero.

36. (3) Since the proton is entering the magnetic field at some angle other than 90°, its path is helix.



Corresponding velocity of the proton along X-axis,

$$v_x = v\cos 60° = 2\times 10^6 \times \frac{1}{2} = 10^6\ \text{ms}^{-1}$$

Due to velocity component v_x, the radius of the helix is described and is given by the relation

$$r = \frac{mv_x}{qB} = \frac{1.6\times 10^{-27}\times 10^6}{1.6\times 10^{-19}\times 0.10} = 0.1\text{ m}$$

Now, $T = \frac{2\pi r}{v_x} = \frac{2\pi \times 0.1}{10^6} = 2\pi \times 10^{-7}$ s

37. (1) $K = \frac{1}{2}mv^2 = eV$ or $v = \sqrt{\frac{2eV}{m}}$

Also, $F = evB = e\left[\sqrt{\frac{2eV}{m}}\right] \times B$

Therefore, $\frac{F}{2F} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{V'}}$

$$\therefore \quad \frac{V'}{V} = \frac{4}{1}$$

38. (4) $\vec{E}$ is parallel to $\vec{B}$ and $\vec{v}$ is perpendicular to both. Therefore, path of the particle is a helix with increasing pitch. Speed of particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad \text{...(1)}$$

Here, $v_y^2 + v_z^2 = v_0^2$

and $v = 2v_0 \Rightarrow v_x = \sqrt{3}v_0$

$$v_x = a_x t \Rightarrow \sqrt{3}v_0 = \frac{qE}{m}t \Rightarrow t = \frac{\sqrt{3}mv_0}{qE}$$

39. (3) Using, impulse = change in linear momentum, we have

$$\int F\,dt = mv \quad \text{or} \quad \int (iBl)\,dt = mv$$

or $\quad Blq = mv \qquad \left(\text{as} \int i\,dt = q\right)$

$$\therefore \quad q = \frac{mv}{Bl}$$

40. (1) $F_{CAD} = F_{CD} = F_{CAD}$

$\therefore$ Net force on the frame $= 3F_{CD}$

$= (3)(2)(1)(4) = 24$ N $\quad (F = ilB)$

41. (3) $p = \frac{2\pi m}{Bq}(v\cos 45°) = \frac{2\pi m}{Bq}(v\sin 45°)$

$\therefore \quad \dfrac{mv\sin 45^\circ}{Bq} = \dfrac{p}{2\pi}$ = radius of helix

42. **(4)** Torque τ acting on a current carrying coil of area A placed in a magnetic field of induction B is given by

$\tau = NIBA \sin\theta$

where I = current in the coil, θ = angle which the normal to the plane of the coil makes with the lines of induction B.

Here, $N = 1, \quad B = 1.5 \times 10^{-2}$ T

$A = 0.05 \times 0.08 = 40 \times 10^{-4}\ \text{m}^2$

$I = 10.0$ amp, $\theta = 90^\circ = \pi/2$

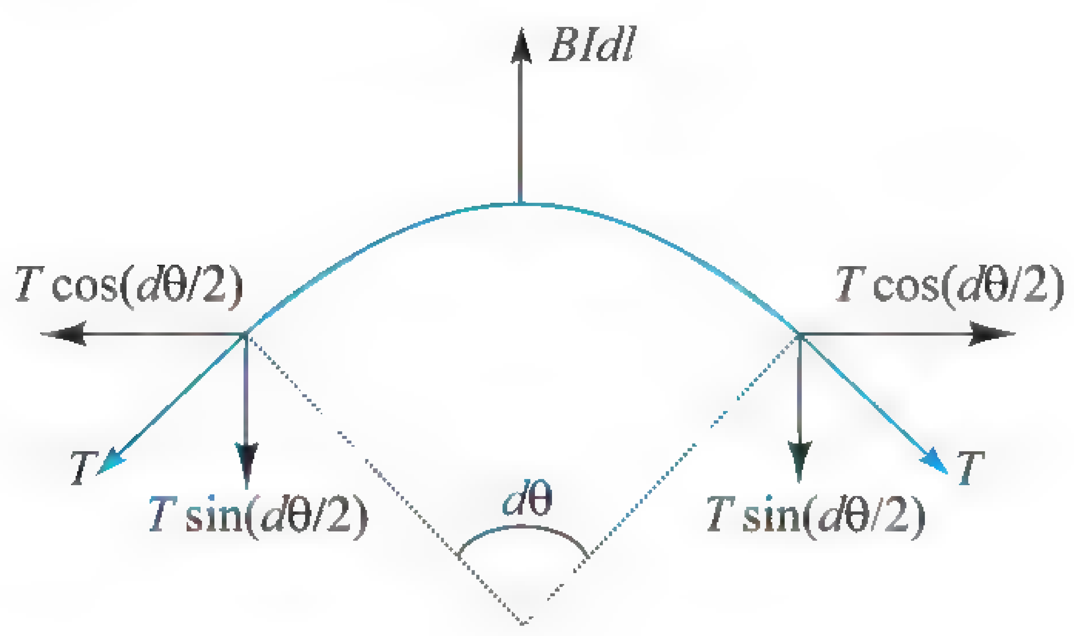
$\tau = (1.5 \times 10^{-2})(10.0) \times (1)(40 \times 10^{-4}) \sin(\pi/2)$

$= 6 \times 10^{-4}$ Nm

43. **(3)** $BI(dl) = 2T\sin(d\theta/2)$

$\Rightarrow \quad BI(rd\theta) = 2T(d\theta/2) \qquad (\theta \text{ is small, } \sin\theta = \theta)$

$\Rightarrow \quad T = BIr = BIl/2\pi$



44. **(1)** Torque due to magnetic field $\tau_{mag} = MB_0 = I\pi R^2 B_0$ ...(i)

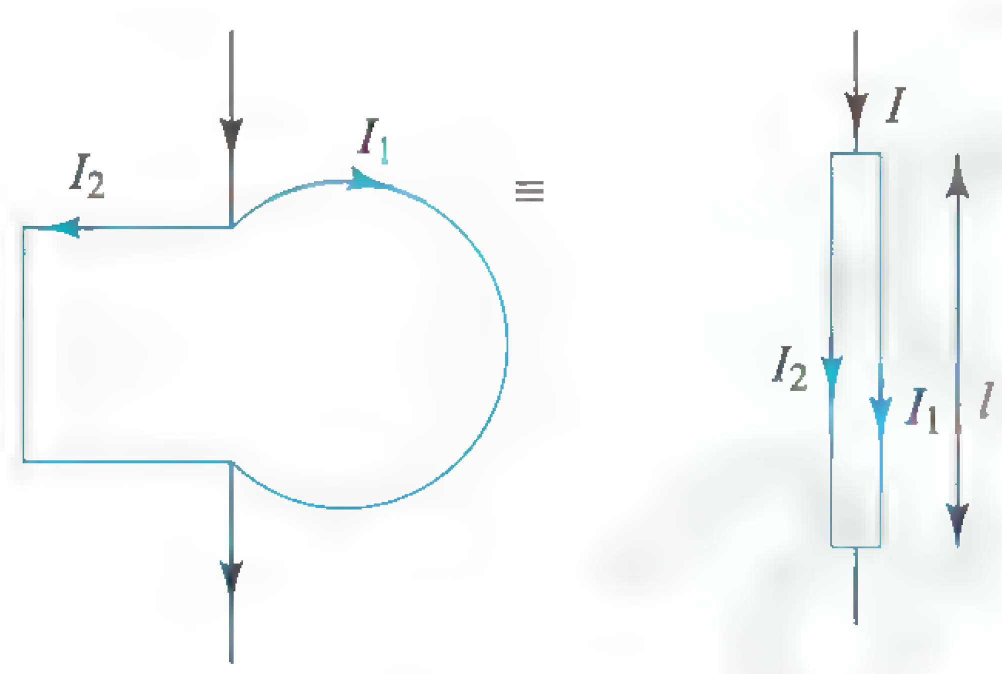
Torque due to weight about the point where string is connected

$\tau_{weight} = mgR$ ...(ii)

If ring remains horizontal, then $\tau_{mag} = \tau_{weight}$

$I\pi R^2 B_0 = mgR \quad \Rightarrow \quad I = \dfrac{mg}{\pi R B_0}$

45. **(2)** $F = BI_1 l + BI_2 l$

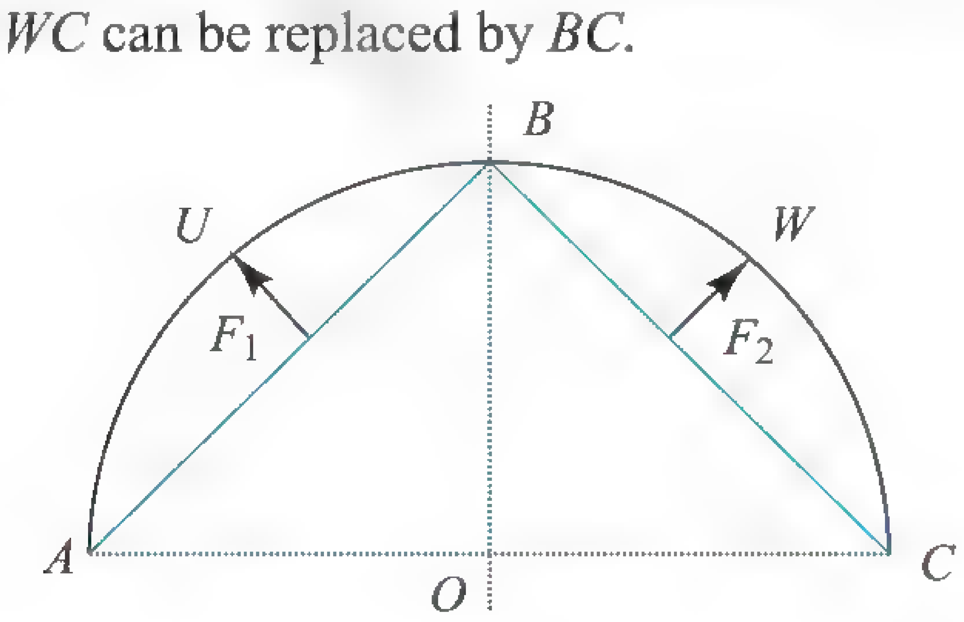


$\Rightarrow \quad F = Bl(I_1 + I_2) = BIl$

46. **(1)** Radius of circular path of the charged particle $r = \dfrac{mV}{qB}$

Here $R < r$, hence the particle will press the outer wall of the pipe hence the force applied by the pipe on the particle should be towards the centre of the pipe.

47. **(3)** The part AUB of the wire can be replaced by straight wire AB and BWC can be replaced by BC.



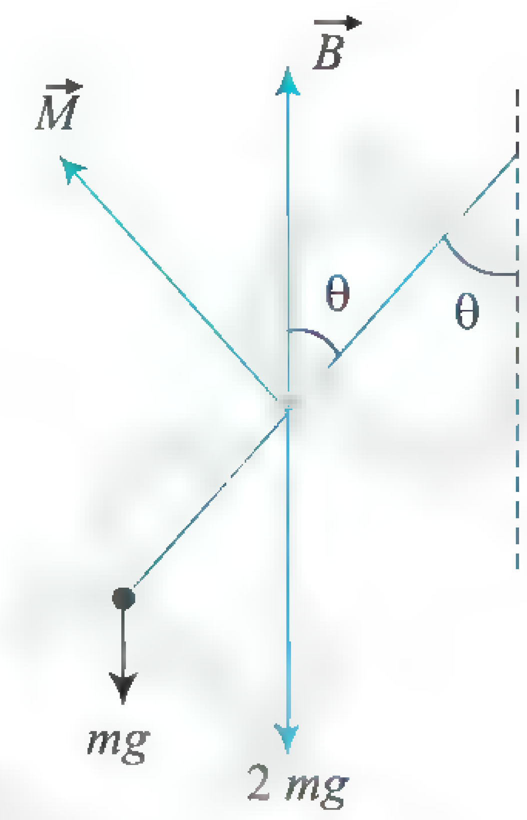
Force experienced by AB is, $F_1 = IB_0\left(\sqrt{2}R\right)$

Force experienced by BC is, $F_2 = I \times 2B_0 \times \sqrt{2}R$

Resultant magnetic force acting on the wire is,

$F = \sqrt{F_1^2 + F_2^2} = \sqrt{10}\, IB_0 R$

48. **(1)** Let a is the side of square.



Torque of current $= MB\sin(90 - \theta)$

$= MB\cos\theta = i\, a^2 B\cos\theta$

Mass of each side $= m = \rho A a$

Torque of gravity $= 2mg\,\dfrac{a}{2}\sin\theta + mg\, a\sin\theta$

$= 2\, mg\, a\sin\theta = 2\rho A g a^2 \sin\theta$

Now, both torques should be same

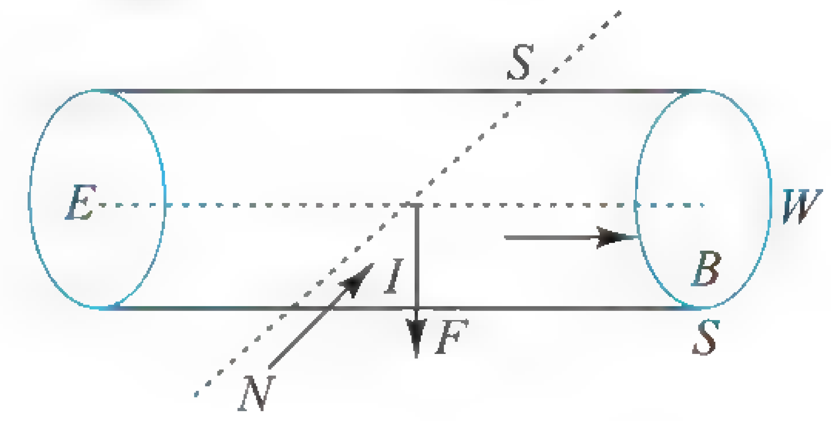
i.e., $\quad ia^2 B\cos\theta = 2\rho A g a^2 \sin\theta$

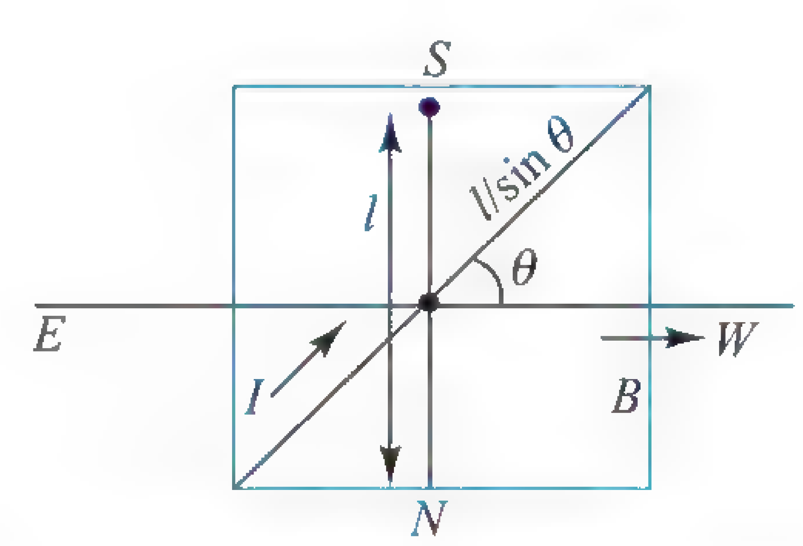
$\Rightarrow \quad \cot\theta = \dfrac{2\rho A g}{iB}$

49. **(3)** Initially: $1.2\ N = 1(\vec{l} \times \vec{B})$ downward

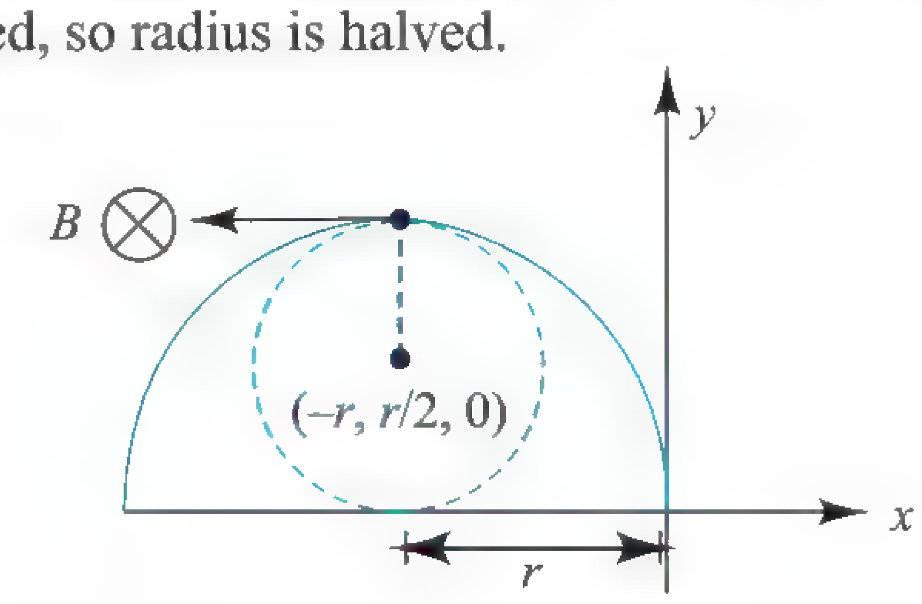
In the given condition:

$F = I\dfrac{l}{\sin\theta}B\sin\theta = IlB = 1.2\ \text{N}$ downward





50. **(2)** Two particles will meet at P. After they meet they will stick together. Momentum mv will remain same. But charge is doubled, so radius is halved.



Finally, both move in dotted circle.

51. **(3)** After two and a half time periods, its x-coordinate will be $2.5P_0$ (so x-coordinate of image will be $17.5P_0$). It will be at distance $2R_0$ (equal to diameter) on the negative z-axis and y-coordinate will be zero.

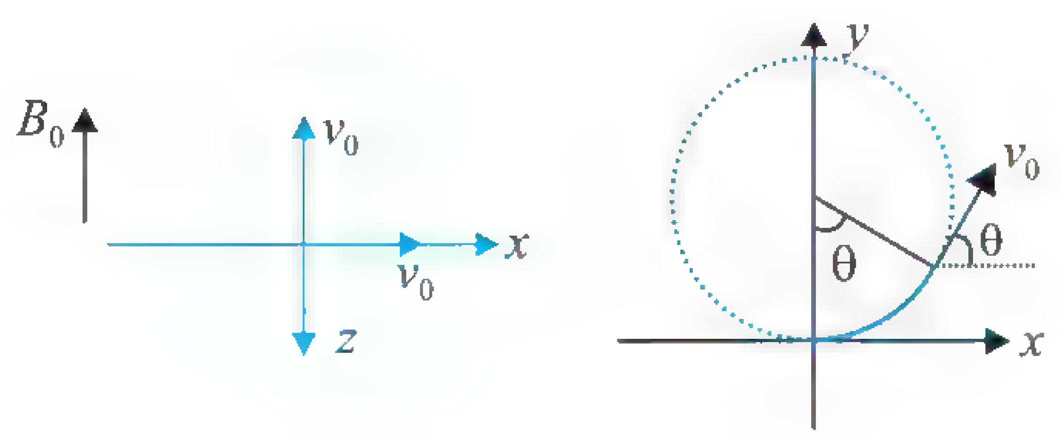
52. (1) Wire *abc* can be replaced by a straight wire *ac* for the computation of force.

Length of $\overrightarrow{ac}$ can be written as,

$$\vec{l} = \vec{r}_C - \vec{r}_A = \left[(\hat{i}+\hat{j})-(\hat{i}+\hat{k})\right]50\times10^{-2} = 0.50\,[\hat{j}-\hat{k}]$$

Required force, $\vec{F} = I\left(\vec{l}\times\vec{B}\right) = 0.6\hat{i}$

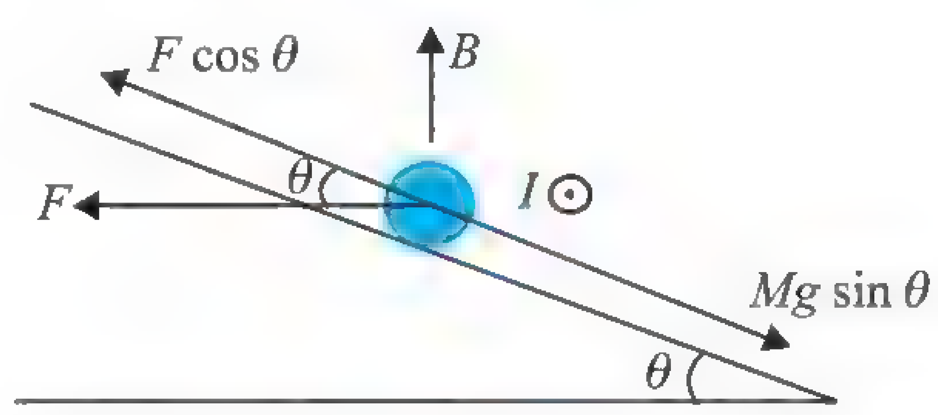
53. (1)



$$\vec{v} = v_0\cos\theta\hat{i} + v_0\sin\theta\hat{j} - v_0\hat{k}$$

where $\theta = \omega t = \dfrac{qB_0}{m}t = \alpha B_0 t$

54. (3) $F\cos\theta = Mg\sin\theta$



$$IBL\cos\theta = Mg\sin\theta \quad\Rightarrow\quad I = \frac{Mg\tan\theta}{LB}$$

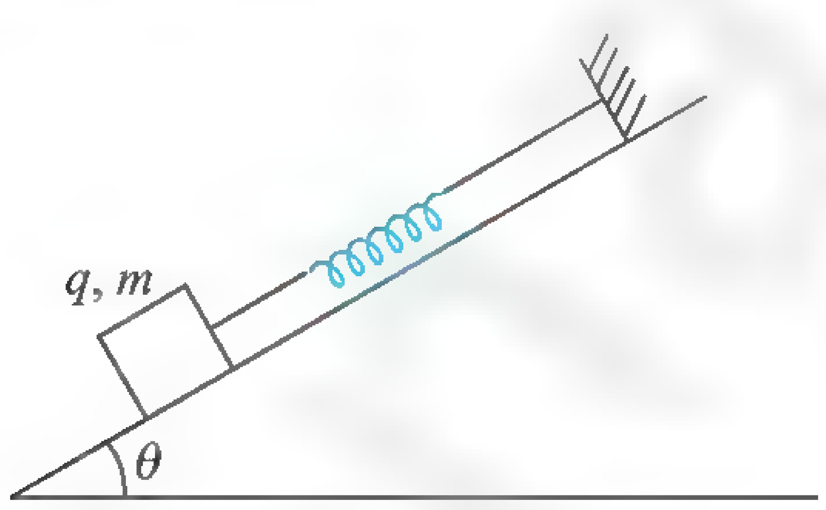
55. (1) Due to torque of magnetic field, ring will rotate about vertical diameter. $\tau = I\alpha \quad\Rightarrow\quad MB = I\alpha$

$$\Rightarrow\quad i\pi r^2 B = \frac{1}{2}mr^2\alpha$$

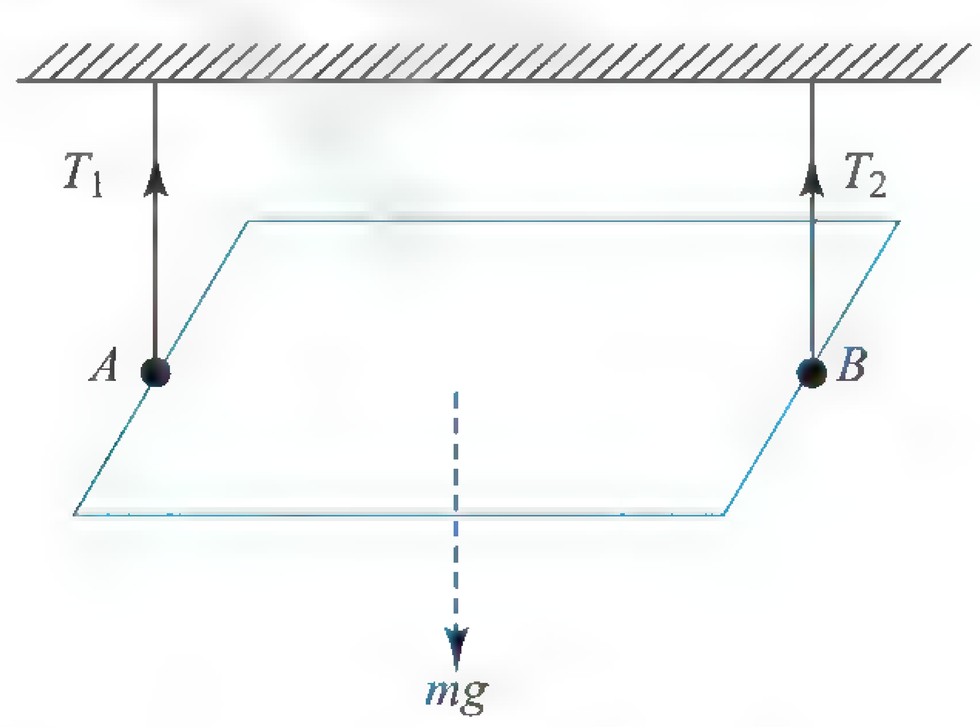
$$\Rightarrow\quad \alpha = \frac{2iB\pi}{m} = \frac{2\times4\times10\pi}{2} = 40\pi\ \text{rad s}^{-2}$$

56. (1) The ratio M/L is always $\dfrac{q}{2m}$

57. (1) There will be no effect of magnetic force on time period because the magnetic force will be perpendicular to the inclined plane.



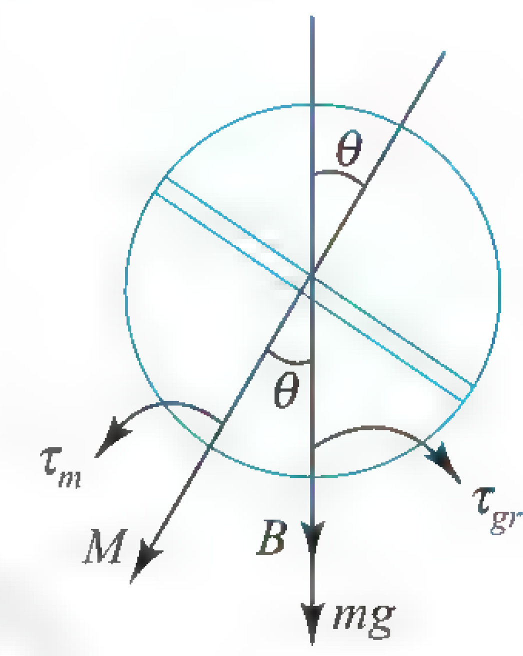
58. (3) Taking moments about point *B* to be zero.



$$T_1.l + iblB = mg\frac{l}{2};\quad T_1 = \frac{mg-2ibB}{2}$$

59. (1) effective length from A to C is $2\sqrt{2}a$

60. (1) The gravitational torque must be counter balanced by the magnetic torque about *O*, for equilibrium of the sphere. The gravitational torque $\tau_m = \pi i r^2 B\sin\theta$



$\therefore\quad \pi i r^2 B\sin\theta = mgr\sin\theta$

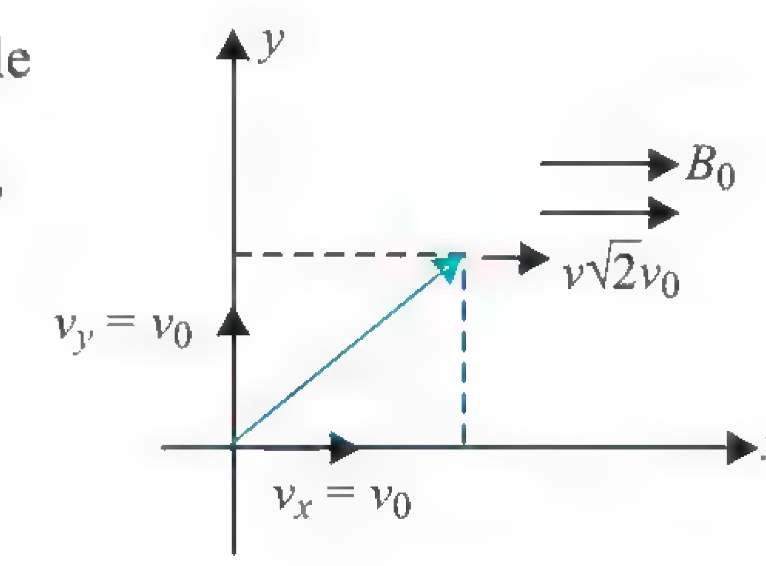
$\Rightarrow\quad B = \dfrac{mg}{\pi i r}$

61. (3) The particles will not collide if

$d > 2\,(r_1 + r_2)$

or $\quad d > 2\left(\dfrac{mv_1}{Bq} + \dfrac{mv_2}{Bq}\right)$ $\quad$ or $\quad d > \dfrac{2m}{Bq}(v_1 + v_2)$

62. (4) $\alpha = \dfrac{q}{m}$, path of the particle will be a helix of time period,



$$T = \frac{2\pi m}{B_0 q} = \frac{2\pi}{B_0\alpha}$$

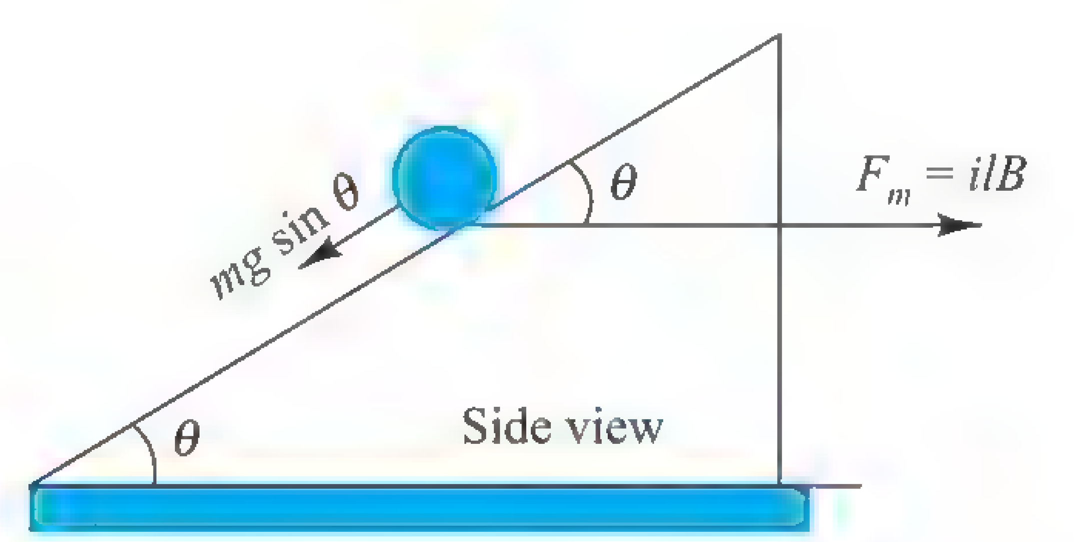
The given time $t = \dfrac{\pi}{B_0\alpha} = \dfrac{T}{2}$

$\therefore$ Coordinates of particle at time $t = T/2$ would be $(v_x\,T/2,\,0,\,-2r)$

Here, $r = \dfrac{mv_0}{B_0 q} = \dfrac{v_0}{B_0\alpha}$

$\therefore$ The coordinates are $\left(\dfrac{v_0\pi}{B_0\alpha},\,0,\,\dfrac{-2v_0}{B_0\alpha}\right)$.

63. (2) Magnetic force acts in the direction shown in figure



Rod will move downward with constant velocity if net force on it is zero.

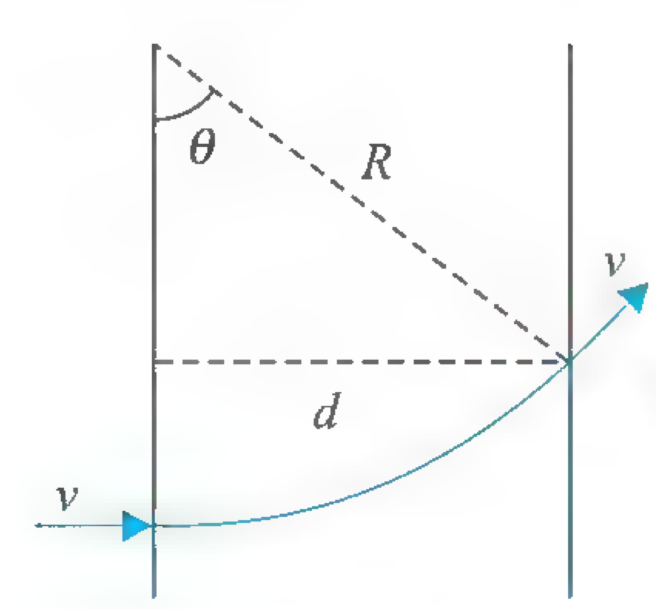
or $\quad F_m\cos\theta = mg\sin\theta$

or $\quad ilB\cos\theta = mg\sin\theta$

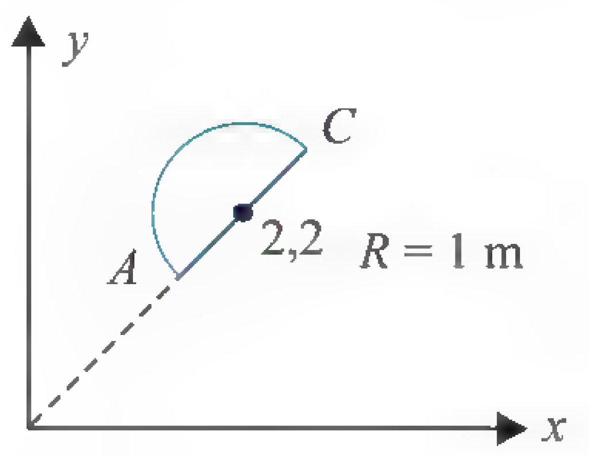
$\therefore\quad B = \left(\dfrac{mg}{il}\right)\tan\theta$

64. (3) $R = \dfrac{mv}{qB}$, $\sin\theta = \dfrac{d}{R} = \dfrac{dqB}{mv}$

$$t = \frac{2\pi m}{qB}\,\frac{\theta}{2\pi} = \frac{m}{qB}\,\theta = \frac{m}{qB}\sin^{-1}\left[\frac{dqB}{mv}\right]$$

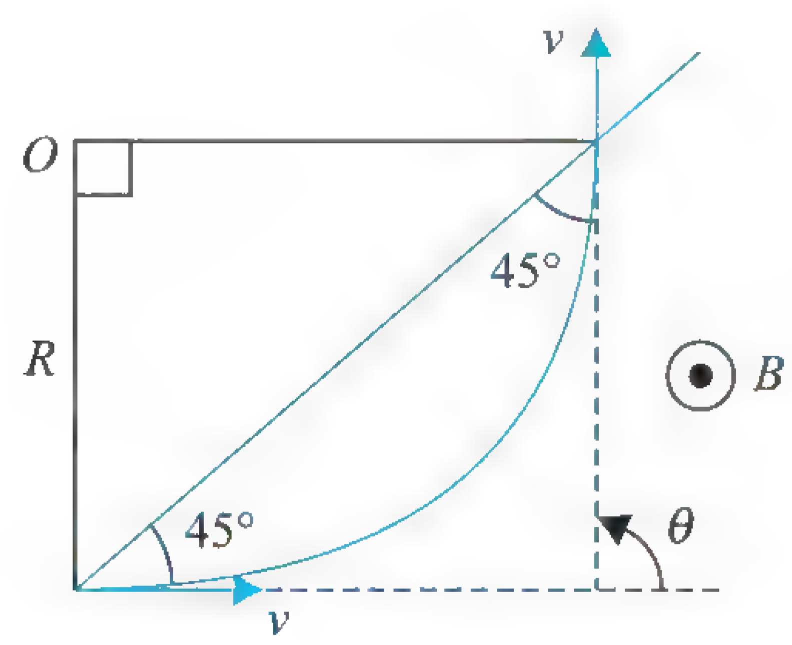


65. (2) $\vec{l} = \vec{AC} = 2R(\cos 45°\hat{i} + \sin 45°\hat{j}) = \sqrt{2}(\hat{i}+\hat{j})$



$$\vec{F} = I\vec{l} \times \vec{B} = 1 \times \sqrt{2}(\hat{i}+\hat{j}) \times (3\hat{i}+4\hat{j}+\hat{k})$$
$$= \sqrt{2}[\hat{i} - \hat{j} + \hat{k}]$$

66. (2) Path of electron is shown in figure. Deviation of electron will be $\theta = 90°$.



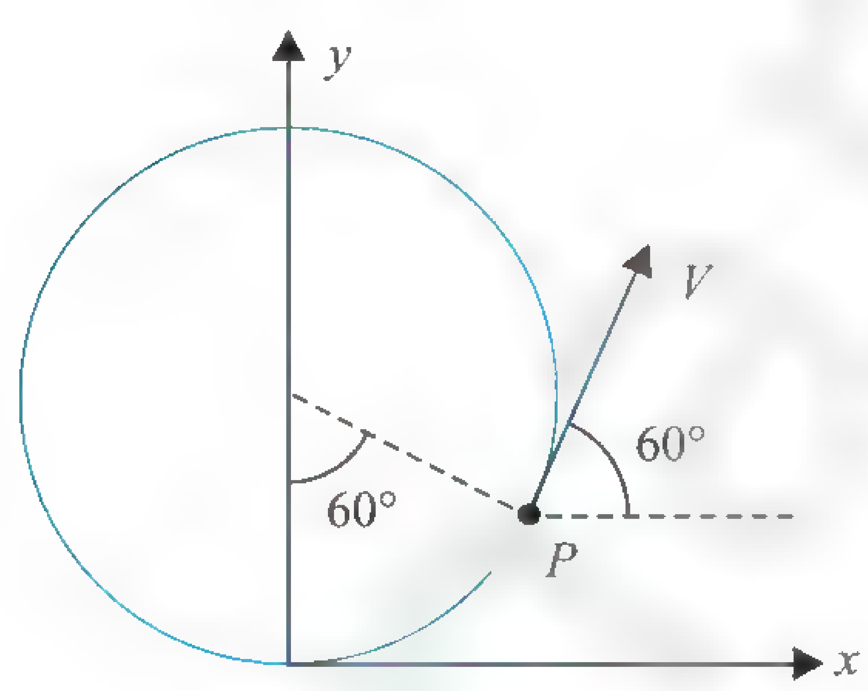
67. (1) Change in kinetic energy is only due to work done by electric field. Hence work done by electric field is change in KE:

$$qEa = \frac{1}{2}m\left[(2v)^2 - v^2\right] \Rightarrow E = \frac{3mv^2}{2qa}$$

68. (1) Time period $T = 2\pi\dfrac{m}{qB} = 1\text{ s}$

Thus, the particle will be at P after $t = \dfrac{1}{6}$ s

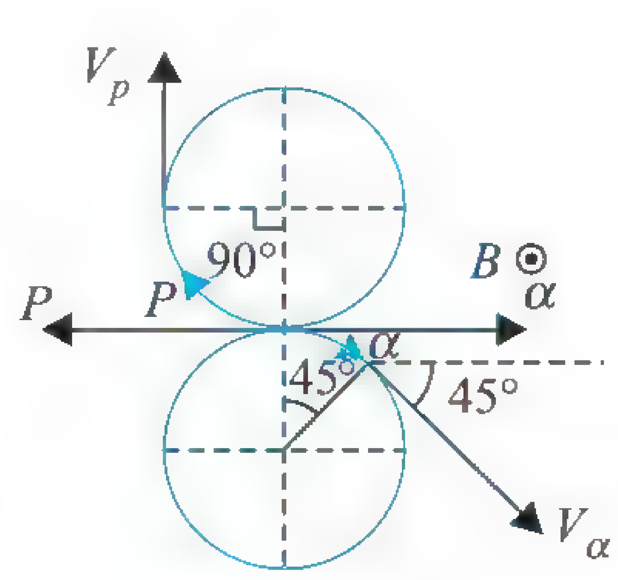
$\therefore \quad \vec{V} = 10(\cos 60\hat{i} + \sin 60\hat{j}) = 5(\hat{i} + \sqrt{3}\hat{j})$ m/s



69. (3) $\omega = qB/m$,

$$\omega_p = eB/m, \omega_\alpha = \frac{2eB}{4m} = 2\omega_P$$

So proton will cover double angle in same time as that of α-particle. Clearly from the above figure angle between v_α and v_p is 135°



70. (2) $\vec{M} = Ia^2[-\hat{i}+\hat{j}]$, $\vec{\tau} = \vec{M} \times \vec{B}$,

Solve to get $\tau = 0$

71. (4) Since the particle is moving undeflected, so magnetic force = electric force

$$\Rightarrow \quad qvB\sin\theta = qE \quad \Rightarrow \quad v = \frac{E}{B\sin\theta}$$

Pitch, $d = v\cos\theta\, T = v\cos\theta\dfrac{2\pi m}{qB}$

$$= \frac{E}{B\sin\theta}\cos\theta\frac{2\pi m}{qB} = \frac{2\pi mE}{qB^2\tan\theta}$$

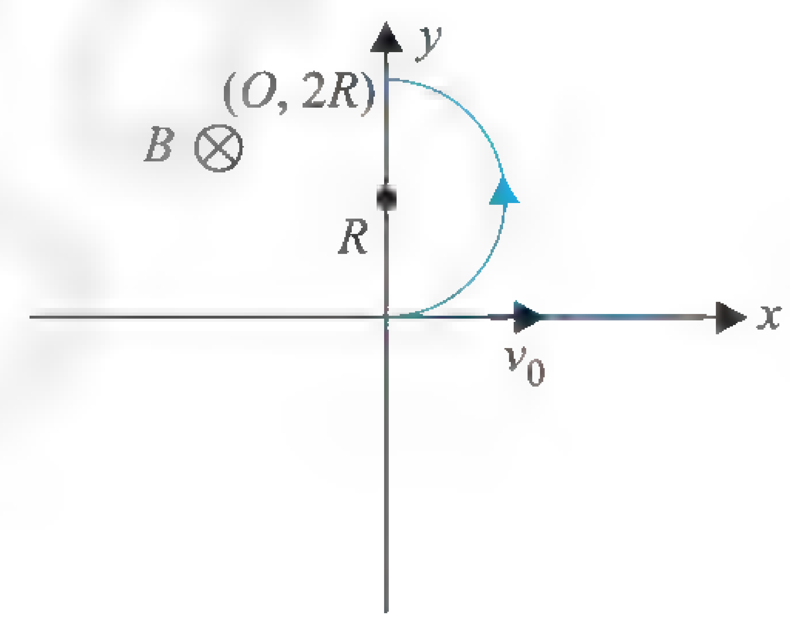
72. (1) Magnetic field is uniform, apply $\vec{F} = I\vec{L} \times \vec{B}$

where $\vec{L} = 0.25\hat{i} + \hat{j}$ and $\vec{B} = 4(\cos 60\,\hat{i} + \sin 60\,\hat{j})$

73. (2) $\vec{B}_A = \dfrac{\mu_0 R}{4\pi R}\hat{k} + \dfrac{\mu_0 I}{2R}\hat{i} - \dfrac{\mu_0 I}{4\pi R}\hat{j}$

$$B_A = \frac{\mu_0 I}{4\pi R}\sqrt{(2\pi-1)^2 + (1)^2} = \frac{\mu_0 I}{4\pi R}\sqrt{2(2\pi^2 - 2\pi + 1)}$$

74. (2) y coordinate $= 2R = \dfrac{2mv_0}{qB}$



75. (4) Since, $QP = r$, $AP = r\sqrt{2}$ and $\angle AQP = 90°$

$AQ = r$ and $\angle QAP = 45°$

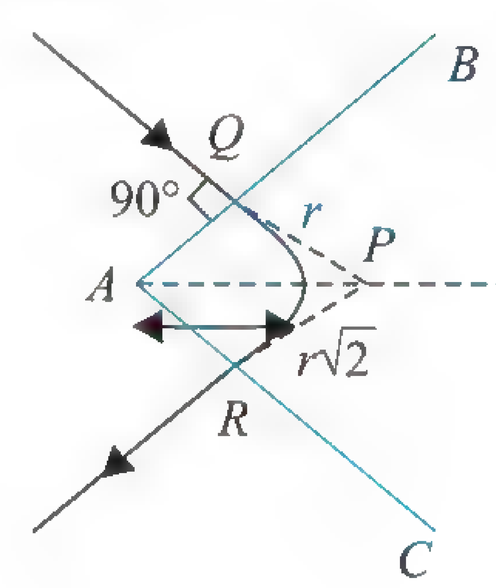
The same logic concludes $AR = r$ and $\angle PAR = 45°$

So, $\angle QAR = 90°$

$$r = \frac{mv}{Bq}$$

Distance travelled $= \dfrac{\pi r}{2}$

Time $= \dfrac{\pi}{2} \cdot \dfrac{mv}{Bq} \cdot \dfrac{1}{v} = \dfrac{m\pi}{2Bq}$



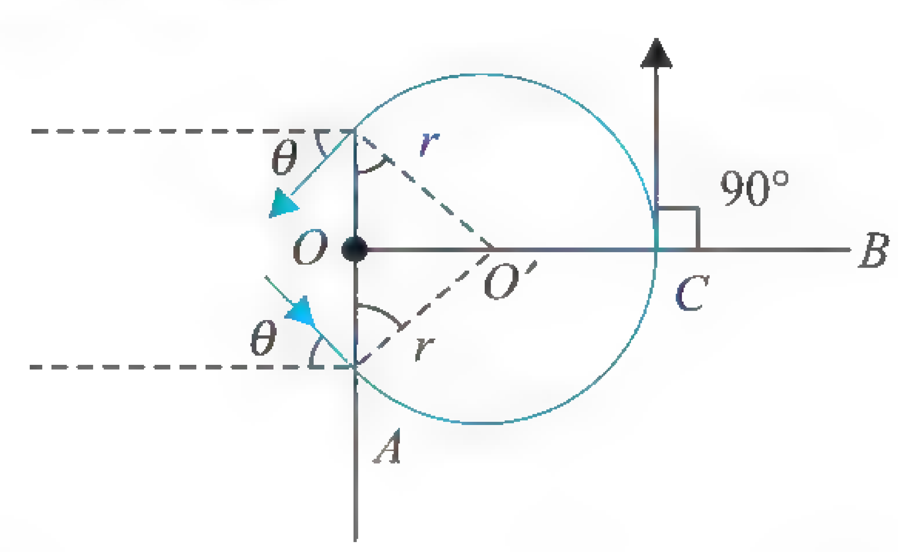
76. (3) Work done $= q\,E.\,2a$ = change in KE

$$q \cdot \frac{mv^2}{qa} \cdot 2a = \frac{1}{2}mv_Q^2 - \frac{1}{2}mv^2$$

$$4v^2 = v_Q^2 - v^2 \quad \Rightarrow \quad v_Q = v\sqrt{5}$$

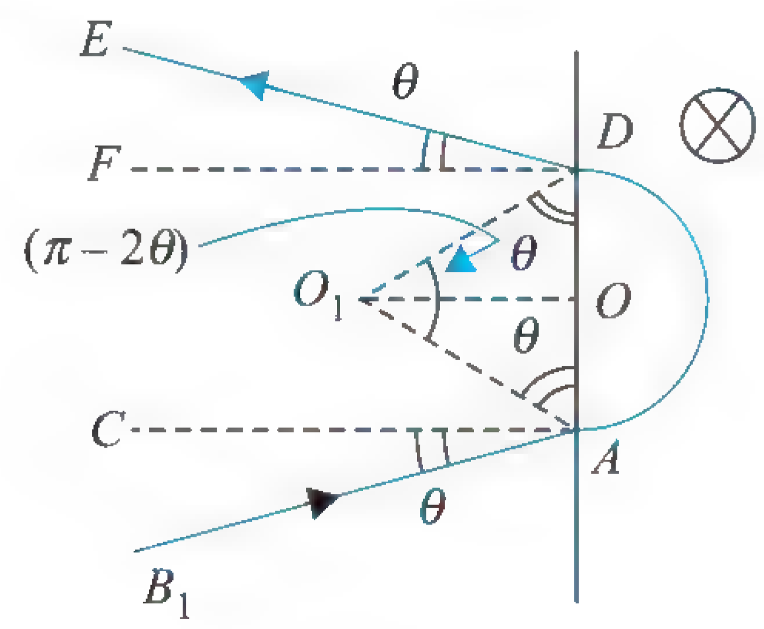
77. (4) $r = \dfrac{mv}{qB} = \dfrac{2K}{qvB}$

Thus, $OA = r\cos\theta$ (given)

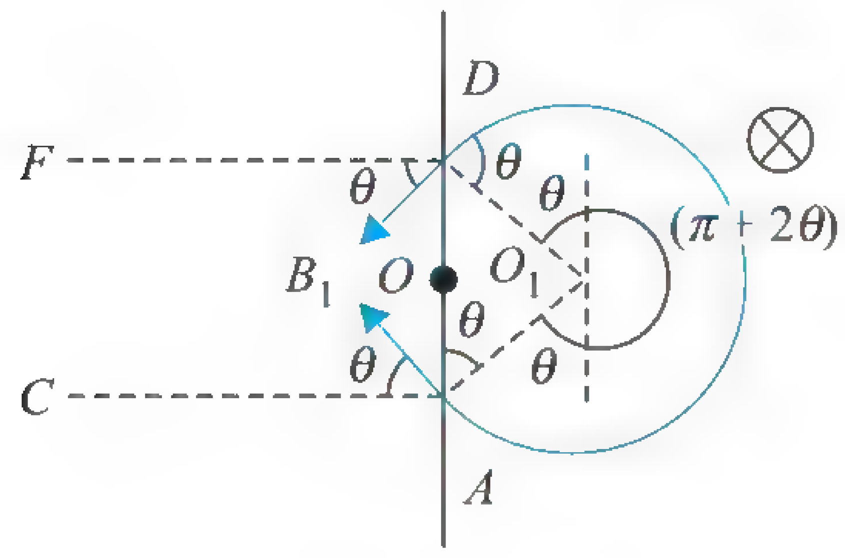


On line OB, centre O' of part of circle (in which particle moves) lies. The tangent at C with OB makes an angle of 90°. Particle leaves circular path at C. Thus, required angle is 90°.

78. (3) The diagrams are self-explanatory.



When θ is negative.



When θ is positive.

79. (1) O will be the midpoint of incident and point of emergence. Hence, $OA = OB$

80. (1) The geometry of the figures of the solution of problem (78) suggests $\phi = \theta$.

81. (1) See diagrams of the solution of problem (78)

When θ is + ve, angle at centre $= \pi + 2\theta$

When θ is −ve, angle at centre $= \pi - 2\theta$

When $\theta = 0$, incidence is normal, a semicircle is completed and

$$T = 2\pi \frac{m}{BQ} \text{ (time period)}$$

When θ is positive, time spent in magnetic field

$$= \frac{T}{2\pi} \times (\pi + 2\theta)$$

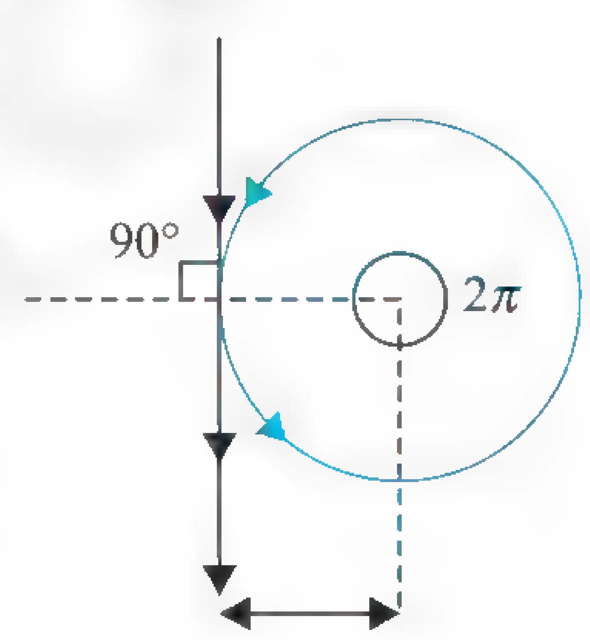
When θ is negative, time spent in magnetic field

$$= \frac{T}{2\pi} \times (\pi - 2\theta)$$

Hence, ratio $= \dfrac{(\pi + 2\theta)}{(\pi - 2\theta)} = \dfrac{(\pi/2 + \theta)}{(\pi/2 - \theta)}$

82. (2) An observation will show that when θ is positive and actually 90°, angle formed will be 2π at the centre and centre of the circle formed by the charged particle will be at a distance of radius of the circle from AD.

It will be also true for θ to be negative. (θ measured anticlockwise from normal AC).



Hence range is $\pm r = \pm \dfrac{mv}{QB}$.

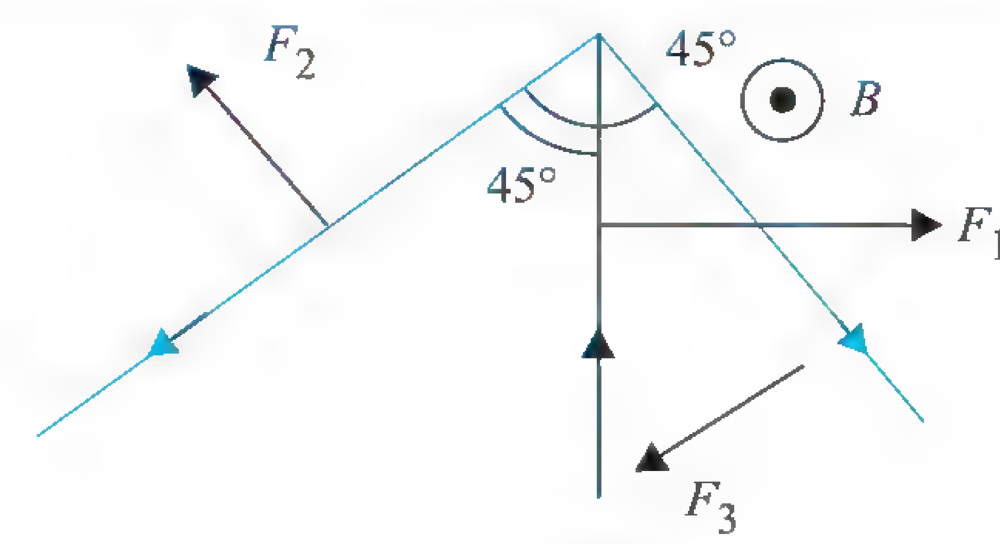
83. (2) The forces will act as shown in figure

Magnitude of F_2 and F_3 will be same.

$$F_2 = F_3 = ILB = 0.5 \times 5 \times 1$$
$$= 2.5 \text{ N} (I_2 = I_3 = 0.5 \text{ A})$$

Resultant of F_2 and F_3

$$= \sqrt{F_2^2 + F_3^2} = 2.5\sqrt{2} \cong 3.5 \text{ N}$$



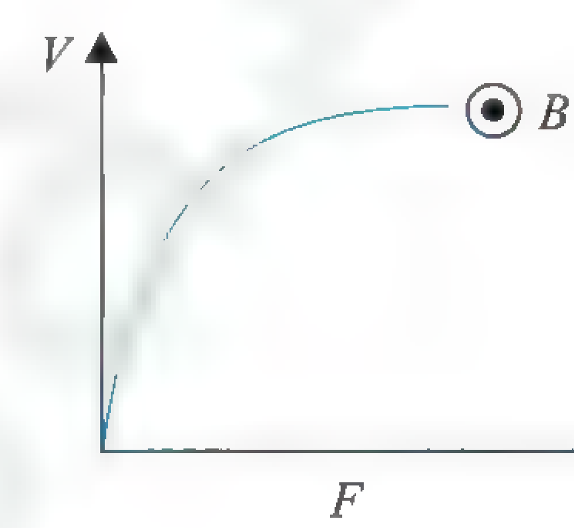
Along negative x-axis

$F_1 = 1 \times 4 \times 1 = 4$ N along positive x-axis

Net resultant force $= 4 - 3.5 = 0.5$ N along positive x-axis.

84. (3) See solution of the previous problem.

85. (2) The force is along x-direction, the system enters the magnetic field with some velocity along y-direction.



So, the system will move in circular path.

86. (3) Since, the system will move in circular path, the velocity will be constant. Hence, the acceleration will be zero.

87. (2) Angle θ for one segment, $\theta = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$

$$M_0 = IA = IA = I\left[r^2\frac{\theta}{2} - r_1^2\frac{\theta}{2}\right]$$

$$= I\left[(2a)^2\frac{\theta}{2} - a^2\frac{\theta}{2}\right] = I \cdot \frac{3a^2\theta}{2}$$

For 4 segments, $M = 4I \cdot 3a^2\dfrac{\theta}{2} = 6I a^2 \theta$

Magnetic moment due to circle $= I\pi a^2$

Total magnetic moment $= 6I a^2\theta + I\pi a^2$

$$= 6I a^2 \times \frac{\pi}{4} + I \cdot \pi a^2 = \frac{5}{2} I \pi a^2$$

88. (1) Force on TP

$$= -F_{\text{loop}} = I\,aB\sin(90° + 60°)\ \frac{IaB}{2} \odot$$

$$F_{\text{loop}} = \frac{IaB}{2} \otimes$$

Alternatively

$$F_{PQ} = IaB\sin(90° + 30°) \otimes = IaB\frac{\sqrt{3}}{2} \otimes$$

$$F_{QP} = I(2a)B\sin 30° \otimes = IaB \otimes$$

$$F_{RS} = I(a)B\sin 60° \otimes = IaB\frac{\sqrt{3}}{2} \odot$$

$$F_{ST} = IaB\sin(90° + 60) \odot = \frac{IaB}{2} \odot$$

So, $F_{\text{net}} = \dfrac{IaB}{2} \otimes$

Multiple Correct Answers Type

1. (1), (2)

If electric field is parallel to magnetic field, the charged particle moving parallel to $\vec{E}$ experiences a force in the direction of $\vec{E}$; due to $\vec{B}$ there will not be any force.

Hence, no deflection.

The particle may go undeflected in the case when forces due to electric field and magnetic field balance each other.

2. (1), (2)

If a charged particle goes unaccelerated in a region containing electric and magnetic fields,

$$|q\vec{E}| = |q\vec{v} \times \vec{B}| \Rightarrow |\vec{E}| = |\vec{v} \times \vec{B}| \Rightarrow E = vB\sin\theta$$

3. (1), (2)

The pairs $\vec{F}, \vec{v}$ and $\vec{F}, \vec{B}$ are always at right angle to each other, because $\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $\vec{v}$. Vectors $\vec{B}$ and $\vec{v}$ may have any angle between them.

4. (1), (2), (3), (4)

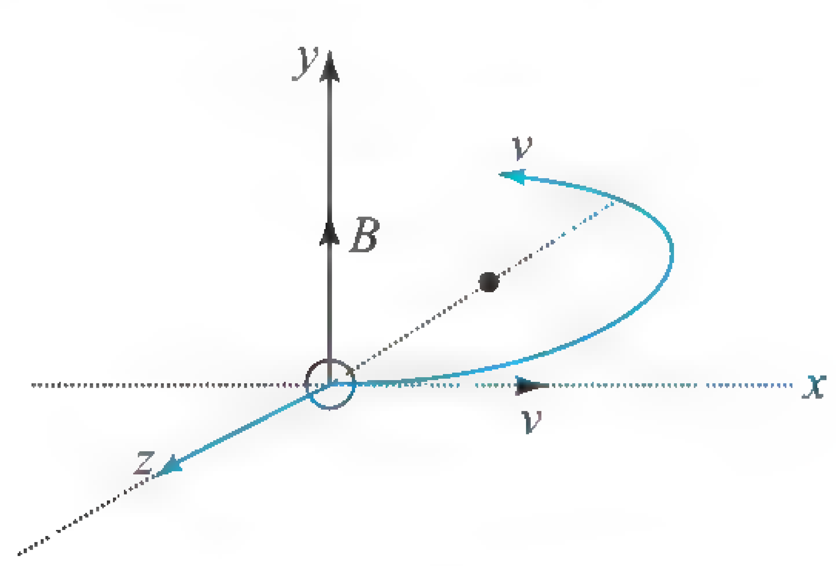
Options (1) and (2) are theoretical facts. As in case of moving charged particle in magnetic field $\vec{F}_{mag} \perp \vec{v}$, hence power associated will be zero (option (3) is correct).

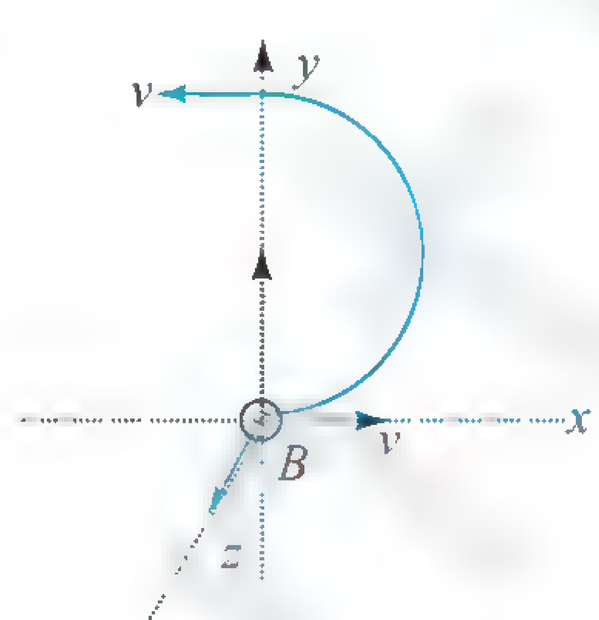
If both the electric and magnetic fields exist: If $\vec{B} \parallel \vec{E}$, the path of the charged particle will be helical.

If $\vec{B}$ is not parallel to $\vec{E}$, the radius of the charged particle will not be constant. Hence, the path will not be circular (option (4) is correct).

5. (1), (2)

Use right hand rule or $\vec{F} = q\vec{v} \times \vec{B}$ for explanation.



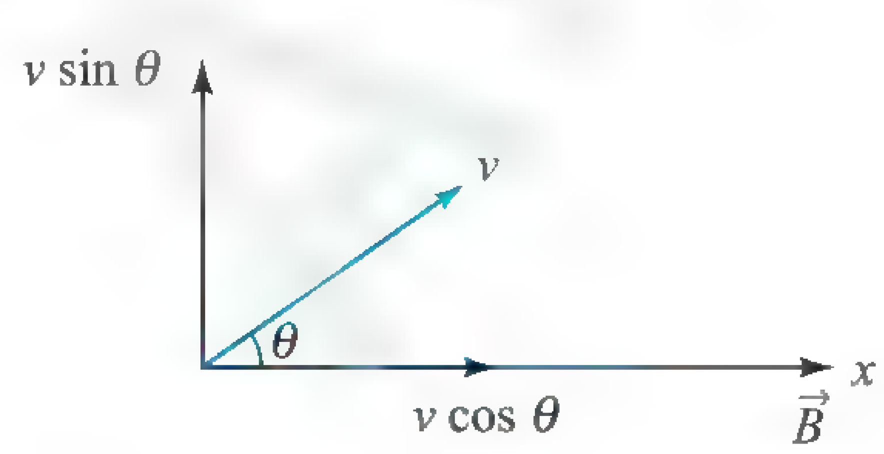


If we apply magnetic field along y-direction, the circular path in x–z plane, the particle can move parallel to magnetic direction.

If we apply the magnetic field in z direction, the plane of circular path will be x–y plane.

6. (1), (4)

When the particle moves in a helical path, the plane of the helical path will be slightly inclined with x-axis. Hence, options (1) and (4) are correct.



7. (1), (2), (3)

Pitch $= \left(\dfrac{2\pi m}{QB}\right) v\cos\theta$ and radius $r = \dfrac{mv\sin\theta}{QB}$

Maximum distance of the particle from the x-axis $= 2r$

$$\therefore \quad \left(\frac{2\pi m}{QB}\right) v\cos\theta = 2\frac{mv}{QB}\sin\theta$$

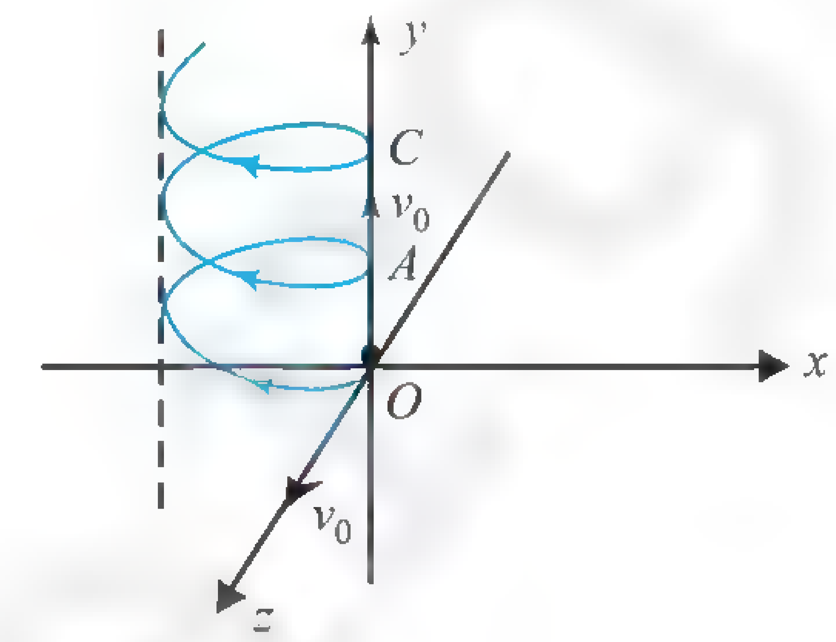
or $\quad \tan\theta = \pi$

Hence, the options (1), (2) and (3) are not correct.

8. (2), (4)

The path of the particle will be helix as shown in figure. Clearly x-coordinate is always negative. z-coordinate can be negative and positive both. x and z coordinate will be zero at the same time at points A, C etc.

$$y = v_0 t \Rightarrow y \propto t$$



9. (1), (3), (4)

As $\vec{F} \perp \vec{v}$, so $\vec{F} \cdot \vec{v} = 0 \Rightarrow m\vec{a} \cdot \vec{v} = 0$

$$\Rightarrow \quad m(2\hat{i} + x\hat{j}) \cdot (3\hat{i} + 4\hat{j}) = 0$$

$$\Rightarrow \quad 6 + 4x = 0 \Rightarrow x = -1.5$$

Let the magnetic field is $\vec{B} = a\hat{i} + b\hat{j} + c\hat{k}$, then from $\vec{F} = q\vec{v} \times \vec{B}$

$$\Rightarrow \quad m(2\hat{i} + x\hat{j}) = q(3\hat{i} + 4\hat{j}) \times (a\hat{i} + b\hat{j} + c\hat{k})$$

$$\Rightarrow \quad 2m\hat{i} - 1.5m\hat{j} = q(3b\hat{k} - 3c\hat{j} - 4a\hat{k} + 4c\hat{i})$$

$$4c = 2m \Rightarrow c \neq 0$$

$$3b - 4a = 0 \Rightarrow b = \frac{4}{3}a$$

a and b both may or may not be zero.

10. (1), (2), (3)

If a charge particle experiences no electromagnetic force, then electric field must be zero (as $\vec{F} = q\vec{E}$). The magnetic field may or may not be zero as a charge particle at rest or moving in the direction of magnetic field experiences no force.

1. When both $\vec{E}$ and $\vec{B}$ are 0, it is undeflected.
2. If $\vec{E}$ is non-zero, it will move in the direction of E and remains undeflected.
3. $\vec{E}$ is non-zero, force will be in direction of E. B is non-zero and V is parallel to $\vec{B}$.
4. $\vec{F} = q\vec{v} \times \vec{B}$ will become zero due to $\vec{B}$ and proton will move in a direction of $\vec{E}$ only.

11. (2), (3), (4)

$\vec{\tau} = \vec{M} \times \vec{B}$ and $U = -\vec{M} \cdot \vec{B}$

Here, $\vec{M}$ and $\vec{B}$ are anti-parallel.

$\therefore \quad \vec{\tau} = \vec{O}$ and $U = +MB$ (maximum)

12. (1), (2)

Let d = distance of the target T from the point of projection. P will strike T if d is an integral multiple of the pitch.

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right) v\cos\theta$$

$$= k\left(\frac{v}{B}\right)$$

where k = constant

Initially, pitch: $d = k\left(\dfrac{v_0}{B_0}\right)$

For the given options if pitch $d' = \dfrac{d}{2}, \dfrac{d}{3}, \dfrac{d}{4}, \ldots$, etc., then charge will hit the target.

13. (1), (3)

In the absence of a magnetic field, the particle will experience gravitational force mg. As a result, the particle will not continue moving in the horizontal direction but will describe a parabolic path. So, a magnetic field must be present and its direction must be perpendicular to the direction of the velocity. The magnetic force experienced by the particle is given by $\vec{F} = q(\vec{v} \times \vec{B})$.

The magnitude of the force is $F = qvB \sin\theta$. If the particle is to move in the horizontal direction, this force must balance the force of gravity, i.e., $mg = qvB \sin\theta$

The minimum value of B corresponds to $\sin\theta = 1$ or $\theta = 90°$. Thus, $mg = qvB$

or $\quad B = \dfrac{mg}{qv} = \dfrac{0.5\times10^{-3}\times9.8}{2.5\times10^{-8}\times6\times10^{4}} = 3.27\text{ T}$

Hence, the correct options are (1) and (3).

14. (1), (2), (3)

To find the Ampere's force on a conductor of any shape, replace the conductor by an imaginary straight conductor joining the two ends of the given conductor. So, if B is in x-direction, then the imaginary straight conductor will be along the field and the force acting on it will be zero. If B is in y direction, then the force will be λBI acting along the z direction. Similarly, if B is in the z direction, then the force will be λBI, acting along the negative y direction.

15. (1), (4)

The magnitude of the magnetic field depends only on the distance from the x-axis. Points A and C are at distances of 1 unit each from the x-axis. Points B and D are at distances of $\sqrt{2}$ unit each from the x-axis. Magnetic field at point D,

$$B = \frac{\mu_0 I}{2\sqrt{2}\pi}$$

It is obvious that field B is inclined at an angle of 45° with the x–y plane.

16. (1), (3)

1. Velocity of the particle can be constant if the electric force balances the magnetic force, i.e., $q\vec{E} = q\vec{v}\times\vec{B}$.
2. If $E = 0$, the particle can trace circular path if $v \perp B$.
3. If $E = 0$, kinetic energy remains constant, because magnetic force is not doing any work.

17. (1), (2), (4)

$$|\vec{v}| = \sqrt{8^2+6^2} = 10\text{ m s}^{-1}$$

Radius of circular path $r = \dfrac{mv}{qB} = \dfrac{10}{2} = 5\text{ m}$

As the magnetic field is in z-direction, hence the path of the charged particle will be circular of radius 5 m in x-y plane; hence options (1) and (2) may be possible.

Time period of circular path $T = \dfrac{2\pi r}{v} = \dfrac{2\pi\times5}{10} = \pi$ seconds

18. (2), (3)

$\vec{V} \perp \vec{B}$

Therefore, path of the particle is a circle. In magnetic field, speed of the particle remains constant. Therefore, distance moved by the particle in time $t = \dfrac{\pi}{B_0\alpha}$ is $v_0 t$ or $\dfrac{\pi v_0}{B_0\alpha}$.

Magnitude of velocity is always v_0, hence option (4) is incorrect.

19. (1), (2), (3), (4)

Option (2) is obvious

$$r = \frac{mv_0}{qB}$$

$$PQ = 2r\sin\alpha = 2\frac{mv_0}{qB}\sin\alpha$$

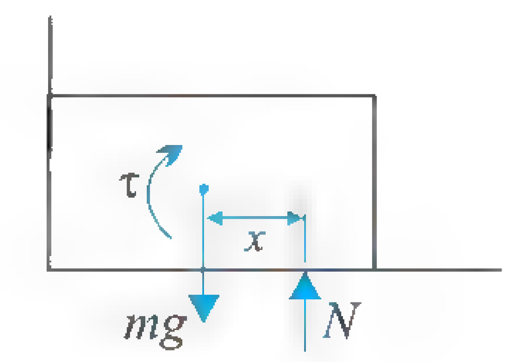
$$\alpha = \beta$$

Time taken $= T = \dfrac{2\pi r}{v}$

$$T = \frac{2\pi m}{qB}$$

For t time, $t = \dfrac{T}{2\pi}(2\pi - 2\alpha) = \dfrac{2m}{qB}(\pi - \alpha)$

20. (2), (4)



Torque of magnetic field: $\tau = MB_0 = Ia^2B_0$

For rotational equilibrium: $\tau = mgx$

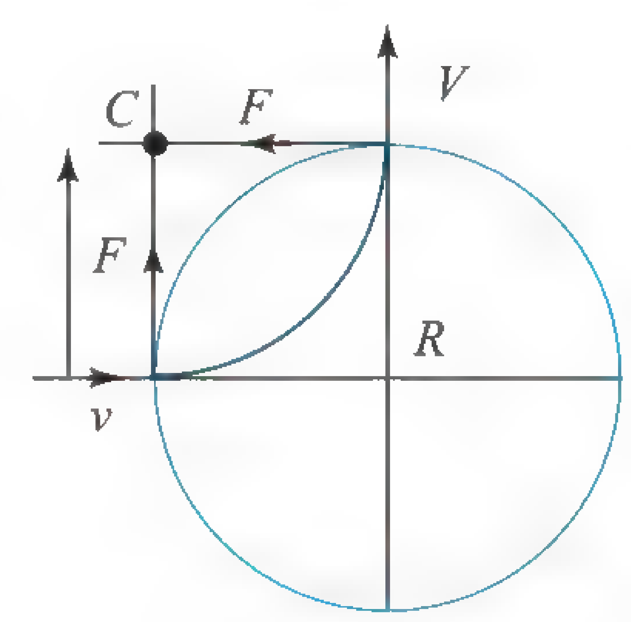
$\Rightarrow \quad Ia^2B_0 = mgx$

$\Rightarrow \quad x = \dfrac{Ia^2B_0}{mg}$

For the block not to topple:

$$x < \frac{a}{2} \Rightarrow I < \frac{mg}{2aB_0}$$

21. (1), (2), (3)



The particle will move along an arc which is part of a circle of radius:

$$r = \frac{mv}{Bq}$$

From figure, we can see that $r = R$

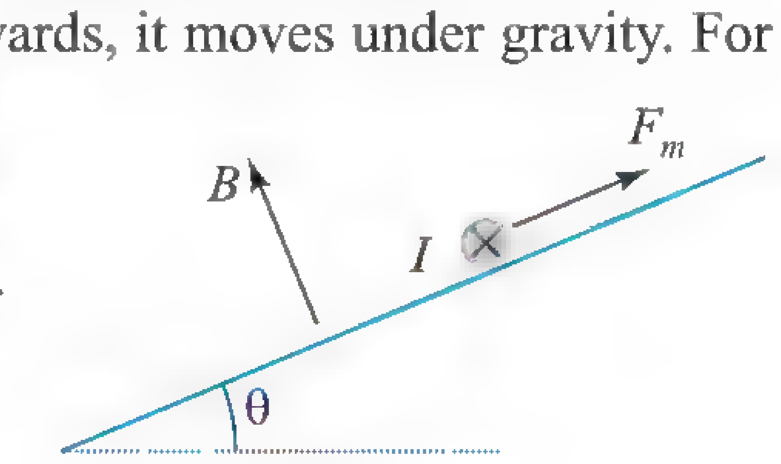
$\therefore \quad R = \dfrac{mv}{Bq}$

$$T = \frac{\pi r/2}{v} = \frac{\pi r}{2v} = \frac{\pi R}{2v} \qquad \left(\because\ r = R = \frac{mv}{Bq}\right)$$

$\therefore \quad T = \dfrac{\pi m}{2Bq}$

22. (1), (2)

When a charge is passed through a wire, a current flows in the wire (for a very small time). As a result of this, the wire experiences a magnetic force for a very small duration, i.e, during the time for which charge has been passed through the wire, due to which it acquires some velocity; then onwards, it moves under gravity. For the wire to move up the incline, the positive charge has to pass through the wire in a direction coinciding with into the plane of paper.



$$F_m = IBl = \frac{dq}{dt}\times B\ell = ma$$

[For this small duration dt, we can neglect gravitational force because I would be very large due to small passage time of charge.]

$\Rightarrow \quad m\dfrac{dv}{dt} = \dfrac{dq}{dt}\times Bl \quad \Rightarrow \quad m\,dv = dq\times B\ell$

$\Rightarrow \quad mv = q\times Bl$

$\Rightarrow \quad v = \dfrac{q\times Bl}{m} \quad \Rightarrow \quad q = \dfrac{mv}{Bl}$

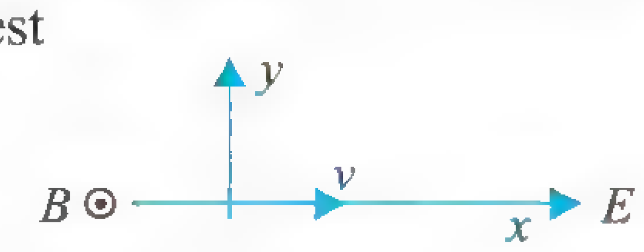
where q is the charge passed through the wire and v is the velocity acquired by the wire just after charge had been passed through it.

From kinematics, $v^2 = 2g \sin\theta\, s$

$$q = \frac{m\sqrt{2gs\sin\theta}}{Bl}$$

23. (3), (4)

(1) If $v = 0$, $E = 0$ then no force will act on particle, hence particle remains at rest



(2) here $v = 0$, $E \neq 0$, due to E the particle will gain velocity in x direction. Due to this velocity it will experience force due to magnetic filed along negative y axis. As a result of it, the particle will move in curved path that will not be a circle

(3) If $v = 0$, $E \neq 0$, then particle will execute cycloidal motion. See theory for this.

(4) If $E = 0$, then path will be circular.

24. (1), (2), (3), (4)

Net force on charge should be zero in region 1,

$$qE = qvB \quad \Rightarrow \quad v = E/B$$

In region 1, net force is zero on particle, hence on work is done on particle in region 1.

In region 2, only magnetic force is there, hence no work is done in this region also.

In region 2: $r = \dfrac{mv}{qB_0} = \dfrac{E/B}{(q/m)B_0} = \dfrac{E}{sBB_0}$

If $l_2 > r$ or $l_2 > \dfrac{E}{sBB_0}$, then particle will complete half revolution in region 2 and come out of it with same velocity in opposite direction.

25. (2), (4)

Time period of motion is given as $T = \dfrac{2\pi m}{Bq} = \dfrac{2\pi}{\alpha B_0}$

At $t = \dfrac{\pi}{\alpha B_0} = \dfrac{T}{2}$

Velocity of particle is $-v_0\hat{i} + v_0\hat{k}$

Speed of charge in magnetic field always remains constant so it will be

$$v = v_0\sqrt{2}$$

At $\quad t = \dfrac{2\pi}{\alpha B_0} = T$

Displacement is equal to pitch which is given as

$$\Delta x = v_0 T = \frac{2\pi v_0}{\alpha B_0}$$

At $t = \dfrac{2\pi}{\alpha B_0} = T$

Distance is given as $\Delta x = v \times T = \dfrac{2\sqrt{2}v_0\pi}{\alpha B_0}$

Linked Comprehension Type

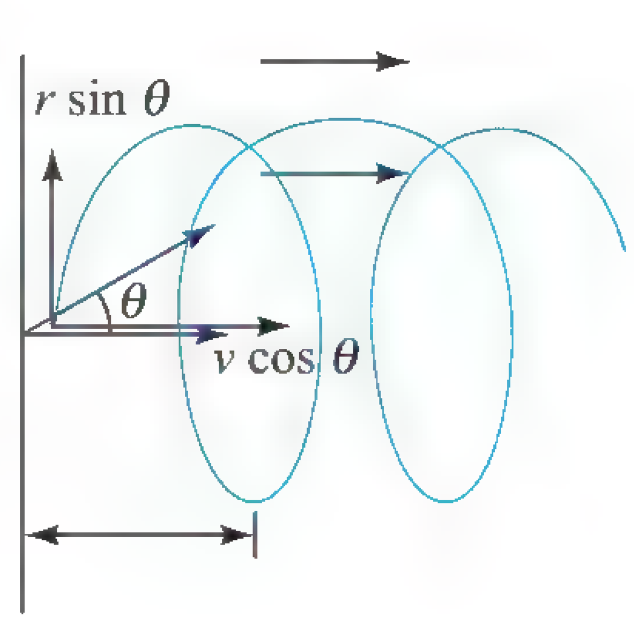
For Problems 1–3

1. (2) **2. (1)** **3. (3)**

The first particle will have a helical path and the second particle will move rectilinearly along the field. For the two particles to meet again and again, $v_{||}T = v'T$

where v' is the speed of the second particle.

$\therefore \quad v' = v_{||} = v\cos\theta$



$$\frac{1}{2}mv^2 = qV \quad \Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$\therefore \quad v' = \sqrt{\frac{2qV}{m}}\cos\theta$$

$$T = \frac{2\pi m}{qB}$$

Distance travelled = pitch

$$\text{Distance} = \sqrt{\frac{2qV}{m}}\cos\theta \times \frac{2\pi m}{qB}$$

$$\text{Distance} = \sqrt{\frac{2Vm}{q}}\,\frac{2\pi}{B}\cos\theta$$

For Problems 4–5

4. (3) **5. (1)**

For the first case: $\vec{F} = q\vec{v} \times \vec{B}$

$\Rightarrow \quad -5\sqrt{2} \times 10^{-3}\,\hat{k}$

$$= 10^{-5} \times \frac{10^6}{\sqrt{2}}\left(\hat{i} + \hat{j}\right) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= \left(\frac{10}{\sqrt{2}}\right)\left[B_z\hat{i} - B_z\hat{j} + \left(B_y - B_x\right)\hat{k}\right]$$

$\Rightarrow \quad B_z = 0,\ B_y - B_x = -10^{-3}\ \text{T} \qquad \text{...(i)}$

Similarly, for the second case:

$$F_2\hat{j} = (10^{-5})(10^6\,\hat{k}) \times \left[\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\right]$$

$$F_2\hat{j} = 10\left(B_x\hat{j} - B_y\hat{i}\right)$$

$F_2 = 10B_x,\ B_y = 0 \qquad \text{...(ii)}$

Using eqs. (i) and (ii), we get $B_x = 10^{-3}$ T

Thus, $\vec{B} = (10^{-3}\ \text{T})\,\hat{i}$

Also, $F_2 = 10B_x = 10^{-2}$ N.

For Problems 6–7

6. (1) **7. (4)**

Particle's acceleration is in y direction,

$$\frac{dv_y}{dt} = \frac{qE}{m} = \text{constant}$$

The motion of the particle is equivalent to circular motion in xz plane with uniform acceleration in y-direction.

Hence, $v_0^2 = v_x^2 + v_z^2 = \text{constant}$

The magnetic force cannot change the magnitude of v_0.

The y-component of velocity,

$$v_y = \frac{qE}{m}t$$

The y-coordinate at time t, $y = \dfrac{1}{2}\dfrac{qE}{m}t^2$

The time period of circular motion in xz plane,

$$T = \frac{2\pi m}{Bq}$$

Let the particle cross y-axis after n rotations, then

$$t = nT = \frac{2\pi mn}{qB}$$

$$\text{Thus, } y_n = \frac{qE}{2m} \times \left(\frac{2\pi mn}{qB}\right)^2 = \frac{2\pi^2 mn^2 E}{qB^2}$$

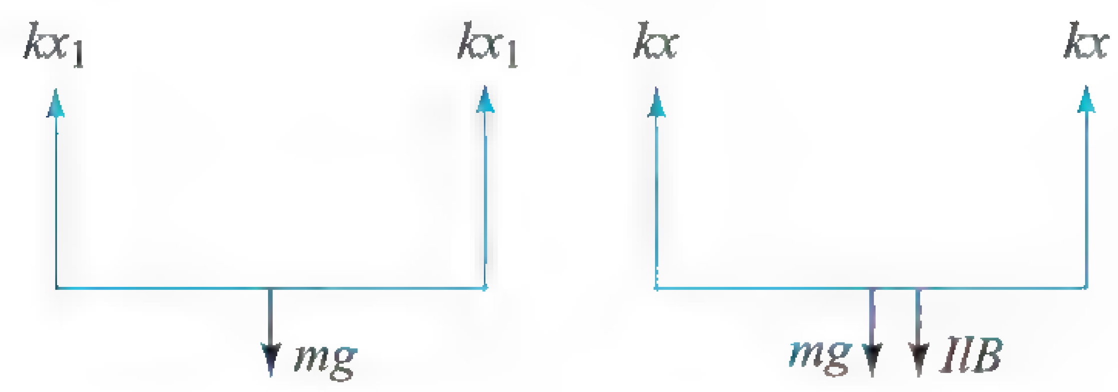
$$\text{As} \quad v_y = a_y t = \left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right) = \frac{2\pi nE}{B}$$

$$\text{Thus, } \tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi nE} \quad \Rightarrow \quad \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$$

For Problems 8–9

8. (2) **9. (3)**

$$I = \frac{\varepsilon}{R} = \frac{24}{12} = 2\text{A}$$



$$\text{Magnetic force} = IlB\sin\frac{\pi}{2} = 2 \times 5 \times 10^{-2} \times B = \frac{B}{10}$$

$$2kx_1 = mg;\ 2k = \frac{mg}{x_1} \quad \text{and}$$

$$2k(x_1 + x_2) = mg + IlB$$

$$2kx_1 + 2kx_2 = mg + IlB$$

$$\frac{mg}{x_1} x_2 = IlB$$

$$10 \times 10^{-3} \times 10 \times \frac{0.3}{0.5} = \frac{B}{10} \quad \Rightarrow \quad B = 600 \times 10^{-3} = 0.6\ \text{T}$$

For Problems 10–11

10. (1) **11. (1)**

(a) $F_1 = mg$

When bar is just ready to levitate,

$$IlB = mg,\ I = \frac{mg}{lB} = \frac{(0.750\ \text{kg})(9.80\ \text{m/s}^2)}{(0.500\ \text{m})(0.450\ \text{T})} = 32.67\ \text{A}$$

$\varepsilon = IR = (32.67\ \text{A})\ (25.0\ \Omega) = 817\ \text{V}$

(b) $R = 2.0\ \Omega$, $I = \varepsilon/R = (817\ \text{V})/(2.0\ \Omega) = 408.5\ \text{A}$

$F_1 = IlB = 92\ \text{N}$

$a = (F_1 - mg)/a = 113\ \text{ms}^{-2}$

For Problems 12–13

12. (1) **13. (4)**

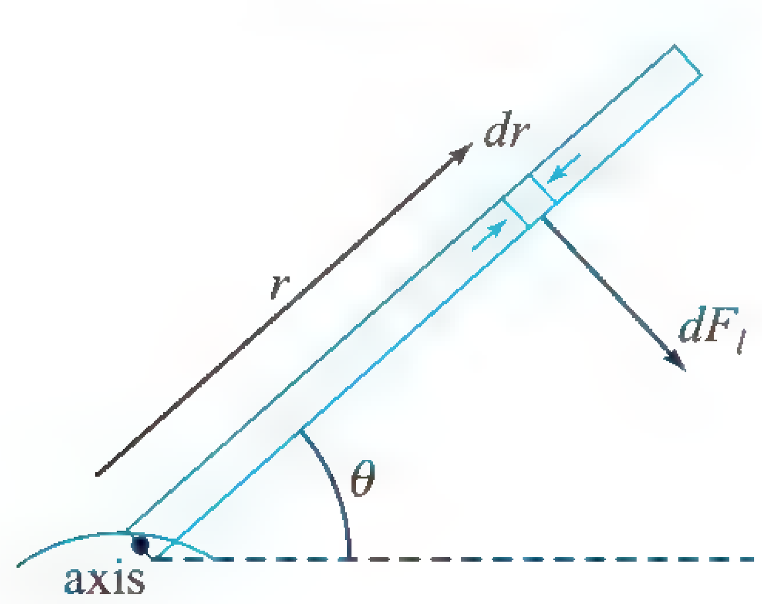
$F = ILB$, to the right.

$$v^2 = 2ad \quad \Rightarrow \quad d = \frac{v^2}{2a} = \frac{v^2 m}{2ILB}$$

For Problems 14–15

14. (1) **15. (1)**

By examining a small piece of the wire (figure), we find:



Divide the rod into infinitesimal sections of length dr.

The magnetic force on this section is $dF_1 = IB\,dr$ and is perpendicular to the rod. The torque dt due to force on this section is $d\tau = rd\,F_1 = IBr\,dr$

The total torque is

$$\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\ \text{N m}^{-1},$$

clockwise. This is the same torque calculated from a force diagram in which the total magnetic force $F_l = ILB$ acts at the center of the rod.

F_1 produces a clockwise torque, so the spring force must produce a counterclockwise torque. The spring force must be to the left, the spring is stretched.

Find x, the amount the spring is stretched:

axis at hinge, counterclockwise torques positive

$$(kx)l\sin 53° - \frac{1}{2}Il^2B = 0$$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50\ \text{A})(0.200\ \text{m})(0.340\ \text{T})}{2(4.80\ \text{Nm}^{-1})\sin 53.0°}$$

$$= 0.05765\ \text{m}$$

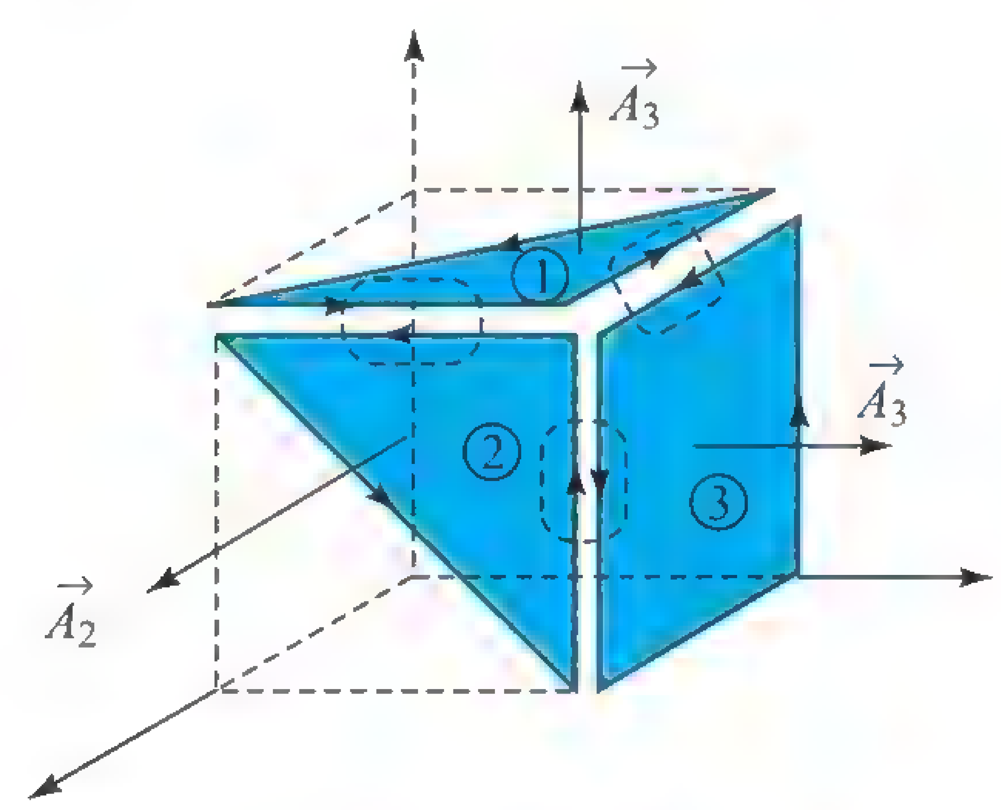
For Problems 16–18

16. (1) **17. (2)** **18. (3)**

The net force on a current carrying loop of any arbitrary shape in a uniform magnetic field is zero.

$$\vec{F}_{\text{net}} = 0$$

The given loop can be considered to be a superposition of three loops as shown in figure. The area vector of the three loops (1), (2) and (3) are



$$\vec{A}_1 = \left(\frac{1}{2} \times 10 \times 10 \times 10^{-4}\right)\hat{j}\ \text{m}^2$$

$$\vec{A}_2 = \left(\frac{1}{2} \times 10 \times 10 \times 10^{-4}\right)\hat{k}\ \text{m}^2$$

$$\vec{A}_3 = \left(10 \times 10 \times 10^{-4}\right)\hat{i}\ \text{m}^2$$

Magnetic moment vector,

$$\vec{\mu} = i\vec{A} = 10(0.01\hat{i} + 0.005\hat{j} + 0.005\hat{k})\ \text{Am}^2$$

$$= (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})\ \text{Am}^2$$

Torque,

$$\vec{\tau} = (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})$$

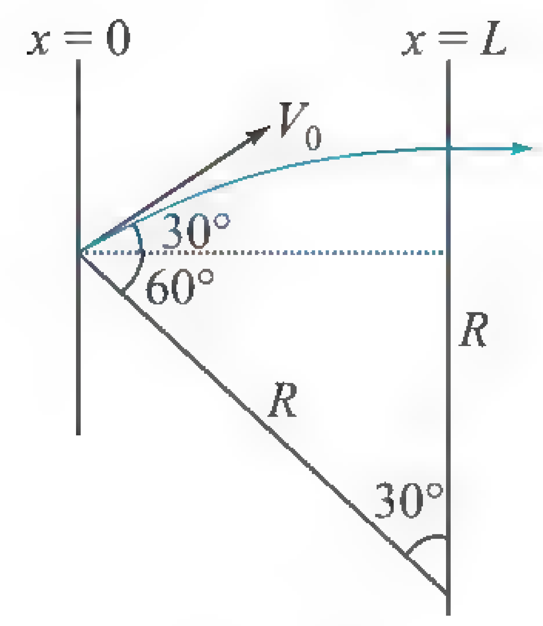
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1 & 0.05 & 0.05 \\ 2 & -3 & 1 \end{vmatrix} = 0.2\hat{i} - 0.4\hat{k}\ \text{Nm}$$

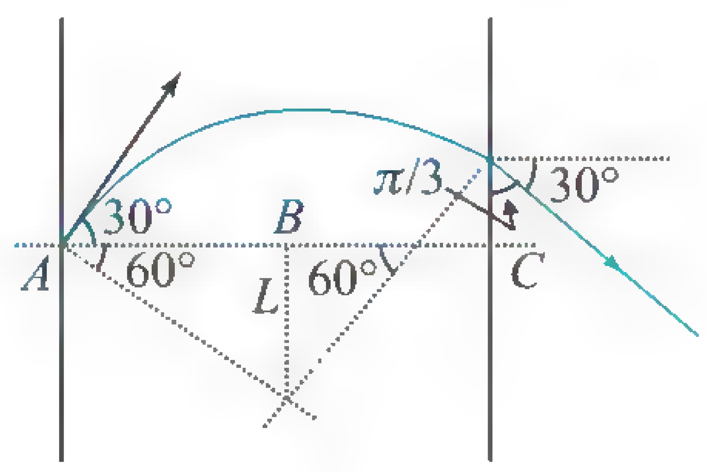
For Problems 19–20

19. (2) **20. (4)**

$\sin 30° = L/R$

$$L = \frac{mV_0}{2qB} = \frac{50 \times 10^{-3} \times 100}{2 \times 1 \times 25} = \frac{1}{10}\ \text{m} = 10\ \text{cm}$$

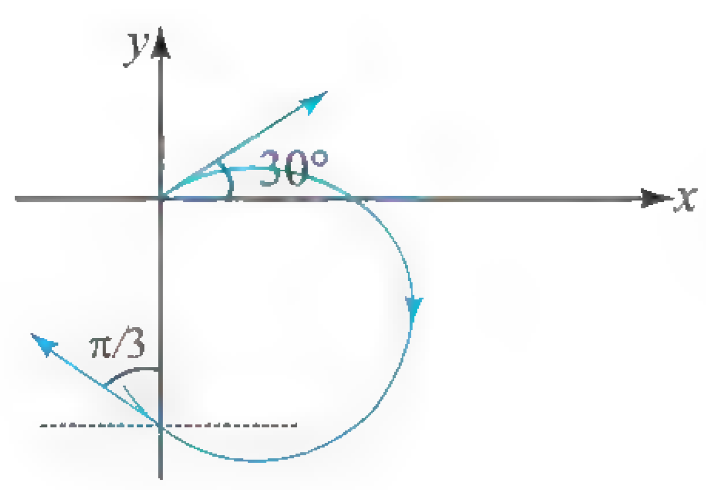




$L = AB + BC$

$= r\cos 60° + r\cos 60° = r$

$$L = \frac{mV_0}{qB} = 20\ \text{cm}$$



⇒ This is minimum L.

For other case (figure):

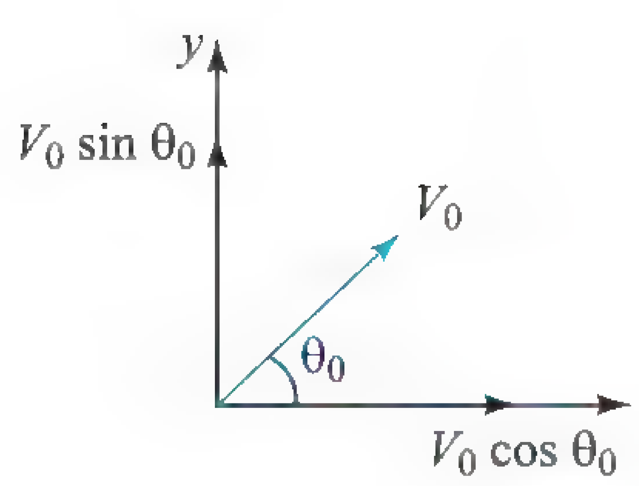
For this case L can extend upto ∞

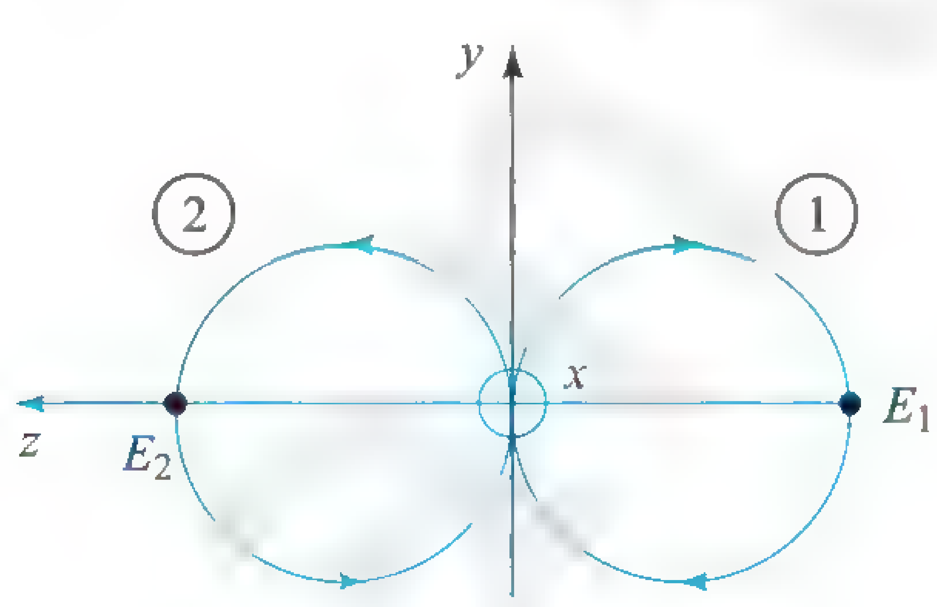
For Problems 21–22

21. (1), (2) **22. (2)**

As the magnetic field is along the x-axis, the magnetic force will be along (–) z-axis at $t = 0$. So, the particle will move in helical path along (1). At $t = T_0$, the direction of field changes, so force becomes along + z-direction, and now the particle will move in helical path along (2). It will be moving along x-axis, so that resulting path will be helical.

At $t = \dfrac{T_0}{2}$, particle will be at E_1.





x-coordinate $= \dfrac{P_0}{2}$ (half of pitch)

y-coordinate = 0 (from figure)

and z-coordinate $= -2R_0$ (from figure)

Hence, (1) is correct.

Similarly at $t = \dfrac{3T_0}{2}$ particle will be at E_2.

∴ The coordinates are $\left(\dfrac{3P_0}{2}, 0, 2R_0\right)$

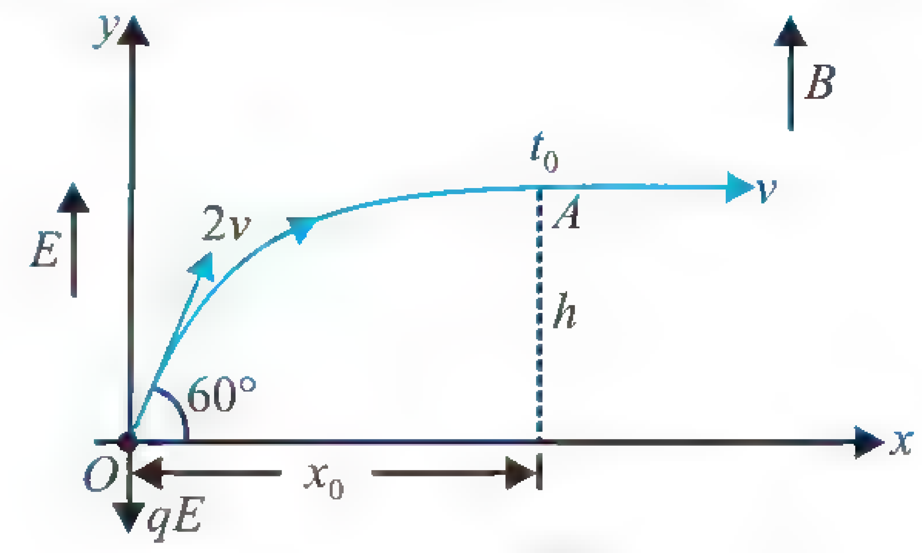
Hence, (2) is correct.

From figure, we can see that distance between two extremes: E_1 and E_2 is $4R_0$.

For Problems 23–25

23. (1) **24. (2)** **25. (3)**

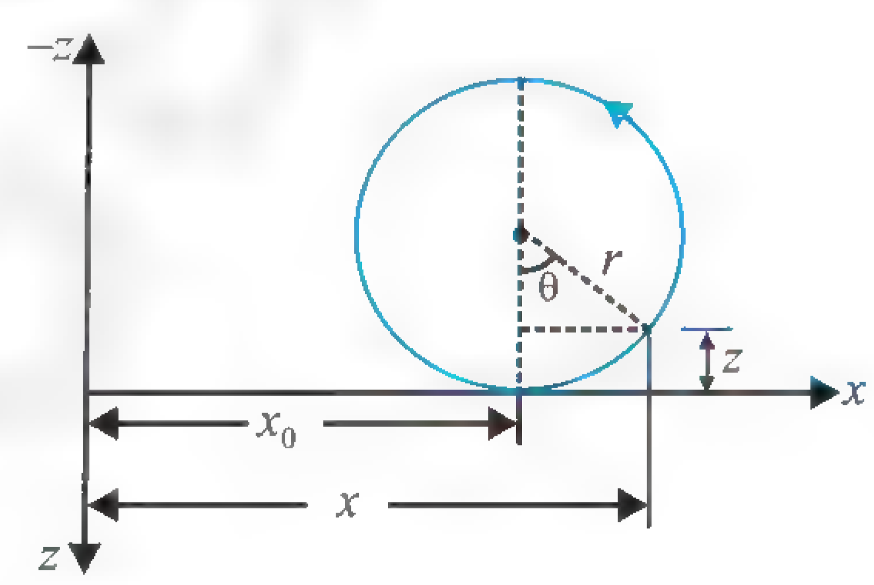
First particle will travel along parabolic path OA. Let time from O to A is t. $a_y = \dfrac{-qE}{m}$



$$x = \frac{\sqrt{3}\,mv^2}{qE} = (2v\cos 60°)\,t_0 \quad\Rightarrow\quad t_0 = \frac{\sqrt{3}\,mv}{qE}$$

$$v_y = u_y + a_y t_0 = 2v\sin 60° - \frac{qE}{m}\frac{\sqrt{3}\,mv}{qE} = 0$$

Hence at point A, velocity will be purely along x-axis and it will be $2v\cos 60° = v$. Now magnetic field is switched on along y-axis. Now its path will be helical as shown below with increasing pitch towards negative y-axis.



$$r = \frac{mv}{qB}$$

$$x = x_0 + r\sin\theta = (2v\cos 60°)t_0 + \frac{mv}{qB}\sin\omega t$$

$$= v\sqrt{3}\frac{mv}{qE} + \frac{mv}{qB}\sin\left(\frac{qB}{m}t\right)$$

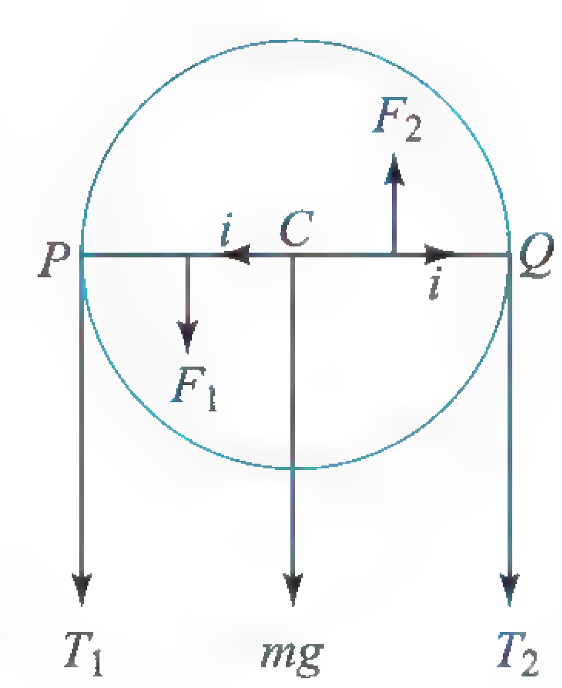
z-coordinate: $z = -(r - r\cos\theta) = -\dfrac{mv}{qB}\left[1 - \cos\left(\dfrac{qB}{m}t\right)\right]$

For Problems 26–28

26. (1) **27. (2)** **28. (2)**

PC and CQ are in Parallel. So, equivalent resistance of the circuit is $R/2$. In static condition, current through the battery is

$I = 2V/R$.



The current through CP and CQ is

$i = I/2 = V/R$

Hence, $F_1 = F_2 = Bir$

Consider the FBD of ring and the blocks

FBD of ring: Consider torque about 'C',

$T_1 r + F_1 r/2 + F_2 r/2 = T_2 r$ ...(i)

FBD of blocks: $T_1 = mg$ and $T_2 = 2mg$

So, $mgr + Bir^2/2 + Bir^2/2 = 2mgr$

or $i = \dfrac{mg}{Br}$ or $\dfrac{V}{R} = \dfrac{mg}{Br}$ or $V = \dfrac{mgR}{Br}$

Net torque of tension

$$= (T_2 - T_1)r = mgr = \frac{BVr^2}{R}$$

For Problems 29–31

29. (1) Angle remains constant, because magnitudes of components of velocities along and perpendicular to magnetic field remain constant.

30. **(2)** $f = \dfrac{qB}{2\pi m} = \dfrac{10^3}{19} \times \dfrac{\sqrt{3800}}{2\pi} = \dfrac{10^4}{\pi\sqrt{38}}$

31. **(4)** Pitch $= TV\cos\theta$

$$= \frac{1}{f}V\frac{\vec{V}\vec{B}}{BV} = \frac{1}{f}\frac{\vec{V}\vec{B}}{B} = \frac{\pi\sqrt{38}\times 400}{10^4\sqrt{3800}} = \frac{4\pi}{10^3}\text{ m} = \frac{\pi}{250}\text{ m}$$

For Problems 32–34

32. **(4)** At speed $v_0 = F_e = F_m$

$$qE = qv_0B \quad \Rightarrow \quad v_0 = \frac{E}{B}$$

Hence (4) is correct option.

33. **(3)** Greater the distance d, larger would be the deviation produced in the path. Hence the particle will be unable to enter the detector for greater distance d, thus narrowing the range of speeds detected.

34. **(1)** For $v = 0,\ a = \dfrac{qE}{m}$

For $v = v_0,\ a = \dfrac{qE - qvB}{m}$

or $\quad a = \dfrac{-qB}{m}v + \dfrac{qE}{m} \quad (y = -mx + c)$

For $\quad v = v_0, a = 0$

For $\quad v > v_0\ a = \dfrac{qBv}{m} - \dfrac{qE}{m} \quad (y = mx - c)$

Hence graph (1) is correct

For Problems 35–37

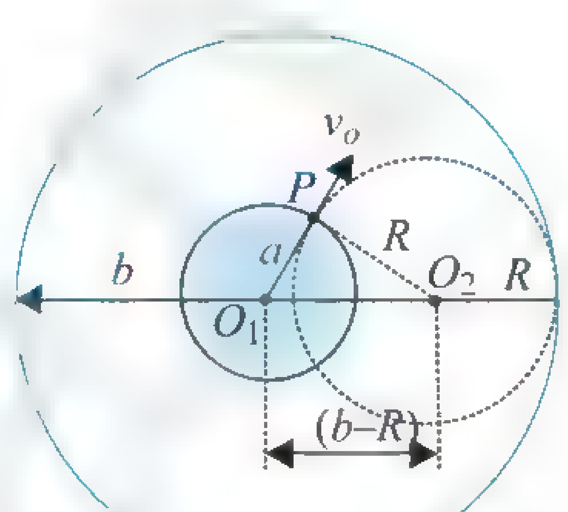
35. **(1)** In the absence of magnetic field, the electron moves in a straight hit to the anode due to the presence of electrostatic force.

Hence choice (1) is correct.

36. **(2)** When $V = 0$, the electron moves in homogeneous static field B. The magnetic field acts orthogonal to the velocity and hence electron moves in a circle.

Hence choice (2) is correct.

37. **(3)** From the figure, we see that in critical core the radius R of the circle satisfies



$$\sqrt{a^2 + R^2} = b - R$$

or $\quad a^2 + R^2 = b^2 + R^2 - 2bR$

or $\quad R = \left(\dfrac{b^2 - a^2}{2b}\right) \quad$ ...(i)

The radius R of the circular path is determined by equating centripetal force and Lorentz force

$$eB_cv_0 = \frac{mv_0^2}{R} \quad \Rightarrow \quad B_c = \frac{mv_0}{eR}$$

Using equation (i)

$$B_C = \frac{2mbv_0}{e(b^2 - a^2)}$$

Hence choice (3) is correct.

For Problems 38–40

38. **(2)** Velocity at A $v = \sqrt{2as}$

$$= \sqrt{2\times\left(\frac{qE}{m}\right)s} = \sqrt{\frac{2\times 1\times 10\times 1.8}{1}} = 6\text{m/s}$$

In magnetic field, speed does not change. Hence particle will collide with speed 6 m/s

39. **(4)** In magnetic field path of the particle is circle. Radius of circular path is

$$r = \frac{mv}{qB} = \frac{(1)(6)}{(1)(5)} = 1.2\text{ m}$$

$$d = 2.4\text{ m} - 1.8\text{ m} = 0.6\text{ m}$$

Since $\ d < r\sin\theta = \dfrac{d}{r} = \dfrac{0.6}{1.2} = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 30°$

$AE = AD - DE = r - r\cos\theta$

$$= r(1 - \cos\theta) = 1.2\left(1 - \frac{\sqrt{3}}{2}\right) = 0.6(2 - \sqrt{3})$$

$FC = BF\tan\theta = \dfrac{0.6}{\sqrt{3}}$

$\therefore \quad$ y-co-ordinate $= AE + FC$

$$= 0.6\left(2 - \sqrt{3}\right) + \frac{0.6}{\sqrt{3}}$$

$$0.6\left[2 - \sqrt{3} + \frac{1}{\sqrt{3}}\right] = \frac{1.2\left(\sqrt{3} - 1\right)}{\sqrt{3}}\text{ m}$$

Hence choice (4) is correct and other choices are wrong.

40. **(1)** Total time $t = t_{OA} + t_{AB} + t_{BC}$

$$t_{OA} = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2sm}{qE}} = \sqrt{\frac{2\times 1.8\times 1}{(1)(10)}}$$

$$= 0.6\sec = 3/5\sec$$

$$t_{AB} = \left(\frac{30°}{360°}\right)T = \frac{1}{12}\times\frac{2\pi m}{qB}$$

$$\frac{(2\pi)(1)}{(12)(1)(5)} = \frac{\pi}{30}\sec$$

$$t_{BC} = \frac{BC}{v} = \frac{0.6\sec\theta}{v} = \frac{0.6\left(\frac{2}{\sqrt{3}}\right)}{6} = \frac{1}{5\sqrt{3}}\sec$$

Hence $\ \dfrac{3}{5} + \dfrac{\pi}{30} + \dfrac{1}{5\sqrt{3}} = \dfrac{1}{5}\left(3 + \dfrac{\pi}{6} + \dfrac{1}{\sqrt{3}}\right)\sec$

Hence choice (1) is correct.

For Problems 41–43

41. **(2)** Since electrons are deflected downward, hence electric field must be upward. The magnetic force in order to cancel the electric force must point upward. From right hand rule, magnetic field must point out of page. Hence choice (2) is correct.

42. **(2)** Since $d_1 = \dfrac{eEL^2}{2mv^2}$ or $\dfrac{e}{m} = \dfrac{2d_1v^2}{EL^2}$

Here $d_1 = 2\times 10^{-3}$ m

$v = 3\times 10^7$ m/s $L = 0.1$m

$E = 1800$ V/m

$$\frac{e}{m} = \frac{2\times(2\times 10^{-3})\times(3\times 10^7)^2}{1800\times(0.1)^2} = 2\times 10^{11}\text{ C/kg}$$

Hence choice (2) is correct.

43. **(1)** In the passage, we have

$$d_1 = \frac{eEL^2}{2mv^2}\text{ and }\frac{e}{m} = \frac{2d_1E}{B^2L^2}$$

By increasing speed of electron to two times, d_1 will become 1/4th. For this magnetic field should be halved, so that we can correctly measure e/m. Hence choice (1) is correct.

Matrix Match Type

1. i. → a., c., d.; ii. → a., c., d.; iii. → a., b.; iv. → a., b.

i. Kinetic energy of the particle can remain constant, if both the fields are present. This is possible if the force due to both fields cancel each other.

Kinetic energy of the particle can also remain constant if only magnetic field is present, because magnetic field does not do any work.

Obviously KE will remain constant if no field is present.

ii. This is possible if either both the fields are present or no field is present. This is also possible if only magnetic field is present and the particle is at rest or moving in the direction of field.

iii. This is possible if electric field is present, magnetic field may or may not be present.

iv. This is possible if only electric field is present and velocity and electric field are along the parallel lines. Magnetic field may also be present if it is parallel to velocity.

2. i.→ a., b., d.; ii. → a., b., c., d.; iii. → a., b., d.; iv. → b.

i. Velocity of the particle may be constant, if forces of electric and magnetic fields balance each other. Then, path of particle will be straight line. Also, path of particle may be helical if magnetic and electric fields are in same direction. But path of particle cannot be circular. Path can be circular if only magnetic field is present, or if some other force is present which can cancel the effect of electric field.

ii. Here, all the possibilities are possible depending upon the combinations of the three fields.

iii. This situation is similar to part (i).

iv. In a uniform electric field, path can be only straight line or parabolic.

3. i. → d.; ii. → a.; iii. → b.; iv. → c.

We know that $r = mv/qB$. So, larger the mass, larger is the radius. Since radius of path '4' is largest, so path '4' should correspond to ion B.

Hence, ii. → a.

Now path '2' should correspond to ion C, because charge on both B and C is negative, so their paths should be on the same side.

Hence, iii. → b.

Now, radii of paths '2' and '3' are same. If path '2' corresponds to C, then path '3' should correspond to ion A, because masses of A and C are same.

Hence, i. → d.

Now, remaining path '1' should correspond to D.

Hence, iv. → c.

4. i. → a., c., d.,; ii. → a., b.; iii → d.; iv. → a.

i. Since $\vec{E} = 0$ and $\vec{B} \neq 0$, so path will be straight line if velocity is parallel to $\vec{B}$. Or path will be circular if $\vec{V} \perp \vec{B}$. Or path will be helical (with uniform pitch) if $\vec{V}$ is at some other angle to $\vec{B}$.

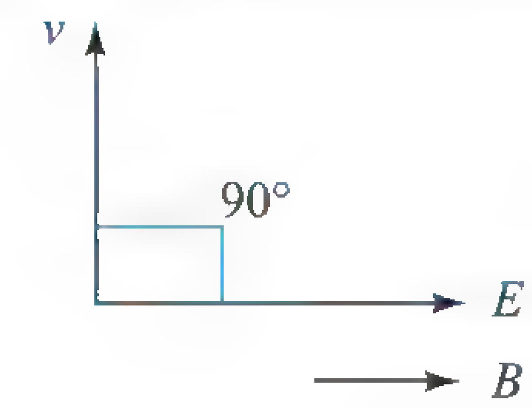
Hence, i. → a, c

ii. Since $\vec{E} \neq 0$ and $\vec{B} = 0$, so path will be straight line if $\vec{v} \parallel \vec{E}$ or parabola otherwise.

Hence, ii. → a, b

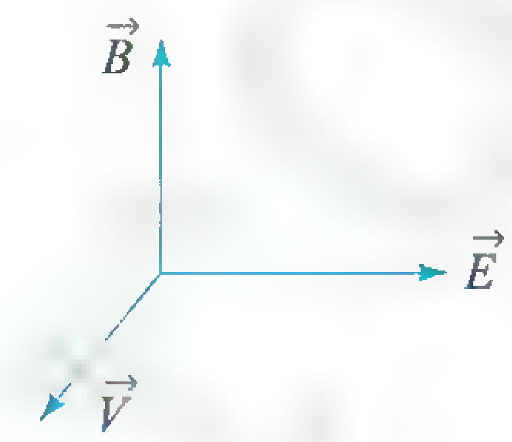
iii. $\vec{E} \neq 0, \vec{B} \neq 0, \vec{E} \parallel \vec{B}$

Helical path with non-uniform pitch



Hence, iii. → d.

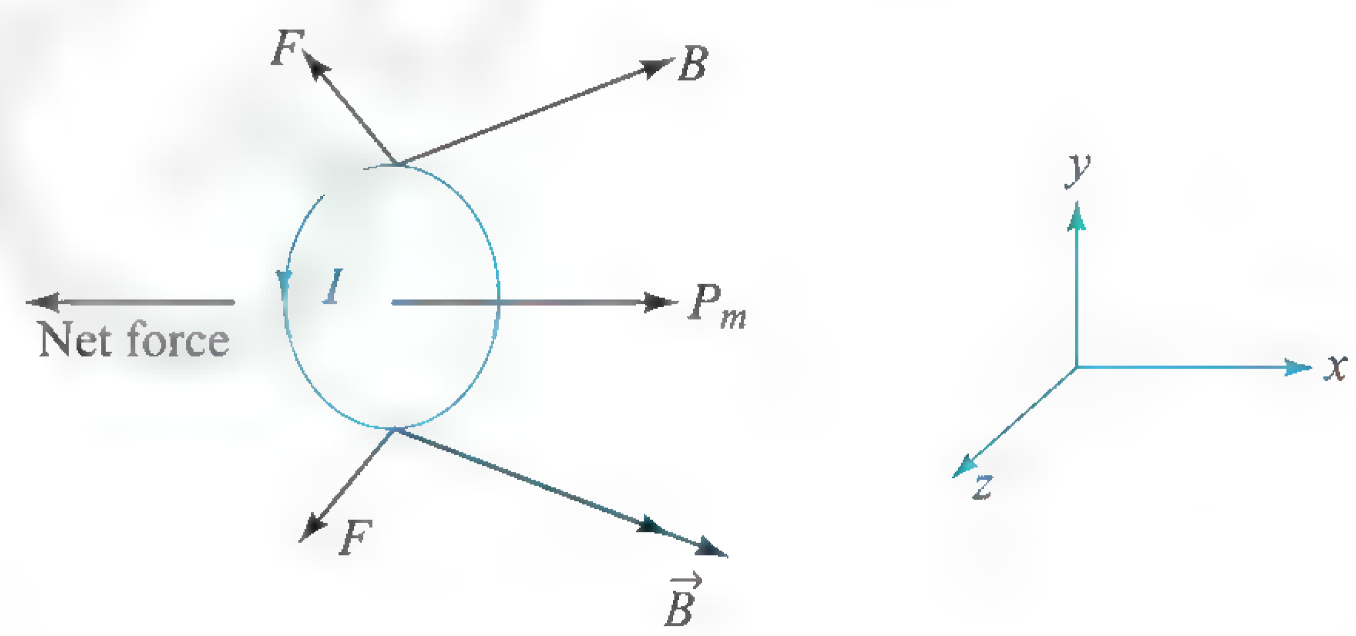
iv. Straight line path if $\vec{V} \times \vec{B} = \vec{E}$



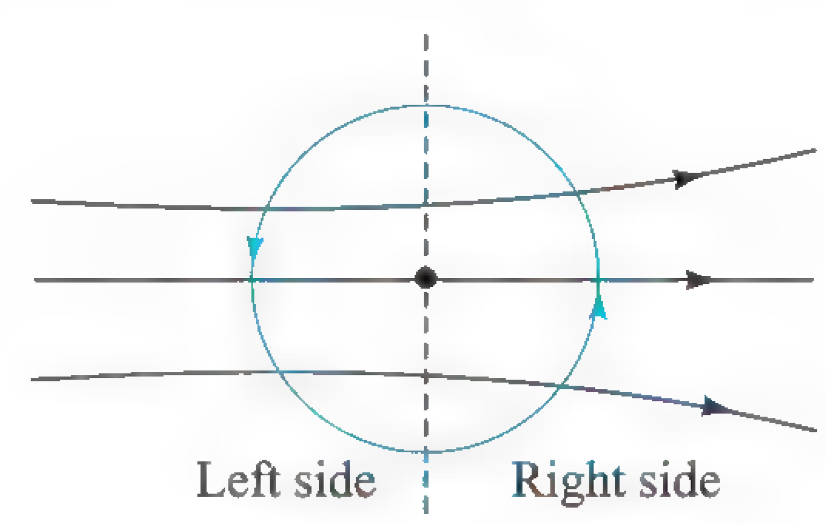
Hence, iv. → a.

5. i. → b.; ii. → a., c.; iii. → a., c.; iv. → a., d.

i.

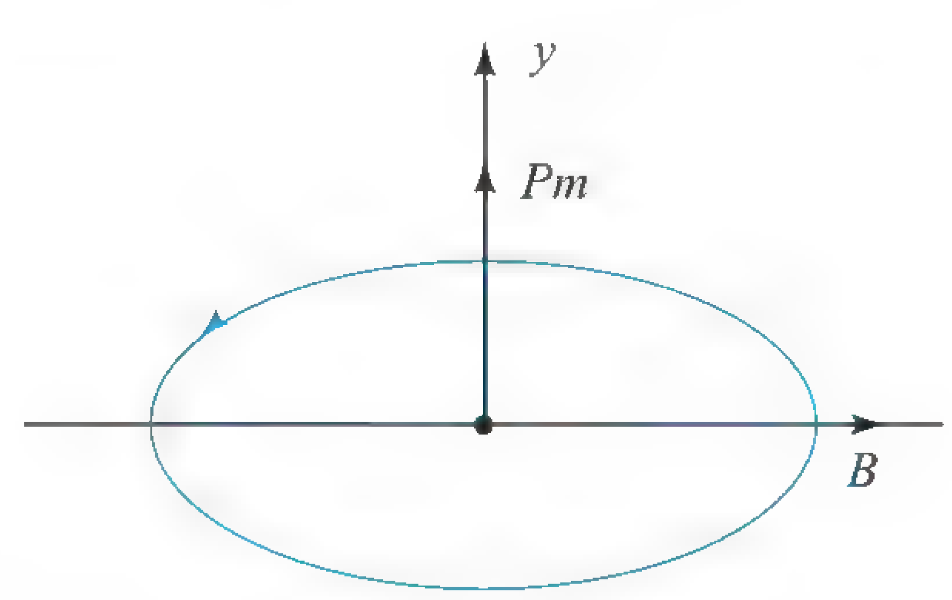


ii. Force on left side is along z-axis and on right side is along $-z$-axis. But former is greater because of higher magnetic field on left side. Hence, net force is along z-direction or along P_m.



iii. Now net force is reversed, but P_m is also reversed.

iv.



Force on left side is more than on right side. And net force will be along y-axis or along P_m.

6. i. → c., d.; ii. → c., d.; iii. → b., c.; iv. → a., c.

i. Because the magnetic field is parallel to x-axis, the force on wire parallel to x-axis is zero. The force on each wire parallel to y-axis is $B_0 \dfrac{i}{2} l$. Hence, net force on the loop is $B_0 il$.

Similar is the situation in case (ii)

Hence, i. → c, d ii. → c, d

iii. Since direction of current from entry point in the loop to exit point in the loop is along the diagonal of the loop, the direction

of external uniform magnetic field is also along the same diagonal. Hence, net force on the loop is zero.

iv. The direction of current from entry point in the loop to exit point in the loop is along the diagonal (of length $\sqrt{2}\,l$) of the loop. The direction of external uniform magnetic field is also perpendicular to the same diagonal. Hence, magnitude of net force on the loop is $B_0 i\left(\sqrt{2}l\right)$. Since force on each wire on the loop passes through centre of the loop, net torque about centre of the loop is zero.

7. i. → a., b., c., d.; ii. → a., b., d.; iii. → d.; iv. → c.

i. If both are zero, then no force will act on the particle and then particle will move with uniform velocity.
If both are non-zero, then it is possible that both the forces acting on the particle cancel each other and particle move in straight line.
If one of them is not zero, then particle can move without change in its direction if it moves along the direction of non-zero field.

ii. The velocity will definitely change if only E is non-zero, otherwise velocity may remain same.

iii. This is possible only if B is non-zero and E is zero.

iv. This is possible only if E is non-zero and B is zero.

8. i. → b., d.; ii. → a., b., c.; iii. → b., d.; iv. → b., c.

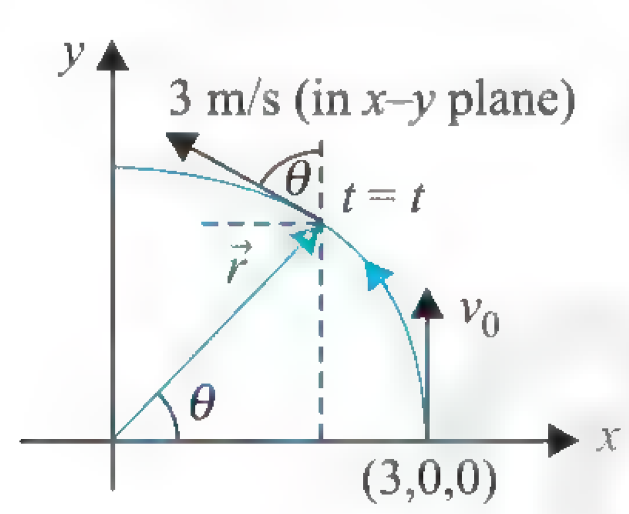
i. If a charged particle at rest experiences a force, then this should be electrostatic force. So electric field should be present, magnetic field may or may not present.

ii. This is possible if (i) both fields are zero (ii) both fields are not zero and their forces balance (iii) $E = 0$, $B \neq 0$ and velocity is parallel to magnetic field.
But if only E is there, then velocity cannot remain same.

iii. speed can vary only if electric field is present. Particle will go undeviated with varying speed if v, E and B all are in same direction. B may also be zero.

iv. For helical path, B should be present, E may or may not be.

9. i. → c., d.; ii. → c., d.; iii. → a.; iv. → b.



$$\theta = \omega t = 1 \times t = t$$

$$\omega = \frac{qB_0}{m} = 1 \text{ rad/s}, \quad R = \frac{mv_\perp}{B_0 q} = 1 \times 3 = 3 \text{ m}$$

$$T = \frac{2\pi}{\omega} = 2\pi \text{ sec}, \text{ pitch } = 4 \times 2\pi = 8\pi \text{ m}$$

$$\vec{r} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\,\hat{k}$$

$$\vec{v} = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + 4\hat{k}$$

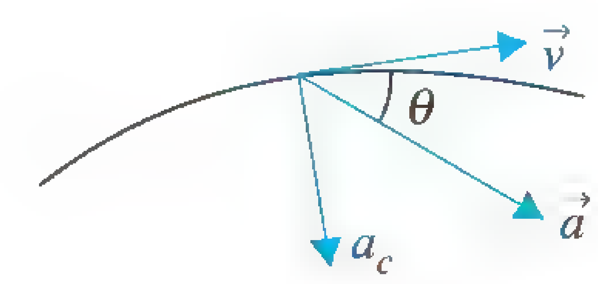
$$\vec{a} = -3\cos t\,\hat{i} - 3\sin t\,\hat{j}$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3\sin t & +3\cos t & +4 \\ -3\cos t & -3\sin t & 0 \end{vmatrix}$$

$$= (+12\sin t)\hat{i} - (+12\cos t)\hat{j} + 9\hat{k}$$

$$= +12\sin t\,\hat{i} - 12\cos t\,\hat{j} + 9\hat{k}$$

$$\Rightarrow |\vec{v} \times \vec{a}| = 15$$



$$a_C = \frac{v^2}{r} \quad \Rightarrow \quad a\sin\theta = v^2/r$$

$$\Rightarrow a\frac{|\vec{v}\times\vec{a}|}{va} = \frac{v^2}{r} \quad \Rightarrow \quad r = \frac{v^3}{|\vec{v}\times\vec{a}|}$$

$$\Rightarrow \text{radius of curvature} = \frac{v^3}{|\vec{v}\times\vec{a}|} = \frac{5^3}{15} = \frac{25}{3}\text{ m}$$

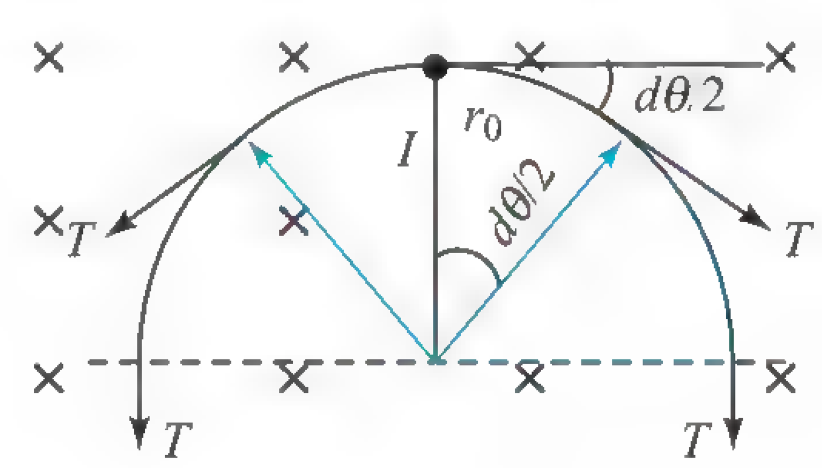
Numerical Value Type

1. (3) $IBl = \mu mg$

$$\Rightarrow \frac{6}{20} \times 0.8 \times \frac{5}{100} = \mu\frac{10}{1000}(10)$$

$$\Rightarrow \mu = 0.12 = 3/25$$

2. (1) For the equilibrium of a small part of semicircular arc subtending an angle of $d\theta$ at the centre,



$$\text{or,} \quad 2T\sin\left(\frac{d\theta}{2}\right) = BIr_0\,d\theta$$

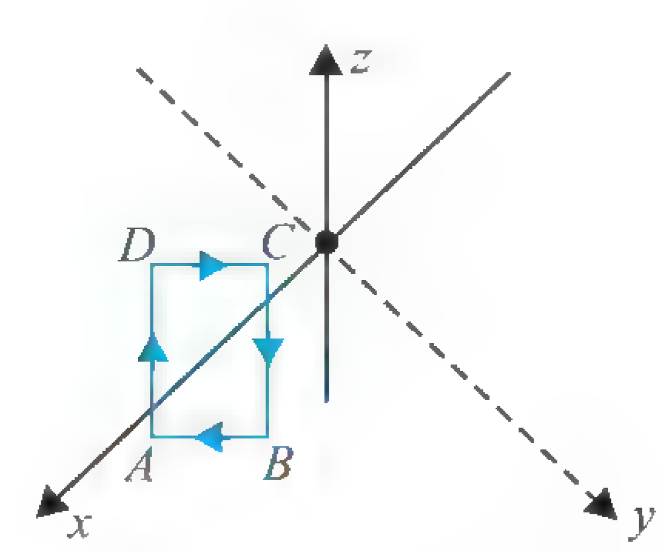
$$B = \frac{T}{Ir_0} = \frac{1.5}{(10)(0.15)} = 1\text{ T}$$

3. (2) $B = 10^4\text{ G} = 1\text{ T}$

For equilibrium, magnetic force should be equal to weight of wire.

$IBl = mg \quad \Rightarrow \quad I \times 1 \times 0.10 = 20 \times 10^{-3} \times 10 \quad \Rightarrow \quad I = 2\text{A}$

4. (2) $\vec{m} = I_2\vec{S}$

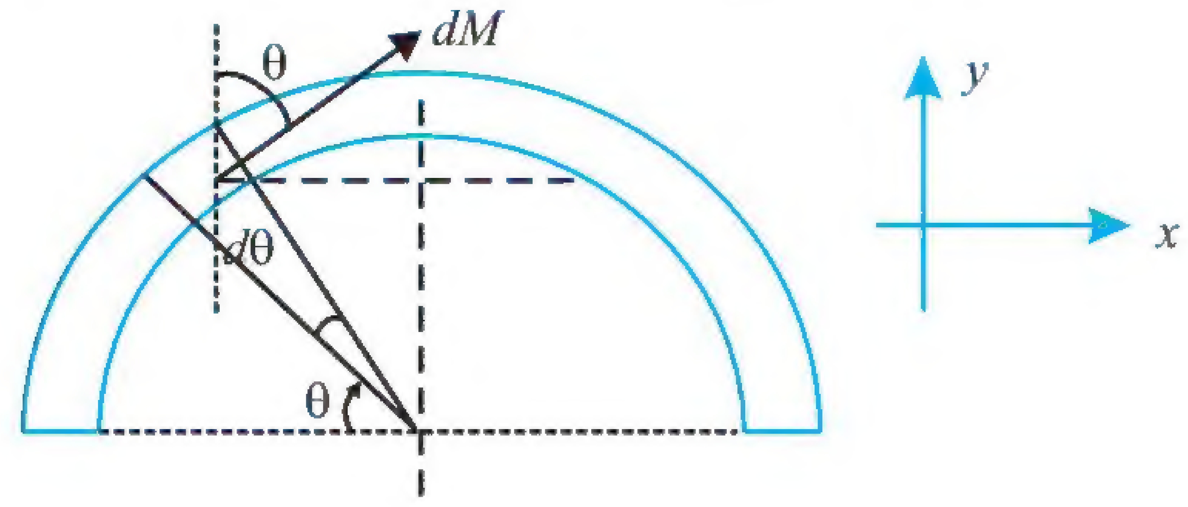


$$\vec{S} = \overrightarrow{BA} \times \overrightarrow{AD}, \ \overrightarrow{BA} = 2d\hat{i} - 2a\hat{j}, \ \overrightarrow{AD} = 2b\hat{k}$$

$$\vec{S} = 2(d\hat{i} - a\hat{j}) \times 2b\hat{k}$$

$$\vec{M} = -4bI(d\hat{j} + a\hat{i}) \quad \Rightarrow \quad |\vec{M}| = 4bI\sqrt{d^2 + a^2} = 2\text{ J/T}$$

5. (5) The magnetic moment of circular current is given by AI, I being the circulating current and A is the area of cross-section; the direction is perpendicular to the plane of current. Now, for an element of toroid of length rdq, its magnetic moment is along the direction of arrow as shown, of magnitude (perpendicular to its cross-section) = current × area × number of turns in the length rdq.



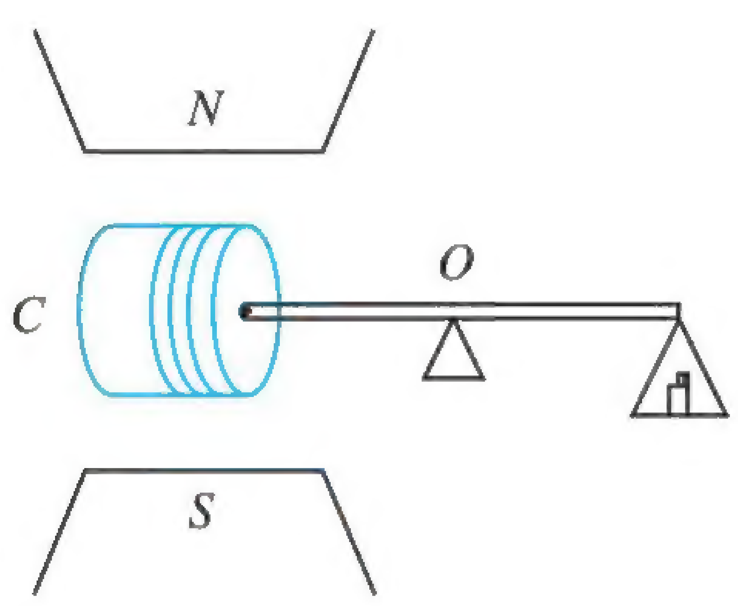
$$dM = I\,\frac{\pi d^2}{4}\left(\frac{N}{\pi r}\,r\,d\theta\right) = \frac{N}{4}\,d^2 I\,d\theta$$

Resolving dM into components along x and y, we get $dM \sin\theta$ and $dM\cos\theta$; components along y from neighbouring elements cancel out to zero, and components along x are added. So

$$M = \int dM \sin\theta = \int_0^{\pi} \frac{N}{4}\cdot d^2 I \sin\theta\, d\theta = \frac{Nd^2 I}{2}$$

Putting the given values we get $M = 5$ Am^2.

6. (4)



Magnetic torque = $NISB \sin(\vec{S}, \vec{B})$ $NISB \sin 90°$

Gravitational torque = $(\Delta m \times g)l$

For equilibrium $NISB = \Delta mgl \;\Rightarrow\; \Delta mgl/NIS$

$$B = \frac{(60\times10^{-6})\times 9.8\times 0.3}{200\times 22\times10^{-3}\times 1\times10^{-4}} = 4\times10^{-1}\text{ T}$$

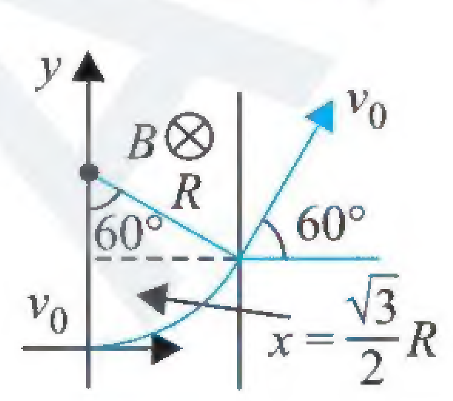
7. (2) Force exerted by air on the rod $= \left(\rho\,\frac{L}{2}\,2R\right)v^2 = \rho LRv^2$

Balancing torque about point O, $NI(\pi R^2)B = \rho LRv^2\,\frac{3L}{4}$

$$\Rightarrow\quad 300\pi IBR = \frac{3\rho v^2 L^2}{4}$$

$$\Rightarrow\quad I = \frac{\rho L^2 v^2}{400\pi BR} = \frac{1}{100}\left(\frac{Lv}{2}\sqrt{\frac{\rho}{\pi BR}}\right)^2 = 0.002\text{ A} = 2\text{ mA}$$

8. (4) The particle will come out of the magnetic field at an angle $\theta = 60°$ with the original direction.

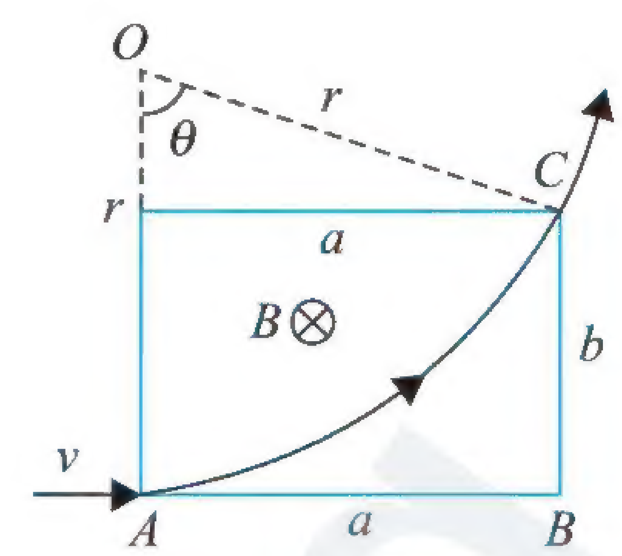


$$\Delta\vec{v} = \left(v_0\cos 60\hat{i} + v_0 \sin 60\hat{j} - v_0\hat{i}\right) \Rightarrow |\Delta\vec{v}| = v_0$$

9. (5) Let speed of the particle is v (speed will remain constant)

$$r = AO = CO = \frac{mv}{qB} \text{ also } \frac{a}{r} = \sin\theta, \frac{r-b}{r} = \cos\theta$$

Solve the above equation to get



$$v = \frac{qB(a^2+b^2)}{2mb} = \frac{10^{-6}\times 1.2(4^2+3^2)}{2\times10^{-6}\times 3} = 5\text{ m/s}$$

10. (2) $v_{\perp} = -\frac{v_0}{2}$ will contribute to circular motion.

$v_{\parallel} = \frac{v_0\sqrt{3}}{2}$ will contribute to helical path. $T = 2\pi\,\frac{m}{B_0 q}$

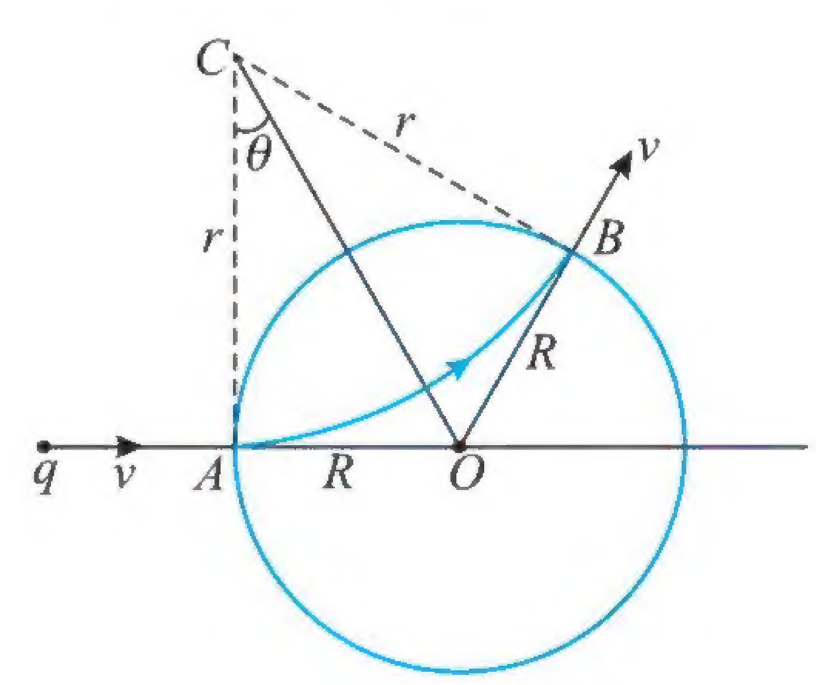
Pitch, $P = v_{\parallel}T = \frac{v_0\sqrt{3}}{2}\,T$

Number of turns $= \frac{R}{P} = \frac{v_0 T\sqrt{3}(2)}{v_0\sqrt{3}\,T} = 2$

11. (60) Radius of circular path of the particle is

$$r = \frac{mv}{qB} = \frac{2\times10^{-3}\times 0.3}{10^{-3}\times 0.2} = 3\text{ m}$$

C is centre of the circular path.



Particle enters the field at A and leaves at B.

$$\tan\theta = \frac{R}{r} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

Deviation $= 2\theta = 60°$.

12. (0.01) F_2 is in y-direction when velocity is along z-axis. Therefore, magnetic field should be along x-axis. So let, $B = B_0\hat{i}$

Given $v_1 = \frac{10^6}{\sqrt{2}}\hat{i} + \frac{10^6}{\sqrt{2}}\hat{j}$

and $F_1 = -5\sqrt{2}\times10^{-3}\,\hat{k}$

From equation $F = q(v\times B)$

We have

$$(-5\sqrt{2}\times10^{-3})\hat{k} = (10^{-6})\left[\left(\frac{10^6}{\sqrt{2}}\hat{i} + \frac{10^6}{\sqrt{2}}\hat{j}\right)\times(B_0\hat{i})\right]$$

$$= -\frac{B_0}{\sqrt{2}}\hat{k}$$

$$\therefore\quad \frac{B_0}{\sqrt{2}} = 5\sqrt{2}\times10^{-3}$$

or $\quad B_0 = 10^{-2}$ T

Therefore, the magnetic field is $B = (10^{-2}\hat{i})$ T

$$F_2 = B_0 q v_2 \sin 90°$$

As the angle between B and v in this case is 90°.

$$F_2 = (10^{-2})(10^{-6})(10^{6})$$

$$= 10^{-2}\text{ N} \approx 0.01\text{ N}$$

13. (1.6) From the equation, $r = \dfrac{mv}{Bq}$

We have $B = \dfrac{mv}{qr}$

Substituting the values, we have

$$B = \frac{(1.67\times10^{-27})(10^{7})}{(1.6\times10^{-19})(6.4\times10^{6})} = 1.6\times10^{-8}\text{ T}$$

14. (2) $F_e = F_m$ or $eE = eBv$

$$\therefore \quad v = \frac{E}{B} = \frac{120\times10^{3}}{50\times10^{-3}} = 2.4\times10^{6}\text{ m/s}$$

Let n be the number of protons striking per second.

Then, $ne = 0.8\times10^{-3}$

or $$n = \frac{0.8\times10^{-3}}{1.6\times10^{-19}} = 5\times10^{15}\text{ m/s}$$

Force imparted = Rate of change of momentum

$$= nmv$$

$$= 5\times10^{15}\times1.67\times10^{-27}\times2.4\times10^{6}$$

$$= 2.0\times10^{-5}\text{ N}$$

15. (3.46) To graze at C

Using equation of trajectory of parabola,

$$y = x\tan\theta - \frac{ax^2}{2v^2\cos^2\theta} \qquad \text{...(i)}$$

Here, $a = \dfrac{qE}{m} = \dfrac{10^{-6}\times10^{-3}}{10^{-10}} = 10\text{ m/s}^2$

Substituting in equation (i), we have

$$0.05 = 0.17\tan 30° - \frac{10\times(0.17)^2}{2v^2\times(\sqrt{3}/2)^2}$$

Solving this equation, we have $v = 2$ m/s

In magnetic field, $AC = 2r$

or $$0.1 = 2r$$

or $$r = 0.05\,m = \frac{mv\cos 30°}{Bq}$$

$$\therefore \quad B = \frac{mv\cos30°}{(0.05)q} = \frac{(10^{-10})(2)(\sqrt{3}/2)}{(0.05)(10^{-6})}$$

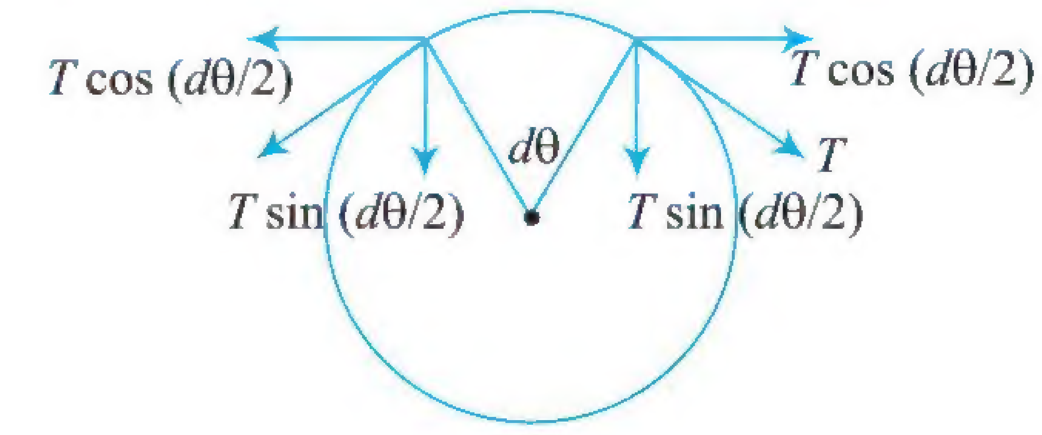
$$= 3.46\times10^{-3}\text{ T} = 3.46\text{ mT}$$

Archives

JEE Advanced

Single Correct Answer Type

1. (3) $2T\sin\dfrac{d\theta}{2} = BIRd\theta$



$$Td\theta = BIRd\theta \qquad \text{(for } \theta \text{ small)}$$

$$T = BIR = \frac{BIL}{2\pi}$$

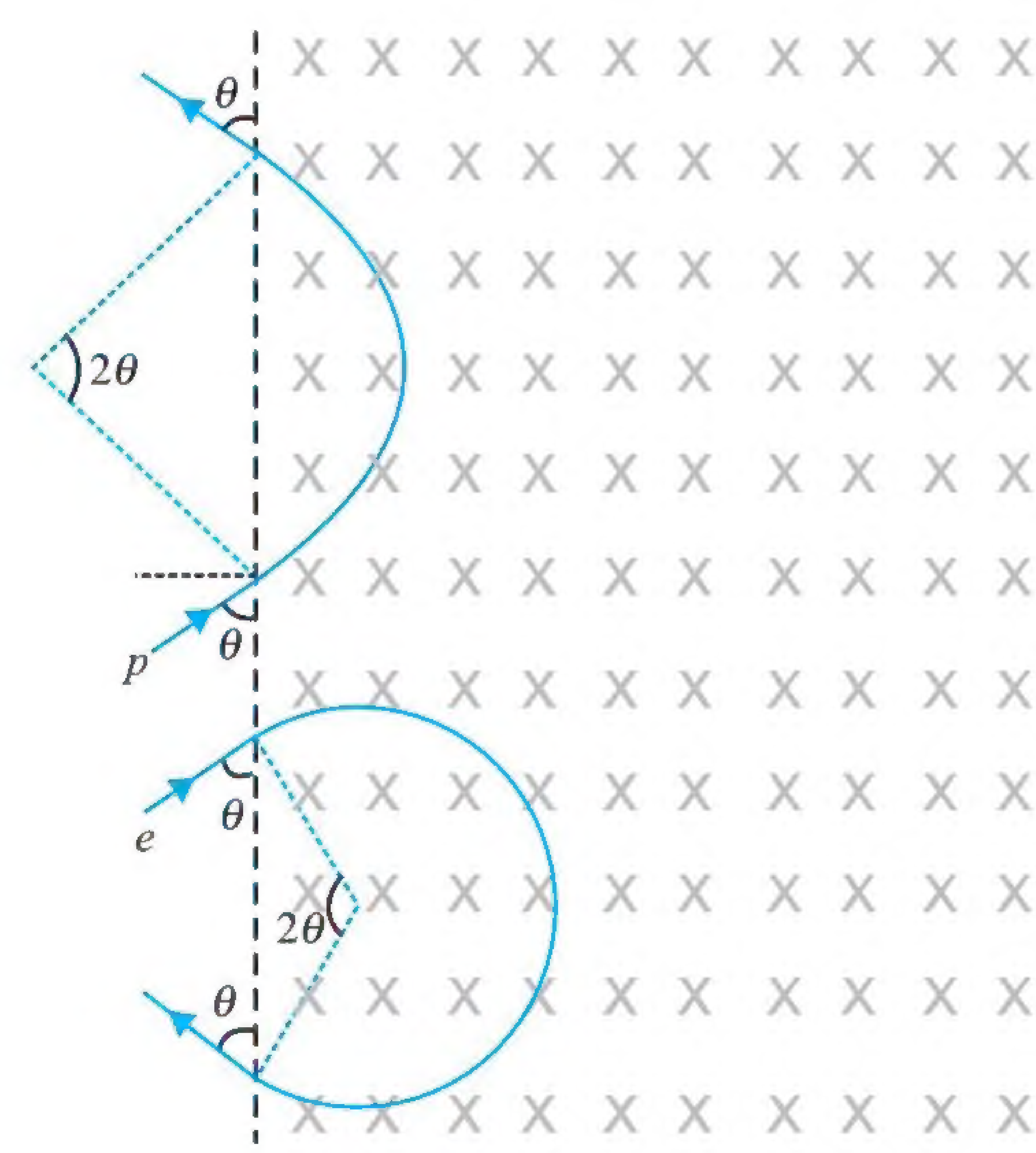
2. (2) Area $= a^2 + 4\times\dfrac{\pi\left(\dfrac{a}{2}\right)^2}{2} = a^2 + \dfrac{\pi a^2}{2}$

$$A = \left(1+\frac{\pi}{2}\right)a^2\hat{k}$$

Multiple Correct Answers Type

1. (2),(4)

$$t_p = \frac{2\theta\times R_P}{v} = \frac{2\theta\times m_P v}{eBv} = \frac{2\theta m_P}{eB}$$



$$t_e = \frac{(2\pi-2\theta)\times R_e}{v} = \frac{(2\pi-2\theta)m_e v}{eBv} = \frac{(2\pi-2\theta)m_e}{eB}$$

$$t_e \neq t_p$$

2. (3),(4)

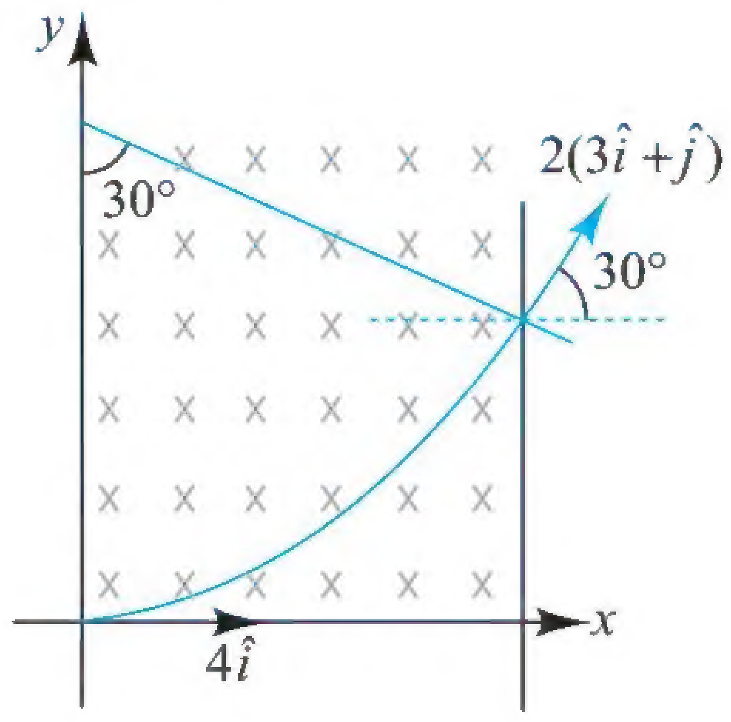
If $\theta = 0°$ then due to magnetic force path is circular but due to force qE_0 (↑) q will have accelerated motion along y-axis. So combined path of q will be a helical path with variable pitch so (1) and (2) are wrong.

If $\theta = 10°$, then due to $v\cos\theta$, path is circular and due to qE_0 and $v\sin\theta$, q has accelerated motion along y-axis so combined path is a helical path with variable pitch (3) is correct.

If $\theta = 90°$ then $F_B = 0$ and due to qE_0 motion is accelerated along y-axis. (4)

3. (1),(3)

So magnetic field is along –ve, z-direction.



Time taken in the magnetic field = $10 \times 10^{-3} = \dfrac{\pi M}{6QB}$

$$B = \frac{\pi M}{6\times 10^{-3} Q} = \frac{1000\pi M}{60Q} = \frac{50\pi M}{3Q}$$

4. (1),(2),(3)

As $\vec{B}$ is uniform

$\Rightarrow$ Wire can be replaced by a straight current carrying conductor.

$\Rightarrow \quad \vec{F} = i(\vec{l} \times \vec{B})$

$\quad\quad = (2(L + R)\hat{i} \times \vec{B})$

if $\vec{B}$ is along x-axis $\quad \Rightarrow \quad \vec{F} = 0$

otherwise $F \propto (L + R)$

5. (1),(2)

The particle will follow circular trajectory inside the magnetic field region. The magnetic field cannot change the magnitude of velocity and momentum.

For longest possible path, the radius of circular motion can be $\dfrac{3R}{2}$.

At farthest point from y-axis, the momentum is directed upwards.

$\therefore \ |\Delta \vec{p}| = \sqrt{2}p$

The radius and hence separation between p_1 and re-entry point is proportional to m, if Q, v, B are same. So option (3) is incorrect.

The particle will return to region only if it completes the half circle.

$\therefore$ Particle will not enter in region 3 and will re-enter region 1 charge in momentum $\sqrt{2}p$.

So option (4) is incorrect.

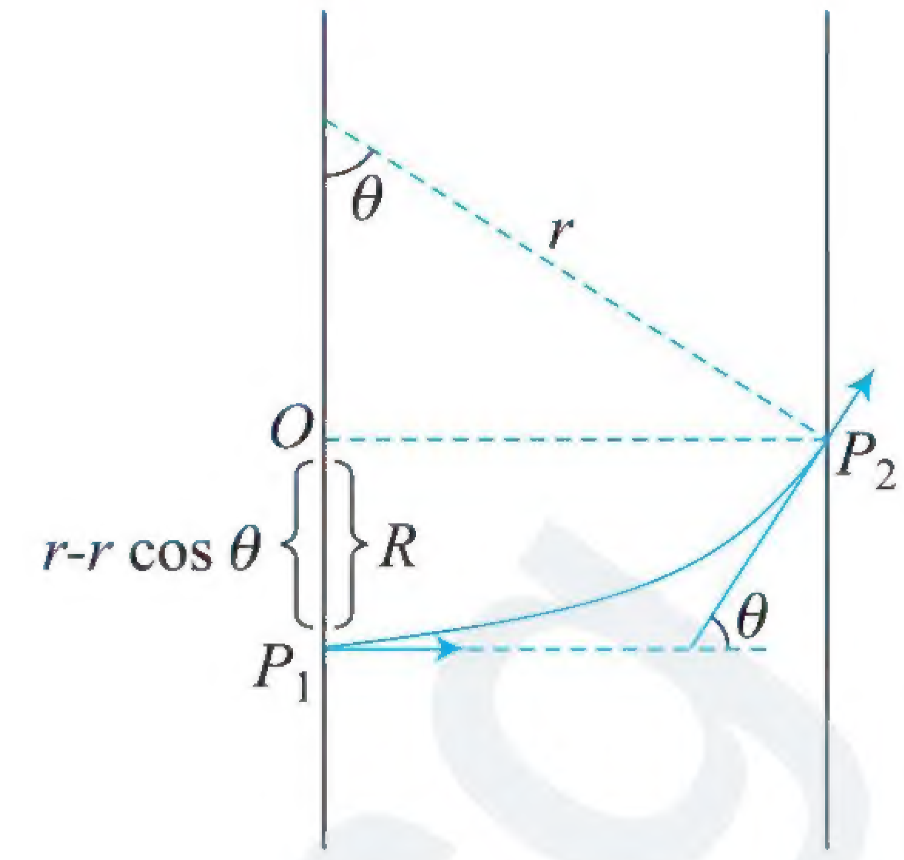
If $\ r \le \dfrac{3R}{2}$

$$\frac{mv}{QB} \le \frac{3R}{2} \Rightarrow \frac{p}{QB} \le \frac{3R}{2}$$

Or, $\quad B \ge \dfrac{2p}{3QR}$

$\therefore$ Option (2) is correct.

If $\ B = \dfrac{8p}{13QR}$; $\ r = \dfrac{p}{QB} = \dfrac{13R}{8}$



It passes through point P_2 if $r - r\cos\theta = R$

$$\text{Here } \sin\theta = \frac{3\frac{R}{2}}{r} = \frac{12}{13} \Rightarrow \cos\theta = \frac{\sqrt{(13)^2 - (12)^2}}{13} = \frac{5}{13}$$

$$\Rightarrow r(1-\cos\theta) = R \ \text{ or } \ \frac{13R}{8}\left(1 - \frac{5}{13}\right) = R$$

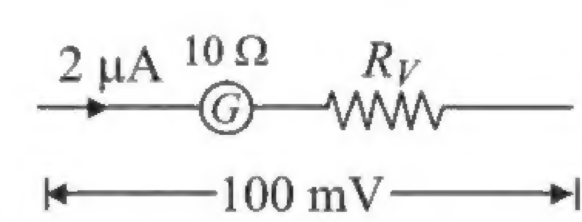
$\therefore$ Particle will enter region 3 through point P_2

$\therefore$ Option (1) is correct.

6. (1),(3)

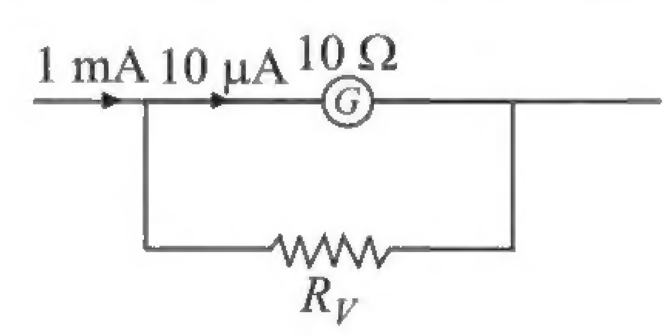
Converting galvanometer into voltmeter

$0.1 = 2\times 10^{-6}\left(10 + R_V\right)$



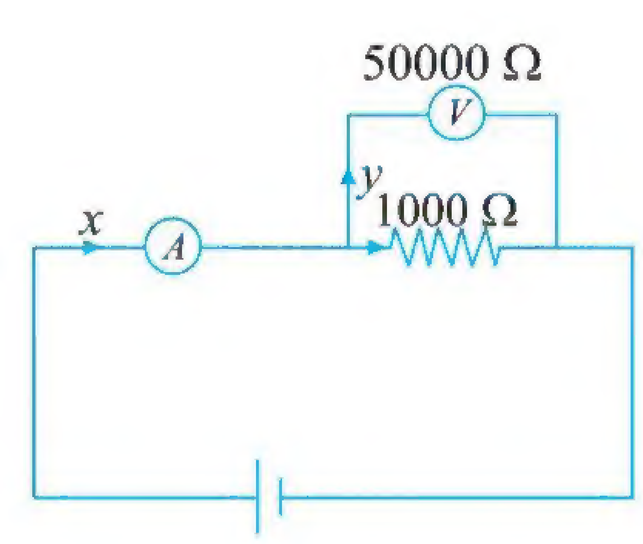
$\therefore \quad R_V = 49990\ \Omega \simeq 50000\ \Omega$

Converting galvanometer into an ammeter



$2\times 10^{-6} \times 10 = 10^{-3} R_A$

$\therefore \ R_A = 0.02\ \Omega$



$y\ 50000 = (x - y)1000$

$\therefore \quad 51y = x \ \Rightarrow \quad \dfrac{y}{x} = \dfrac{1}{51}$

Reading $= \dfrac{y\ 50000}{x} = \dfrac{50000}{51}\ \Omega \simeq 980\ \Omega$

Linked Comprehension Type

1. (1) Larger the magnetic field, smaller the critical temperature.

2. (2) If 0 < B < 7.5 T, then 75 K < T_c(B) < 100 K

Matrix Match Type

1. (4) For constant velocity, acceleration of particle should be zero. Hence net force should be zero.

$$qvB = qE \Rightarrow v = \frac{E}{B} \quad (\text{for } a = 0)$$

Electric field and magnetic field should be perpendicular.

Option (4): $\vec{v} = \frac{E_0}{B_0}\hat{y},\ \vec{E} = -E_0\hat{x},\ \vec{B} = -E_0\hat{z}$ (for electron)

$$\bar{F}_B = q(\vec{v}\times\vec{B}) = -e\left(\frac{E_0}{B_0}\hat{y}\times B_0\hat{z}\right) = -E_0\hat{x}$$

$$F_E = E_0\hat{x}$$

2. (2) $\vec{v} = 0,\ \vec{E} = -E_0\hat{y},\ \vec{B} = B_0\hat{y}$ (Proton)

$F_B = 0$ only force along $-y$ axis is acting due to electric field alone.

3. (4) $\vec{v} = 2\frac{E_0}{B_0}\hat{x};\ \ \vec{E} = E_0\,\hat{z};\ \vec{B} = B_0\hat{z}$ (Proton)

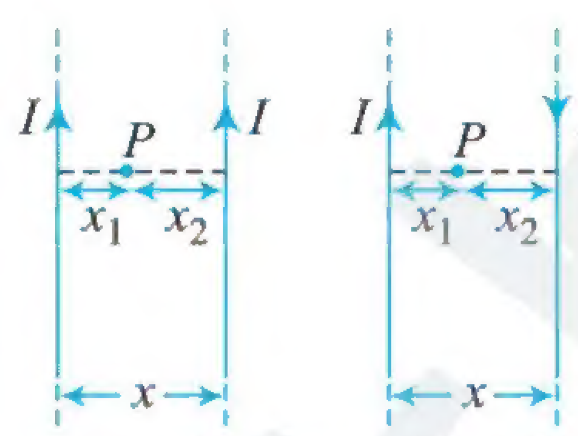
F_B = along $-y$ axis

F_E = along $+z$ axis

So condition for helical path is satisfied.

Numerical Value Type

1. (3) $X_1 = \frac{X_0}{3}$ and $X_2 = \frac{2X_0}{3}$



Case I Case II

$$r = \frac{mu}{qB}$$

Hence ratio of radian

$$\frac{R_1}{R_2} = \frac{B_2}{B_1}$$

Case I

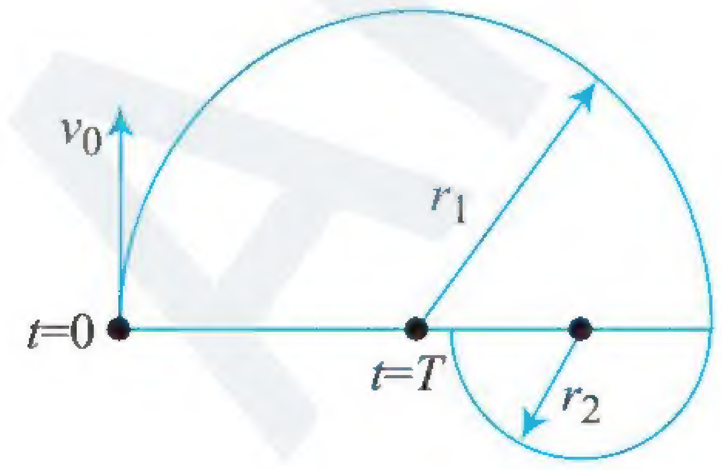
$$B_1 = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x_1} - \frac{1}{x_2}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{3}{x_0} - \frac{3}{2x_2}\right) = \frac{3\mu_0 I}{4\pi x_0}$$

Case II

$$B_2 = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x_1} + \frac{1}{x_2}\right) = \frac{9\mu_0 I}{4\pi x_0} \Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = 3$$

2. (2) The path of the particle motion should be as follows.

We have, radius of a charged particle moving in magnetic field.



$$r_1 = \frac{mv}{qB_1},\quad r_2 = \frac{mv}{qB_2}$$

Since $B_1 = \frac{B_2}{4}$ $\quad\therefore\quad r_1 = 4r_2$

Time in $B_1 \Rightarrow \quad \frac{\pi m}{qB_1} = t_1$

Time in $B_2 \Rightarrow \quad \frac{\pi m}{qB_2} = t_2$

Average speed along x-axis $\langle v_x\rangle = \frac{\int |\vec{v}_x|\,dt}{\int dt} = \frac{d_1 + d_2}{t_1 + t_2}$

Total distance along x-axis

$$d_1 + d_2 = 2r_1 + 2r_2 = 2(r_1 + r_2) = 2(5r_2)$$

Total time $T = t_1 + t_2 = 5t_2$

$\therefore$ Average speed $= \frac{10r_2}{5t_2} = 2\frac{mv}{qB_2}\times\frac{qB_2}{\pi m} = 2$

3. (5.55)

Number of turns $n = 50$ turns

Area $A = 2\times10^{-4}\ \text{m}^2$

Magnetic field $B = 0.02\,\text{T}$

Torsion constant of wire $K = 10^{-4}$

Full scale deflection angle, θ $Q_m = 0.2$ rad

Resistance of galvanometer $R_g = 50\,\Omega$

Range of ammeter $I_A = 0 - 1.0\,\text{A}$

We know torque on the coil, $\tau = MB = C\theta$

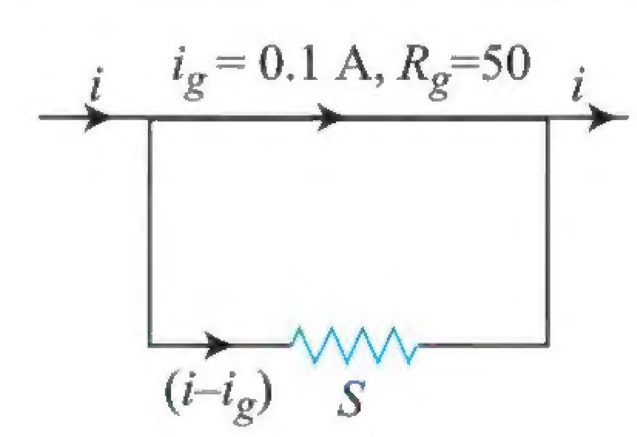
Magnetic moment of the coil $M = ni_g A$

It means $BINA = C\theta$

$$0.02\times i_g\times50\times2\times10^{-4} = 10^{-4}\times0.2\times10$$

$$i_g = 0.1\ \text{A}$$

To convert galvanometer into an ammeter, we need to apply a shunt resistance in parallel with the galvanometer.



From figure we can write $i_g\times R_g = (\text{i} - i_g)\,S$

or $\quad 0.1\times50 = (1 - 0.1)\,S$

which gives $S = \frac{50}{9} = 5.55\ \Omega$

Hence $\quad S = 5.55\,\Omega \quad$ or $\quad 5.56\ \Omega$

4. (4) Radius of a charged particle in magnetic field,

$$r = \frac{mv}{qB} = \frac{\sqrt{2mqV}}{qB}$$

$$\frac{P^2}{2m} = K.E = qV$$

$$\frac{r_s}{r_\alpha} = \sqrt{\frac{32}{1}\times\frac{2}{4}} = 4 \Rightarrow \frac{r_s}{r_\alpha} = 4$$